ECONOMIC STRUCTURE, INTERNATIONAL INCOME DIFFERENTIALS AND LONG-RUN GROWTH

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Abstract

In a well-known paper on the structure of development, Leontief (1963) showed that a country’s degree of economic development could be assessed by the relative completeness of its economic structure. This pattern of development is modelled in this paper. Within a context of international trade, and assuming that the world distribution of labour is given, the model yields that a sufficiently large differential between the economic structures of developed (Northern) and underdeveloped (Southern) economies induces Southern countries to accept lower prices for their basic goods and endure lower incomes. Economic diversification in the South is analysed as an outcome of technological imitation. The model yields that, even if technological knowledge is a public good, a minimum level of quality education is necessary for long-run growth to take place in the South.

I. Introduction

1.1 Facts and questions

Year after year the World Bank reports large gaps in per capita real income across countries. Analysts who have estimated comparable measures of the coun-
tries’ product also report such differences (Summers and Heston, 1991). To explain these income gaps is a goal as old as the science of economics.

It is nowadays generally acknowledged that underdevelopment may be overcome by a sustained process of industrialization. This is perhaps the main lesson from the last two centuries of economic development (Murphy, Schleifer and Vishny, 1989). The recent experiences of the newly industrialized countries from Southeast Asia support this viewpoint. These previously underdeveloped countries were able to make an economic take off, reaching the status of industrialized countries and achieving high living standards. Most underdeveloped countries however have been unable to follow suit. One of the main tasks for development economists is to explain why.

1.2 The vision of Leontief

In order to make a contribution to the search for answers, the vision of Leontief on “The Structure of Development” (Leontief, 1963) is modelled in this paper. Hence, it is convenient to start by reviewing the main ideas of Leontief’s 1963 paper.

Based on a rigorous analysis of input-output matrices of developed and underdeveloped countries, Leontief finds that technologies are relatively invariable; each sector exhibits a relatively invariable relationship between the inputs it receives from other sectors and its contribution to total product of the economy. According to Leontief, each sector technology is some kind of “recipe” that allows the transformation of some “ingredients” into the sector’s product. This technological feature defines the structuralist character of Leontief analysis.1

Leontief also reveals that the most developed economies are structurally quite similar: “displayed in the input-output table, the pattern of transactions between industries and other major sectors of the system shows that the more developed the economy, the more its internal structures resembles that of other developed economies” (op. cit., p. 163). Besides, Leontief finds that the larger and more developed is the economy, the more complete and articulated is its economic structure.2

With respect to underdeveloped economies Leontief observes an important characteristic: “their input-output tables show that in addition to being smaller and poorer they have internal structures that are different because they are incomplete, compared with the developed economies” (op. cit., p. 163). Hence, according to Leontief, a country’s degree of economic development could be assessed by the relative completeness of its economic structure.

Since a country’s lack of development can be compensated by importation of those goods that it does not produce but needs to consume, Leontief pays special attention to the countries’ profile of international trade. His comparative analyses yield that underdeveloped and developed countries are asymmetrically related in the world markets. The underdeveloped countries are characterized by structural lacks and specialize in primary goods, whilst the developed countries are characterized by structural completeness and specialize in manufacturing products.
In order to compare the economic structures of countries, Leontief sorts the sectors in the input-output table according to the degree of backward technological integration. The ensuing triangulation of intermediate transactions reveals the country’s internal economic structure. By means of this analytical procedure, Leontief showed how similar were the internal economic structures of the United States and the developed countries of Western Europe in the sixties.

The implication for economic development of this structuralist vision of the economy is straightforward: long-run economic development tends to follow a sequential industrialization process by which inputs are developed first. As in Marx (1867), who believed that the more industrialized countries pointed out the path of development to the less progressive countries, Leontief also believed that, given the country mix of resources and the available technologies, the essence of the process of development was to create an economic system as similar as possible to the systems of the United States, Western Europe and, in its moment, the Soviet Union.

At about the same time Leontief was making his analysis, some countries of Southeast Asia, namely South Korea, Taiwan, Hong-Kong and Singapore, were beginning their industrial takeoffs. As if they were following Leontief advice, their processes of economic development were characterized by quick economic diversification through copying and adapting technologies from developed countries. In a “miraculous” way these previously underdeveloped countries rapidly became industrialized economies. Japan’s economic “miracle” started even before World War I; it also followed the same pattern of catching up with the more advanced countries (Amsden, 1989; Landes, 1998). Thus, it seems that the wisdom of Leontief deserves careful consideration.

1.3 Modelling the structure of development

The second section of this paper models the vision of Leontief on economic development. Afterwards the model is used to analyse the relationship between developed and underdeveloped countries in world markets. Under some specific conditions, the model generates a gap in real per capita income across countries due to terms of trade losses for underdeveloped economies. The first necessary condition for this outcome is a sufficiently large differential between the economic structures of developed and underdeveloped economies; the second and obviously necessary condition is that international labour mobility is restricted or prohibited.

Put in a nutshell, the rationale for this result is the following. Given the structuralist condition of the economy, labour demand increases with economic diversification. Thus, if the range of sectors is too low (economic diversification is insufficient), labour demand may be too little for the available labour force, inducing a downward wage adjustment.4

Our model is a hybrid of neoclassical and structuralist character. It is neoclassical insofar as economic agents are optimisers, and the technologies are characterized by substitutability among factors of production. The model is also structur-
alist because each economic sector has a particular technology which is defined by the size of the set of intermediate goods used in the sector’s production.

Because of this technological feature, intermediate goods transactions exhibit a triangular pattern in an input-output table whose sectors are ordered according to the degree of backward technological integration. In this way the model captures the phenomenon of input-output deepening.

