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AN ENDOGENOUS GROWTH MODEL OF SOCIAL SECURITY AND THE SIZE OF THE INFORMAL SECTOR*

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Abstract:

Fully-funded pension systems are claimed to be less distortionary of labor allocation decisions than pay-as-you-go systems. This paper explores this issue in the framework of an overlapping generation model of endogenous growth where capital has an external effect on labor productivity and workers move from a formal to an informal sector in response to social security reforms. The two main features of the model are the following. First, as the social return on capital is sufficiently bounded away from zero, the economy can never be dynamically inefficient due to overaccumulation, but the private rate of return on capital may well be lower than the rate of growth. Labor market distortions determining the size of the informal sector are not necessarily lower in a fully funded system. Second, to the extent that a pension reform increases the share of output produced in the formal sector, both productivity and the rate of return on capital rise. Because of conflicting income and substitution effects, the change in consumption and growth cannot be determined unambiguously. Nonetheless, simple numerical exercises show the potential importance of the effects under consideration.

1. *Introduction*

This paper discusses a model of growth, social security and sectoral allocation of labor in the analytical framework of a stylized overlapping generation model of endogenous growth. The traditional set-up of the model is amended by relating

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intergenerational transfers to the decision to allocate labor between two productive sectors, using different technologies. One sector employ both capital and labor and is subject to social security regulation (formal sector); the other sector, less productive, only employs labor and is totally unregulated (informal sector).

The two main features of the model are as follows. First, for a given labor allocation, technology in both sectors is such that the social rate of return to capital is a positive constant. Since the rate of return does not decrease with the size of capital, our economy can never be dynamically inefficient due to overaccumulation (as in the Diamond model, Diamond [1965]). Nonetheless, in the tradition of endogenous growth model, capital has an external effect on labor productivity, so that the return on capital is not totally appropriable by private investors. In a long run equilibrium, the market capitalization rate may well be lower than the rate of capital accumulation. If this is the case, *at the current intertemporal prices,* a pay-as-you-go social security system on average guarantees a higher pension than a fully-funded system. To the extent that workers link current contributions to future pension benefits at the margin, labor market distortions determining the size of the informal sector are not necessarily lower in a fully-funded system.

Second, in traditional OLG models, an increase in pension benefits affects consumption by raising both individual and aggregate wealth. Numerical simulations, however, show that the magnitude of this effect is rather small. In our framework, as labor moves from one sector to another in response to a change in pension and contribution rates, both productivity and the rate of return on capital decrease. Because of conflicting income and substitution effects, the change in consumption and growth cannot be determined unambiguously. Nonetheless, simple numerical exercises show the potential importance of the effects under consideration.

In recent years, reforms aimed at promoting pension funds, under either public or private management[1], have been increasingly advocated by appealing to the growth-enhancing properties of fully-funded pension systems. The traditional argument hinges on the negative effects on saving of a systematic flow of intergenerational transfers under a pay-as-you-go system. In addition to such an argument, it has been stressed that pension funds may enhance growth also by improving the efficiency of both the financial and the labor markets.

Pension reforms strengthen the incentives to liberalize markets and to reduce the magnitude of financial repression. Pension funds, the argument goes, provide particularly valuable resources in the process of reforming domestic financial markets, because of both the magnitude and the quality of their demand for financial assets, supposedly directed towards long-term investment instruments[2]. How and whether pension funds foster the process of financial liberalization is an open issue at both theoretical and empirical level.

The second argument highlights the change in the set of incentives faced by firms and workers in their resource allocation decisions as individual social security contributions are more directly linked to future benefits. The powerful effects of these incentives are widely studied at micro-economic level. Yet, much lesser attention is paid at aggregate level, disregarding a potentially relevant macro-economic dimension of pension policies. This issue is the main focus of our analysis.

The size of the informal sector is surprisingly large not only in developing countries, but also in the industrialized world. In Italy, for example, the "irregular" sector is estimated to produce about 16% of value added in 1990 (70% of the agricultural output, 6% of industrial output, 36% of the building sector output and 22% of services) (Rey [1993]). These estimates led to an extensive revision of national income accounts in the eighties. In developing countries, available estimates of informal sector employment in urban areas vary between an average of 30% for a sample of relatively high-income countries and 50% for a sample of low-income countries (Turnham 1988). As social security contributions are one of the main components of labor costs, it is well understood that the informalization of the production process allows firms to reduce their costs substantially. In the case of Latin America, recent estimates point out that the tax wedge on labor costs imputable to social security is as high as 20% for small firms (Tokman [1992]).