1.4 Modelling technological imitation in the South

The third section of this paper is aimed at modelling the economic diversification of underdeveloped economies. To begin with, it is assumed that developed economies (the North from now on) have more diversified economies. It is also assumed that economic diversification is proportional to the knowledge of technologies (the stock of available “recipes”). Given the triangular technological structure of all past, present and future economic sectors, developed countries are assumed to be the only innovators. Since the North has the knowledge of all current technologies it has the comparative advantage in R&D. On the other hand, there are no innovations in the South because agents do not know how to produce the more sophisticated goods that the North already produces. Southern agents copy the Northern technologies following the ranking of goods given by the degree of backward technological integration, i.e. the next technological transfer corresponds to that good whose technology requires all inputs already available in the South. Inputs are produced first because learning new technologies requires the knowledge of all the required inputs’ technologies (in order to learn new “recipes” it is necessary to know how to make the required “ingredients”).

In this economy, technological transfers are free because technological knowledge is freely available to anyone who understands sciences and technology. Hence, in this model education is the ultimate mechanism explaining the process of technological diversification in the South.

Sustained economic growth in the South is possible due to the productivity gains derived as externalities from the process of economic diversification. The model yields that even if all technological knowledge is a public good, some minimum level of quality education in the South is necessary for long-run economic growth to take place.

Some caveats are in order. Just for simplicity of the analysis it is assumed that education has no effect on the private productivity of labour. In addition, since any transfer of technology is assumed to be free, its equilibrium price has to be nil. Thus in a competitive environment no agent invests in education, economic diversification ceases, and economic growth is zero. Under this scenario, a government has to subsidize education in order to have an educated workforce, and induce economic growth. Even if it is assumed that education has positive effects on private labour productivity, there would be a welfare gap between the competitive equilibrium path and the optimal growth path due to the external effects of education on economic diversification. Thus subsidies to education would be Pareto improving. This feature has been carefully analysed in well-known
models of human capital accumulation through education (e.g. Lucas, 1988) and thus it is not our main interest. The important result of our model is that even with government subsidies to education, the agents may decide not to get educated if the level of quality education is too low. This possibility may define a poverty trap.

1.5 Related literature

Let us point out that Hirschman’s theory of “unbalanced” economic growth is directly related to Leontief vision. Hirschman (1958, 1986) claimed that investing in those sectors with strong forward and backward technological links might bring about economic diversification and economic growth. He also argued for the possibility of sequential instead of simultaneous solutions to the process of industrialization.

As with the static section of the model, the dynamic section stands on the shoulders of giants. If division of work increases labour productivity, as in the pin factory of Adam Smith (1776), there may be social gains in productivity coming from the proliferation of economic activities across the society. Romer (1987, 1990), Rivera-Batiz and Romer (1991), Grossman and Helpman (1991), and Aghion and Howitt (1992), among others, have explored this intuition and have shown the possibility of sustained growth in models where technological innovation permanently increases the social division of labour.

These authors however do not capture the phenomenon of input-output deepening since they assume ab initio identical technologies for all inputs. Thus the main contributions of our model to the analysis of economic development is the triangular structure of the input-output matrix and the sequential process of industrialization that this structure imposes.

Hence, by combining economic diversification and input-output deepening, the present model yields dynamic productivity gains. Moreover, the unit cost of the goods decrease with the degree of backward technological integration and are thus more intensively used. This feature is also found in a learning-by-doing model by Young (1991). However, there is no rationale for this technological feature in Young’s model, it is just assumed as an axiom.

Our model is related to a strand of economic literature which analyses the problems of adapting advanced Northern technologies to the needs of the South (Atkinson and Stiglitz, 1969; Basu and Weill, 1998; Acemoglu and Zilibotti, 2001). Acemoglu and Zilibotti build an economic model based on the mismatch between the skill-biased technological innovation developed by Northern countries for Northern economic conditions—abundant skilled labour—, and the scarcity of skilled labour in the Southern countries. Their model shows how technologies designed to be operated by skilled workers yield a lower productivity when operated by unskilled workers in the South. Hence, this technological mismatch leads to productivity gaps and output differences between the North and the South.

This line of research is likely to be complementary to our structuralist approach rather than contradictory. Their models focus on the mix of human capital
between skilled and unskilled labour. Our model focuses instead on the stock of human capital given by the knowledge of technologies.

The present model is also related to the literature on innovation and technological imitation (Romer, 1987, 1990; Rivera-Batiz and Romer, 1991; Grossman and Helpman, 1991; Barro and Sala-i-Martin, 1995; Acemoglu and Zilibotti, 2001). This literature is strongly influenced by the Schumpeterian vision of purposeful entrepreneurs investing both in technological innovations and imitation. By contrast, some theories of technological diffusion have been built on the assumption that human capital accumulation raises agents’ ability to absorb technologies (Nelson and Phelps, 1966). Hence, without denying the importance of economic incentives, it can be claimed that the public good characteristic of knowledge (Romer, 1986), the external effects of education (Lucas, 1988), the learning-by-doing mechanism of enhancing productivity (Arrow, 1962; Lucas, 1998, 1993; Matsuyama, 1991; Young, 1991), and the possibility of knowledge transmissions through technological linkages (Barstelman, Caballero and Lyons, 1991), are also strong development mechanisms which diversify the economy, enhance total factor productivity, and increase economic growth. With these arguments as background, this paper concentrates on education as a mechanism of inducing international technological diffusion.

In order to support this approach it is worth mentioning the detailed and painstaking work of economic historians, the analyses of development economists, and the empirical statistical analyses of economists. The work of Alice Amsden (1989) on the economic takeoffs of South Korea and Japan, shows the crucial role of education and learning-by-doing in the development of the newly industrialized economies. In his work on the wealth and poverty of nations, David Landes (1998) also shows that education and openness to new ideas, as well as learning-by-doing and industrialization, are requisites for economic success. Some analyses based on cross-country growth regressions had shown consistently the significant positive effect of educational attainment and quality education on economic growth (Barro, 1989, 1991; Levine and Renelt, 1992; Barro and Lee, 1993; Barro and Sala-i-Martin, 1995).

Finally, our model has some precedents. A dynamic model of the structuralist vision of development is explored in the Ph.D. dissertation of the author (Ortiz, 1993), and in an aggregative model of economic growth whose industrial diversification is characterized by input-output deepening (Ortiz, 1996). Our current model is a modified version of the latter.

The modifications are as follows: first, physical capital accumulation is dropped in order to simplify the dynamic analysis –human capital accumulation remains as the single engine of growth by considering technological imitation as a by-product of education–; second, it is assumed that the representative consumer derives utility from the consumption of any good; third, international trade between developed and underdeveloped economies is explicitly considered. A static version of this new model is found in Ortiz (2001). In this paper the static part of the model is broadened to include an analysis of how the terms of trade adjust to equilibrate the world markets.
II. The Model

2.1 The basic set up

The economic structure is represented instantaneously by an input-output matrix augmented with the vector of workforce allocation (see Figure 1). There is no joint production and all sectors (and goods) are indexed according to the degree of backward technological integration between 0 and N. This integration is assumed to increase linearly with the sector’s index: the sector j only uses as intermediate inputs the goods with lower index. This feature guarantees that the input-output matrix is perfectly triangular. The intermediate inputs of any sector can be read vertically off the input-output matrix. The labour force is indexed according to its allocation among sectors.

FIGURE 1

INPUT-OUTPUT MATRIX
The technology of each activity is defined by the following production function:

\[ X_j = L_j^{\alpha} \int_0^1 X_{ij}^{1-\alpha} \, dj, \quad 0 < \alpha < 1, \]  

where \( X_j \) is the gross output of good \( j \), \( X_{ij} \) is the intermediate consumption of good \( i \) in sector \( j \) (\( i < j \)), and \( L_j \) is the workforce allocated to sector \( j \). The technology is characterized by constant returns to scale and high substitutability among intermediate inputs; the marginal rate of technical substitution between any two intermediate inputs is given by \( 1/\alpha > 1 \). Equation (1) implies that all goods are produced with the same technology, the only difference comes from the size of the range of intermediate inputs used by each sector.

The labour force is assumed to be constant and normalized to 1. At any given moment a fraction \( m \) of the labour force is offered inelastically. The equilibrium between demand and supply of labour is given by the following equation:

\[ \int_0^1 N_j \, dj = m, \]  

where \( N \) measures the current range of existing goods.

All goods are perishable and all of them are suitable for final consumption. Hence, the gross demand of the \( i \)-th good is made up of intermediate demands and final consumption. The market equilibrium of the \( i \)-th good satisfies:

\[ X_i = \int_i^N X_{ij} \, dj + C_i, \]  

where \( C_i \) is the final demand for the \( i \)-th good. Note that the \( i \)-th sector is integrated forward only with sectors of higher backward integration (\( X_{ij} > 0 \) for \( j > i \); \( X_{ij} = 0 \) for \( j \leq i \)).

It is assumed that the representative consumer derives utility from the consumption of any good and maximizes the discounted stream of utility over an infinite horizon. The objective function is defined as follows:

\[ \int_0^\infty e^{-\rho t} u \left( \{C_i(t)\} \right) \, dt, \]  

where \( \rho \) is the discount rate, \( u(\cdot) \) is the instantaneous utility function and \( \{C_i(t)\} \) is the vector of current final consumption over the range \([0, N(t)]\).

Now, a specification for instantaneous preferences is required. Let us assume the following modified constant elasticity of substitution utility function:

\[ u(\{C_i\}) = \begin{cases} 
\left( \int_0^N C_i^{\gamma} \, dj \right)^{(1-\varepsilon)/\gamma} - 1, & \text{for } 0 < \gamma < 1, \ \varepsilon > 1, \\ 
(1/\gamma) \ln \left( \int_0^N C_i^{\gamma} \, dj \right), & \text{for } 0 < \gamma < 1, \ \varepsilon = 1.
\]
where $\varepsilon$ is the intertemporal elasticity of substitution of the given bundle of goods—this elasticity only makes sense by combining equations (4) and (5)–, and $\sigma \equiv 1/(1-\gamma)$ is the intratemporal elasticity of substitution among goods.

Although the CES function is usually assumed to be homogeneous of degree 1 ($\varepsilon^{-1} = 0$), the utility function is assumed to be strictly concave ($\varepsilon^{-1} > 0$) with a high intertemporal elasticity of substitution ($\varepsilon \geq 1$). These functional forms imply that the representative consumer experiences diminishing marginal utility with respect to any given bundle of goods. This assumption ensures an interior solution to the dynamic path. It is also assumed that the intratemporal elasticity of substitution among goods is high ($0 < \gamma < 1$, or $\sigma > 1$). This inequality is necessary for a positive marginal utility from diversification ($\gamma > 0$), and also for obtaining well-behaved demand functions for individual goods ($\gamma < 1$).

The above equations complete the static model. Before characterizing the corresponding equilibrium it is convenient to define the technology of human capital accumulation. This will provide the dynamics of the model.