In our work we do not delve into the construction of a theory of informality in production. A theoretical assessment of the role of social security, together with other forms of regulation, in explaining the emergence of an informal economy is a very interesting direction of research, but it is not the purpose of this paper. The reader will notice that in our specification the informal sector does not necessarily disappear if labor regulation is totally abolished[3]. All we require is that, at the margin, social security affects the choice to allocate labor between the two sectors. An increase of pensions associated with a higher social security contribution rate distorts labor allocation to the extent that, *at the margin,* workers do not consider their tax payment appropriable in the future at the market capitalization rate.

The plan of the paper is as follows. Section 2 presents both the supply side and the demand side of the model. The demand side of the model is fairly well-known; nonetheless, for completeness, it will be discussed in detail in section 2.2. Section 3 analyzes the steady-state effects on growth and consumption of alternative pension reforms. Section 4 provides a numerical a simulation of the model, while the last section discusses the main results of the model when some of the simplifying assumptions of the analysis are relaxed.

2. *The model*

2.1. *Supply side: a two sector model*

Sectoral output

The supply-side of the model has two sectors characterized by perfectly competitive markets with free entry. All firms produce a single homogeneous good, Y, but while the production technology in the first sector requires both capital and labor, the production in the second sector is carried out with the exclusive use of labor. Denoting by L(t) the total labor force in the economy, measured in physical units of labor-time, the sectoral allocation can be summarized by the proportion of workers in the first sector

$$\sigma_L(t) = \frac{L_1(t)}{(L_1(t) + L_2(t))} \qquad (1)$$

In the tradition of the endogenous growth literature, we assume the presence of an external effect of the existing capital stock on labor efficiency (Sheshinsky [1967], Romer [1986]). Thus, labor in the production function is measured in efficiency units "J" which do no coincide with raw labor-time

$$J(t) = \varepsilon(t) L(t) \tag{2}$$

where $\varepsilon(t)$ is the economy-wide capital-labor ratio

$$\varepsilon(t) = \frac{K(t)}{L(t)} \tag{3}$$

In the presence of perfectly competitive markets, firms consider $\varepsilon(t)$ as independent of their own decisions. They fail to see the link between their own investment and employment decisions and the efficiency of the labor force.

The production function in the formal sector is a standard constant return to scale function in labor and capital. Denoting by a lower case **k** the ratio of capital to labor efficiency-units, the representative firm technology is

$$Y_{i1} = F[K_{i1}, J_{i1}] = J_{i1} f\left(\frac{K_{i1}}{J_{i1}}\right) = J_{i1} f(k_{i1}) \tag{4}$$

where the subscript 1 refers to the first sector. Thanks to linear homogeneity of the production function, the sectoral output Y_1 can be expressed in terms of the sectoral allocation of labor σ_L only

$$Y_1 = K\sigma_L f\left(\frac{1}{\sigma_L}\right) \equiv \alpha_1 K \Phi[\sigma_L]; \qquad \Phi' > 0,\ \Phi'' < 0,\ \Phi(0) = 0,\ \Phi(1) = 1 \tag{5}$$

For a given σ_L, output is linear in K; by construction, the newly-defined parameter α_1 is the social productivity of capital when the whole labor force is allocated in the first sector[4].

Technology in the informal sector is linear in labor efficiency units. Production by individual firms

$$Y_{2j} = \alpha_2 J_{2j} \tag{6}$$

can be easily aggregated at sectoral level, as a function of labor allocation:

$$Y_2 = \alpha_2 \frac{K}{L} L_2 = \alpha_2 (1 - \sigma_L) K \tag{7}$$

Capital is the only factor which is ultimately productive, even if part of it is embodied in labor. The above expression highlights the fact that moving labor force away from the formal sector diverts the ultimately productive input, (embodied) capital, from the first production process to the second.

Notice that the wage rate per efficiency unit in the informal sector, equal to α_2 (*i. e.*, $w_2 = \alpha_2$), will determine the (appropriately defined) net wage rate in the whole economy.

Aggregate output

As overall output is the sum of gross production across the two sectors

$$Y = (\Phi(\sigma_L)\alpha_1 + (1 - \sigma_L)\alpha_2) K \equiv A[\sigma_L, \alpha_1, \alpha_2] K \tag{8}$$

the aggregate production function is still linear in capital (a so-called AK technology), whereas the linear coefficient A is now a weighted average of productivity in the two sectors[5]. Hereafter, we assume that, from a social point of view, the informal sector be technologically less productive than the formal one:[6]

$$\alpha_2 < \alpha_1 \tag{9}$$

Capital depreciates at the constant rate δ.

Two main characteristics of the aggregate production function deserve attention. First, for a given σ_L, the social return to capital A[.] is independent of the size of economic activity; therefore, dynamic inefficiencies due to overaccumulation (as in Diamond [1965]) can never occur. Second, the external effect of capital on labor productivity reduces the private *vis-à-vis* the social return to capital –investor are not rewarded for increasing the average quality of the labor force, nor they receive any compensation for the capital embodied in labor supplied to the informal sector.