Human capital is simply the accumulated knowledge of techniques defined by the number of existing sectors (goods): $N(t)$. It is assumed that the economy’s human capital is small compared to more advanced economies. It is also assumed that technological knowledge is non-rival and non-excludable. Hence, the economy specializes in appropriating foreign techniques. This process requires educated agents. Furthermore, the appropriation of new techniques requires new skills. Thus, the process of economic diversification continues as long as the agents allocate some effort to education. Since knowledge is not subject to depreciation, the technology of education is defined by the following function:

$$\dot{N}(t) = \delta N(t) [1 - m(t)],$$

where a dot over a variable denotes a time derivative. Thus the rate of creation of new sectors (goods) is proportional to the current level of knowledge, $N(t)$, and the amount of effort allocated to education as measured by the fraction of workforce which is not working, $1 - m(t)$. The parameter $\delta$ is an index of productivity in education. This transition equation of human capital is taken from Uzawa (1965) and Lucas (1988). Its linearity with respect to the current level of human capital implies unbounded human capital accumulation as long as economic agents assign some time to study.

Given the possibility of education, the society faces an intertemporal trade-off: it pays to invest in education today—working less and producing a lower output—in order to enjoy a broader range of goods tomorrow. This assumes, of course, that the productivity in education is sufficiently high: the rate of diversification of goods must be high enough to compensate for the lower level of current consumption.

Additionally, for an interior solution of the dynamic problem the instantaneous utility function should be concave in its arguments, namely the set of goods currently available. That is why it is assumed a high intertemporal elasticity of substitution ($\sigma \geq 1$).
2.2 The instantaneous equilibrium

The representative consumer maximizes his instantaneous utility, equation (5), subject to the instantaneous budget constraint which is defined by the following expression:

$$\int_0^N p_i C_i \, di = mw,$$

where $w$ is the wage rate, $mw$ is current income, and $p_i$ is the price of the i-th good.

The consumer takes as given income and prices, generating the following relative demand function:

$$\frac{C_i}{C_j} = \left(\frac{p_i}{p_j}\right)^{-\sigma}, \quad \sigma \equiv 1/(1-\gamma) > 0. \quad (8)$$

Note that relative demands fall with relative prices.

Firms’ profits in sector $j$ are defined as follows:

$$\pi_j = p_j X_j - w L_j - \int_0^j p_i X_{ij} \, di.$$ 

Due to the assumption of constant returns to scale one can aggregate the firms in each sector. In order to maximize profits, firms in sector $j$ choose the amount of labour force to be hired and the intermediate inputs from the range $[0, j]$. The factor demands are calculated assuming the wage and the input prices as given. Thus, the first order conditions for profit maximization are the following:

$$L_j = \alpha p_j X_j / w, \quad (9)$$

$$X_{ij} = \left[(1-\alpha) \frac{p_j}{p_i}\right]^{1/\alpha} L_j, \quad i \in [0, j]. \quad (10)$$

The solution for the price structure is obtained as follows. Substitution of equations (9) and (10) into equation (1) yields

$$p_j^{-1/\alpha} = (a/w) \int_0^j p_i^{1-1/\alpha} \, di, \quad a \equiv \left[\alpha^\alpha (1-\alpha)(1-\alpha)^{1/\alpha}\right]^{1/\alpha} > 0. \quad (11)$$

Differentiating this expression with respect to $j$ gives

$$\frac{d p_j}{d j} = -\frac{a}{w} p_j^2.$$
The price of the i-th good is derived by integrating between 0 and i:

$$p_i = \frac{w}{\alpha a_i}. \quad (12)$$

Infinity is the only meaningful price for good 0 because its output is nil: non integrated sectors do not produce output [see equation (1)]. This feature is used for deriving equation (12). Hence prices decrease asymptotically towards zero with the degree of backward technological integration. Besides, relative prices are fixed: $p_i p_j = j/i$.

Given the structure of relative prices one can solve for the technical coefficients. Substitution of equation (12) into equation (9) yields the technical coefficient for labour in sector j:

$$\frac{L_i}{X_j} = \frac{1}{a_j}. \quad (13)$$

Now, the intermediate input coefficients of sector j are obtained by combining equations (10), (12) and (13):

$$\frac{X_{ij}}{X_j} = \frac{1 - \alpha}{\alpha} \frac{i^{1/\alpha}}{j^{1/\alpha}}, \quad \text{for all } i \in [0, j]. \quad (14)$$

The last two equations show that given the degree of technological integration, j, the technical coefficients are “fixed” as in a Leontief technology. Note, however, that technological coefficients are not assumed fixed. Actually, intermediate inputs in each activity are assumed to be good substitutes. Fixed technological coefficients in this model are due to fixed relative prices. This feature, in turn, is due to the assumption that the range of the intermediate goods set used by each sector is fixed. Thus, in this economy the workers learn only one way of making each good and the “recipes” are never modified.

Let us solve now for the final demand of the i-th good. By combining equations (7), (8) and (12) one obtains

$$C_i = \alpha a \sigma (i/N)^\sigma m. \quad (15)$$

The final demand structure is biased in favour of sectors with high backward technological integration (i close to N). This result is not surprising as relative prices fall with the degree of backward integration [see equation (12)]. The bias in the final demand structure is stronger the higher the intratemporal elasticity of substitution.

Equation (15) also implies that the final demand structure shifts in favour of newer goods as the number of sectors increases. Thus the final demand for sec-
tors with a low degree of backward technological integration (i ≈ 0) becomes negligible. Again, the higher is the elasticity of substitution the stronger is this effect.

Let us solve now for the structure of gross demands. Substitution of equations (14) and (15) into equation (3) yields

\[ X_i = \frac{1}{\alpha} i^{1/\alpha} \int_i^N \frac{X_j}{j^{1+1/\alpha}} dj + \frac{\alpha}{N^\sigma} m. \]  

(16)

Differentiating twice with respect to i yields

\[ \frac{d^2 X_i}{d i^2} = a \frac{\sigma}{N^\sigma} \left( \frac{\alpha \sigma - 1}{N^\sigma} \right) i^{\sigma-2} \]

This is a second order differential equation whose general solution has the form

\[ X_i = \phi_0 + \phi_1 i + \phi_2 i^\sigma, \]

where \( \phi_0, \phi_1 \) and \( \phi_2 \) are constant coefficients to be determined. By substituting this solution into equation (16) the coefficients are identified. Hence, the solution for the gross demand of the i-th good adopts the following form:

\[ X_i = a \frac{\sigma}{\sigma - 1} \left[ (1-\alpha) \left( \frac{i}{N} \right) + (\alpha \sigma - 1) \left( \frac{i}{N} \right)^\sigma \right] m. \]

(17)

From this equation it is deduced that the economic structure profile depends on the relationship between the elasticity of intratemporal substitution in final consumption, \( \sigma \), and the output elasticity of labour, \( \alpha \).

The final demand for a particular good always increases with the degree of backward economic integration, \( i \), because highly integrated sectors produce cheaper goods. Given "fixed" technological coefficients [see equation (14)], the gross demand tends to increase with final demand. However, the bias of the final demand structure towards highly integrated goods needs not determine the bias of the gross demand structure: even if the final demand for lower integrated goods is negligible, they are still required as intermediate inputs in the production of highly integrated sectors. These derived demands will be higher the larger is the intensity of intermediate input in the production technology, i.e. the lower \( \alpha \). Thus, if the bias toward final goods is not too high (the elasticity of substitution is not too high), and production is intensive in intermediate goods (\( \alpha \), low), so that \( \alpha \sigma < 1 \), the gross demand may be biased towards sectors with an intermediate degree of technological integration. On the other hand, high elasticity of substitution and/or low production intensity in intermediates, so that \( \alpha \sigma > 1 \), determine a bias in gross demand towards highly integrated sectors.
Now, the labour demand in sector $j$ is deduced by combining equations (13) and (17):

$$L_j = \frac{\sigma}{\sigma - 1} \left[ (1 - \alpha) + (\alpha \sigma - 1) \left( \frac{j}{N} \right)^{\sigma-1} \right] \frac{m}{N}.$$  

(18)

Figure 2 shows the different possibilities of labour allocation across sectors.

**FIGURE 2**

**EMPLOYMENT STRUCTURE**

The structure of employment is clearly related to the structure of gross demand. Even goods with the lowest backward technological integration are demanded at least as intermediate inputs. Thus they require some allocation of labour. If production intensity in intermediates is high, the labour demand is biased towards sectors with low technological integration (the labour profile is downward sloping); if production intensity in intermediates is low, the labour demand is biased towards sectors with high technological integration; in the borderline case all sectors hire identical number of workers.

Given the final demand structure one can solve for the instantaneous level of utility. For simplicity the case of logarithmic preferences ($\varepsilon = 1$) is chosen. However, the main results are not significantly changed by allowing for a higher degree of intertemporal substitution (see the Appendix). Thus, plugging equation (15) into equation (5), for $\varepsilon = 1$, yields
\[ u = \ln \left[ \beta m N^{\sigma/(\sigma-1)} \right], \quad \beta = \alpha a^{-1/(\sigma-1)} > 0. \] (19)

The instantaneous level of utility depends on the fraction of the labour force allocated to production (m), and the range of existing goods in the economy (N). Equation (19) shows why it is natural to assume a high degree of intratemporal substitutability among goods (\( \sigma > 1 \)): only in this case the society’s welfare increases with the range of available goods, N.

2.3 The open economy case

Let us consider the model in the context of international trade (refer to Figure 3). Two economic blocks, South and North, are initially in autarky and afterwards they are joined through international trade. Each block is made up of many small countries, so that good prices are competitively determined in world markets. There are no transport costs. The population is mobile within the countries but international migration is prohibited. It is assumed that the North owns a higher level of human capital and thus has a more diversified economy; i.e. the South produces N goods and the North produces N* goods, such that N* > N > 0. From now on all variables related to the North will be denoted with an asterisk.
With international trade the prices of identical goods are equalized. Given common production activities, the factor price equalization theorem implies that the wage rate is identical across countries. Thus, relative price solutions are the same as in the closed economy case because they are determined solely by the degree of backward technological integration [equation (12)].

The next step is the determination of the world gross demands. As Figure 3 shows the world demands are given by

\[ X_i^w = X_i + X_i^* = \int_i^N X_{ij}^* \, dj + \int_i^{N^*} X_{ij} \, dj + C_i + C_i^*, \quad i \leq N, \]

\[ X_i^w = X_i^* = \int_i^{N^*} X_{ij}^* \, dj + C_i + C_i^*, \quad N < i \leq N^*, \tag{20} \]

where the superscript \( W \) denotes world demand. Note that world demand for goods within the range \([N, N^*]\) includes only the intermediate demands of the North since the South does not require this range of goods. However, the world demand for these goods should include the final demands from the South as well as those from the North.

Let us stress the fundamental asymmetric relationship between the South and the North. Whilst the North may be specialized in goods with higher backward integration, it nevertheless can produce the goods with lower backward integration which the South produces. However, the South cannot produce the higher backward integrated goods because of its lack of human capital.

The solutions for the world demands require the analytical expressions for the components of the equations (20). Let us start with the final demands. All consumers share the utility function [see equation (5)], and all of them have access to the consumption of \( N^* \) goods. It means that the South can consume goods it does not produce through international exchange. The final demands are given by the following equations which are equivalent to equation (15):

\[ C_i = \alpha \sigma (i / N) \, m \, L, \]

\[ C_i^* = \alpha \sigma (i / N^*) \, m^* \, L^*, \tag{21} \]

where \( m \) is the fraction of the workforce in productive activities in the South, and \( L \) is the workforce in the South. Variables with asterisk correspond to the North.

Now, labour demand and intermediate inputs are proportional to the gross output in each sector [equations (13) and (14)]. Substituting these demands into the equations (20) yields the solution for the world gross demand of the i-th good:

\[ X_i^w = \frac{\alpha \sigma}{\sigma - 1} \left[ (1 - \alpha) \left( \frac{i}{N^*} \right) + (\alpha \sigma - 1) \left( \frac{i}{N} \right) \right] (m \, L + m^* \, L^*), \tag{22} \]
which is analogous to the solution in the closed economy case [see equation (18)]. Equation (22) applies to all goods within the range [0, N*]. Hence, there is no discontinuity of the world demand structure at the level of the N-th good, as one might believe by looking at Figure 3. The clue for this feature is the smoothness of the world final demand structure. Hence, given “fixed” intermediate input coefficients, the gross demand structure is smooth as well.

The solution for the world demand for labour from the industry j is derived by combining the solution for the world gross demands and equation (13):

\[
L_j^w = \frac{\sigma}{\sigma-1} \left[ (1-\alpha) + (\alpha \sigma - 1) \left( \frac{j}{N^*} \right)^{\sigma-1} \right] \frac{mL + m^*L^*}{N^*}.
\]

(23)

Then, integrating between 0 and i and dividing by the world labour demand, mL + m^*L^*, one deduces

\[
D \left( \frac{i}{N^*} \right) = \frac{\sigma}{\sigma-1} \left[ (1-\alpha) \left( \frac{i}{N^*} \right) + \frac{\alpha \sigma - 1}{\sigma} \left( \frac{i}{N^*} \right)^{\sigma} \right].
\]

(24)

which is the fraction of the world labour demand associated with the range of economic activities [0, i]. The line OET in Figure 4 depicts the evolution of this fraction for the whole range of activities [0, N*]. The line is always increasing and its extreme values are D(0/N*) = 0 and D(N*/N*) = 1. In the illustration it is assumed that \( \alpha \sigma > 1 \), so that the world labour demand increases more than proportionally with the degree of backward technological integration.

FIGURE 4
INTERNATIONAL ALLOCATION OF LABOUR
Of special importance for this analysis is the fraction of the world labour demand derived from the range of activities [0, N], D(N/N*), which corresponds to the ratio NE/NZ in the Figure 4. As South does not produce goods with a backward technological integration higher than N, D(N/N*) represents the maximum possible share in the world labour demand for the South.

If the fraction of the world labour supply from the South is denoted by $S = mL/(mL+m^L* L^*)$, there are three possibilities (refer again to Figure 4):

1. If the South supplies the fraction of the labour supply $S_1$, the North employs a fraction of its workforce equal to the ratio $AE/AZ$ in activities with backward integration lower than N; the remainder, given by the ratio $EZ/AZ$, is employed in activities with higher backward integration. The actual distribution of output supply (with common backward integration lower than N) is not determined.

2. If the South happens to supply the fraction of labour $S_2$, it will be specialized in activities with backward integration lower than or equal to N. The North, of course, will be specialized in activities with backward integration higher or equal to N. In this case there will be only one activity in common, the marginal activity with backward integration equal to N.

3. If the South provides the fraction of labour supply $S_3$, it will be completely specialized in products with backward integration lower than or equal to N. The North will be completely specialized in activities with backward integration higher than N. In this situation the factor price equalization theorem does not apply. The Southern wage will fall in order to correct the excess supply of labour in the South given by the distance EC, and the Northern wage will increase in order to correct the excess demand for labour in the North given by the same distance. Without migration restrictions Southern workers would move to the North, otherwise wages are adjusted. Moreover, prices are also adjusted because they are proportional to wages [equation (12)], i.e. the South suffers losses in terms of trade. Note as well that in this situation the Southern excess supply of labour, EC, is larger the lower the level of human capital in the South, N, relative to the level of human capital in the North, N*. Thus the adjustment of wages and prices is stronger the lower is $N/N^*$.

It is difficult to obtain analytical solutions for the model when wages are not equalized across countries because of the discontinuity of the relative price structure [equation (12)]. However, it is possible to prove that in the margin an international gap of relative prices and wages adjusts the world markets. Let us think of the second case above, the marginal case when the world labour demand from N economic activities equals the Southern labour supply, $D(N/N*) = S$. In this situation the factor price equalization theorem still holds and wages are internationally equalized. Now, it is already known from equation (12) that $N/N^*$ represents the price of the good $N^*$ relative to the price of the good N. Let us denote this relative price by $q (= p_{N^*} p_N = N/N^*)$. Hence the world labour market equilibrium in the marginal case can be rewritten as $D(q) = S$. Given that the South
cannot produce goods with a backward integration higher than N, an adjustment of the price q preserves the world labour market equilibrium when S increases above D(N/N*). Proof: differentiating the equilibrium equation with respect to S and taking into account that D(q) is defined by equation (24) for i = N, it is deduced that

\[
\frac{dq}{dS} = \frac{(\sigma - 1)/\sigma}{(1 - \alpha) + (\alpha \sigma - 1) q^{\sigma - 1}} > 0.
\]

Given that \(\alpha < 1, \sigma > 1\) and \(q < 1\), numerator and denominator of the last expression are positive in sign. Thus, a marginal increment of the Southern labour supply from the limit case where the factor price equalization still holds, leads to higher relative prices for the North in order to equilibrate the world markets.

This analysis implies an important property (refer again to Figure 4): if the relative supply of labour from the South is high, so that the Southern wage is below the Northern wage, the South may increase its real income by increasing its human capital level relative to the human capital level of the North. The ensuing economic diversification increases labour demand in the South thus inducing higher wages. The same claim could be applied to a small group of Southern countries following a development strategy based on economic diversification.

This analysis sheds light on the “miracle” of the recently industrialized countries from Southeast Asia. The model helps to understand an apparent paradox of these countries’ experiences: even though their labour supply was thought initially as “unlimited”, the newly industrialized countries experienced quick increases in real wage from the beginning of their industrial takeoffs (Amsden, 1989).

III. Growth Paths

The existence of an externality effect in economic productivity derived from economic diversification implies a divergence between the dynamics of a competitive (decentralized) economy and the dynamics of the economy under the command of a benevolent authority.

Two considerations are necessary for defining the competitive equilibrium dynamics. First, economic diversification has no effect on the private productivity of labour. Second, given that a technological transfer from abroad is free—techniques are public goods—, the equilibrium price of this activity is zero. Thus foreign techniques are transferred as a by-product of labour activity with no additional remuneration. Hence, in a competitive equilibrium no worker assigns time to education and future technological transfers are suspended. Under these circumstances the economy has no motion at all, welfare growth is zero, and the economic structure is fixed.

In order to find the optimal growth path it is assumed that some economic authority internalizes the externality effect with subsidies to education. Under this
assumption the representative consumer maximizes equation (4) subject to the instantaneous utility function [equation (19)] and the transition equation of education [equation (6)]. The Hamiltonian equation associated with this problem is the following

\[
H(\cdots) = \text{Max} \left\{ \ln \left[ \beta m(t) N(t)^{\sigma/(\sigma-1)} \right] e^{-\rho t} + \lambda(t) N(t) \left[ 1 - m(t) \right] \delta \right\}
\]

where the arguments of the Hamiltonian are \( m(t), N(t) \) and the multiplier \( \lambda(t) \). The first order conditions for maximization are the following:

\[
H_m(\cdots) = 0 : \quad e^{-\rho t} m(t)^{-1} = \delta \lambda(t) N(t), \quad (25)
\]

\[
\dot{\lambda}(t) = - H_N(\cdots)
\]

\[
= - \left( \frac{\sigma}{\sigma - 1} N(t)^{-1} e^{-\rho t} + \lambda(t) \left[ 1 - m(t) \right] \delta \right). \quad (26)
\]

The equilibrium paths of this economy should satisfy the following transversality condition:

\[
\lim_{t \to \infty} \lambda(t) N(t) = 0. \quad (27)
\]

Let us proceed to find the equilibrium. By combining equations (6), (25) and (26) the differential equation that drives workforce allocation is deduced:

\[
\frac{\dot{m}(t)}{m(t)} = -\rho + \frac{\delta \sigma}{\sigma - 1} m(t).
\]

The phase picture corresponding to this equation is in Figure 5. Rest points are \( m = 0 \), and the following steady state equilibrium:

\[
m^* = \frac{\rho}{\delta} \frac{\sigma - 1}{\sigma}. \quad (28)
\]

Under the assumption of an interior solution, \( m^* \) is the only solution consistent with the transversality condition. Hence there is no transitional dynamics in this model, i.e. forward-looking agents choose at once the level of labour supply \( m^* \) given by equation (28).
With logarithmic preferences (\(\varepsilon = 1\)), and a high elasticity of intratemporal substitution among goods (\(\sigma > 1\)), the workforce allocation to productive activities is always positive (\(m^* > 0\)). On the other hand, the allocation of time to education might be positive (\(m^* < 1\)), if the following inequality holds: \(\delta > \rho(\sigma-1)/\sigma\). This means that given some degree of impatience, \(\rho > 0\), the workforce will get educated if the degree of intratemporal substitutability among goods is high (\(\sigma > 1\)) and the education system is sufficiently efficient. If the last condition does not hold, i.e. \(\delta < \rho(\sigma-1)/\sigma\), no time is allocated to education.

If an interior solution exists the transversality condition boils down to the requirement that the discount factor be positive, \(\rho > 0\).

The above analysis implies a relationship between labour supply, education efficiency and welfare gains (refer to Figure 6).

Below the threshold level of education efficiency, no education takes place and hence economic diversification does not progress. For high levels of education efficiency, some effort is allocated to education (the labour supply is lower), the number of sectors increases at the following rate:

\[
\frac{\dot{N}}{N} = \delta - \frac{\sigma-1}{\sigma} \rho, \tag{29}
\]

and the utility level increases permanently:

\[
\dot{u} = \frac{\sigma \delta}{\sigma-1} - \rho. \tag{30}
\]
At this point one should note that equation (30) implies that the growth rate of welfare gains falls steadily towards zero. This is a consequence of assuming an intertemporal elasticity of substitution equal to 1 ($\varepsilon = 1$); however, if this elasticity is larger than one ($\varepsilon > 1$), the growth rate of welfare gains falls asymptotically towards the following positive minimum (see the Appendix):

$$\lim_{t \to \infty} \frac{\dot{u}(t)}{u(t)} = (\varepsilon - 1) \left( \frac{\sigma \delta}{\sigma - 1} - \rho \right).$$

The intertemporal elasticity of substitution measures the willingness to postpone consumption today for consumption tomorrow; thus a higher elasticity reflects a propensity to allocate a higher level of effort in education, which yields a higher rate of welfare growth.

From the previous analysis it is deduced that the relationship between economic growth and education efficiency is not linear. Below some threshold level of education efficiency there is no economic growth; above this level the rate of economic growth tends to decrease, but it converges in the long-run to some value that increases with educational efficiency.

IV. Concluding Comments

Some endogenous growth models suggest that per capita real income differentials between developed and underdeveloped countries may be partially explained by the relative stage of economic diversification (Romer, 1987, 1990; Grossman...
and Helpman, 1991). In addition, the present model suggests that countries characterized by low economic diversification and a large labour force may be bound to accept lower prices for their basic goods in international markets and endure lower per capita real incomes. The rationale for this result is as follows: given a structuralist characteristic of the economy—stability of intersectoral linkages—labour demand increases with economic diversification; thus, if the range of sectors is too low (economic diversification is insufficient), labour demand may be too little for the available labour force, inducing a downward wage adjustment. The same analytical tools help to explain why newly industrialized countries experienced quick increases in real wages from the beginning of their industrial takeoffs: their rapid process of diversification increased labour demand for a given labour supply, therefore these countries experienced an upward wage adjustment.

The dynamic analysis of the model helps to explain why countries with low educational attainment or low school-enrollment ratios, which are usually taken as good proxies for human capital accumulation, have less diversified economies, have lower real incomes and grow slowly.

Historic evidence on the crucial requirement of educated workforce for industrialization is profusely offered by Landes (1998). Evidence of the relationship between economic structure and real income is found in Leontief (1963), Chenery, Robinson and Syrquin (1986), and Syrquin and Chenery (1989). Evidence on the positive correlation between economic growth and education is found in Barro (1989, 1991), Levine and Renelt (1992), and Barro and Sala-i-Martin (1995). The last authors also report a significantly positive correlation between economic growth and the ratio of public spending to GDP. Barro and Sala-i-Martin claim that this ratio could be viewed as a rough proxy for school quality after controlling for years of schooling. In addition, from an econometric comparison across sub-Saharan African countries, Barro and Sala-i-Martin conclude that “the fast growers tend to have less government consumption and market distortions, and more educational spending and investment” (op. cit., ch. 12, p. 449). This result is also consistent with the present model.

The present model delivers endogenous welfare growth if and only if educational efficiency is high. Consequently, given the external effects of education on economic diversification, the model implies that public intervention in the provision of education enhances economic growth. Once prepared the agents are able to copy and adapt foreign technologies. On the other hand, failure to educate agents precludes economic diversification and long run economic growth collapses. These features of the model may help to explain why so few underdeveloped countries had been able to follow the path of development of the recently industrialized countries.

In spite of the importance of industrialization, the model does not support policies aimed at erecting protective barriers in underdeveloped economies, but it does support public investment in quality education and economic policies aimed at diversifying the economy.
The intertemporal elasticity of substitution was fixed to 1 in this paper. Here it is shown that the main dynamic results of the third section are not affected provided that this elasticity be high, i.e. $\varepsilon > 1$ [see equation (5)].

Substitution of equation (15) into equation (5), for $\varepsilon > 1$, yields the instantaneous level of utility:

$$u(m, N) = \frac{1}{1 - \varepsilon^{-1}} \left\{ \left[ \alpha a^{\frac{1}{1-\sigma}} m N^{\frac{\sigma}{1-\sigma}} \right]^{1-\varepsilon^{-1}} - 1 \right\}.$$ 

For $\varepsilon = 1$, the last equation collapses to the equation (19). Now, as before the agents of the economy want to maximize the discounted sum of utility [equation (4)], subject to the transition equation of education [equation (6)]. The first order conditions for this problem are the following:

$$e^{-\rho t} \frac{\partial u(m(t), N(t))}{\partial m(t)} = \frac{\lambda(t)}{N(t)} \delta,$$

$$\frac{\dot{\lambda}(t)}{\lambda(t)} = -\delta \left( 1 + \frac{m(t)}{\sigma - 1} \right),$$

$$\lim_{t \to \infty} \frac{\dot{\lambda}(t)}{N(t)} = 0,$$

where $\lambda$ is the shadow value of the stock of knowledge. Following the same procedure as in the third section, the workforce allocation to productive activities is deduced:

$$m = \varepsilon \left( \frac{\sigma - 1}{\sigma} \frac{\rho}{\delta} - 1 + \varepsilon^{-1} \right).$$

Afterwards the rate of utility growth is also deduced:

$$\frac{\dot{u}}{u} = \frac{(1-\varepsilon^{-1})}{\sigma - 1} \frac{\alpha a^{\frac{1}{1-\sigma}} m N^{\frac{\sigma}{1-\sigma}}}{\alpha a^{\frac{1}{1-\sigma}} m N^{\frac{\sigma}{1-\sigma}} - 1} \frac{\dot{N}}{N}.$$
Where
\[
\frac{\dot{N}}{N} = \varepsilon \left( \delta - \frac{\sigma - 1}{\sigma} \rho \right).
\]

From these equations it is deduced that the interiority condition is as before \( \delta > \rho (\sigma - 1)/\sigma \); hence the conclusions of the third section are valid here. Notice also that as the number of sectors grows, the growth rate of utility falls towards the minimum value shown in equation (30').

Notes
1. The structuralist vision is common to a well-known group of development economists (see Hirschman, 1958; Chenery and Syrquin, 1975); Chenery, Robinson and Syrquin, 1986; Syrquin and Chenery, 1989; and Syrquin, 1994, among others).
2. This is the key characteristic of industrialization which is called input-output deepening: “As countries industrialize, their productive structures become more ‘roundabout’ in the sense that a higher proportion of output is sold to other producers rather than to final users” (Chenery, Robinson and Syrquin, 1986, p. 57). Input-output deepening refers to the following related processes: “(...) first, a shift in output mix toward manufacturing and other sectors that use more intermediate inputs; and second, technological changes within a sector that leads to a greater use of intermediate inputs” (idem).
4. In a different world labour market disequilibrium might be solved by unemployment, but in our economy markets clear, which is a more suitable long-run assumption.
5. The strategic importance of quick and opportune supply of intermediate inputs (Porter, 1990), and the existence of transaction costs (tariffs, transport costs, corruption, etc.), may also explain why structural change proceeds orderly from goods requiring a shallow use of intermediate inputs towards goods requiring a higher intensity of intermediate inputs.

References