2.2. *Demand Side*

The analytical framework for the demand side of the model is a Yaari-Blanchard model of overlapping generations, as developed by Weil [1989] and Buiter [1988,1992]. The endogenous growth version of such a model may run into a problem pointed out by Jones and Manuelli [1992] in the context of discrete-time OLG models.

Jones and Manuelli [1992] noticed that technologies which would generate sustained steady-state growth rates in a representative agent model may not do so in OLG models. The reason is that the endowment of the young generations may constrain the amount of capital that old generations are able to sell in order to finance consumption in their late days. The technological side of the model provides a strong engine for growth, *i. e.*, persistently high productivity of accumulated factors of production. However, the very accumulation process rapidly dwarves the endowment of newly born people, which becomes a binding constraint on the rate of growth.

In the absence of a proper life-cycle, a related issue arises in our Yaari-Blanchard model from the fact that accumulated factors are the ultimate source of productivity. Suppose that all externalities were internalized, so that both the social and the private marginal product of raw labor are zero. In a competitive setting, capital income would exhaust output. Since in the absence of intergenerational bequest and gift motives the newly born generations are endowed exclusively with raw labor, in such a scenario they would not be able to come into play. Endogenous growth would not peter out, but each new generation would starve until death, the time of which, luckily enough, is by construction independent of their diet.

The external effect of capital on individual productivity captures the idea that production requires human skills and knowledge. To the extent that these goods are non rival and non excludable, and can be freely acquired from the economic *milieu*, new generations see their endowment at birth increase with the level of economic activity. Such a feature of OLG model of endogenous growth is often poorly understood. In a representative agent model, any reduction of the share in output of factors which are not productive from a social point of view increases growth and welfare. Thus, to the extent that the productivity of labor hinges on the external effect of capital, it is desirable to reduce the share of labor income.

In our OLG model, factor shares are strictly interwoven with the endowment of new generations at birth. Because of the external effect of capital on their productivity, young generations live out of "rents" from a social planner perspective. Nonetheless, in the early stage of their lives, they also have the highest marginal propensity to save out of labor income. The interaction of these two elements generates a much richer set of possible results than the monotonic relation between factor distribution and the rate of capital accumulation that characterizes the representative agent version of our model.

The demand side of the model is well known. The main analytical steps are nonetheless presented in what follows.

2.2.1. *The individual problem*

At each instant t, individuals born at time $s \leq t$ solve the following problem

$$\text{Max}_{[c(s,v),\, \sigma_L(s,v)]} E_t \int_t^{\infty} \frac{1}{1-R}\, c(s,v)\, e^{-\rho(v-t)}\, dv \tag{10}$$

subject to the budget identity

$$\frac{d}{dt} a(s,t) \equiv [r(t) + \lambda]\, a(s,t) + x(s,t) - \tau(s,t) + p(s,t) - c(s,t) \tag{11}$$

and the appropriate solvency constraint. Individual consumption c(s,v) is required to be always non-negative; E_t is the expectation operator conditional on the information at time t; the non-negative parameter R is the reciprocal of the intertemporal elasticity of substitution (elasticity of instantaneous marginal utility): the desire to smooth consumption over time grows with R; the pure rate of time preference is instead denoted with a $\rho > 0$. The term a(s, t) denotes the stock of individual financial wealth, while x (s, t), τ (s, t), and p(s,t) are the instantaneous flows of individual labor income, social security contributions and pension payments, respectively.

Uncertainty in the model stems from the fact that each individual faces a constant instantaneous probability of death λ, independent of age and calendar time. The presence of a term "λa (s, t)" in the budget identity depends on perhaps the best-known feature of the Yaari-Blanchard model, that is, the existence of efficient annuities markets. Each consumer signs a contract with an insurance company, according to which she continuously receives some instantaneous rate of return "q" on her financial wealth in exchange for her entire wealth when she dies. The annuity industry is perfectly competitive and risk neutral. While at an individual level the time of death is uncertain, a constant death rate implies that a fraction λ of each generation dies at each instant in time. In the absence of aggregate uncertainty, the rate of return "q" will be equal to the death rate as competition among insurance companies drives profits to zero.

The term x (s, t) in the budget identity is labor earning gross of taxes. While labor time is inelastically supplied, individual labor income depends on the individual allocation of time between the two sectors, σ_L (s, t). Letting w* (t) denote the individual wage income gross of taxes, we can write gross labor earning as

$$x(s,t) = [w_1(t)\,\sigma_L(s,t) + w_2\,\sigma_L(s,t)]\,\varepsilon(t)\,L(s,t) \equiv w^*(t)\,j(s,t) \tag{12}$$

Notice that j (s, t) is the amount of labor power in efficiency units of an individual as old as (t–s), as opposed to L (s, t), which is the corresponding amount of labor time. In a model where agents supply labor over their whole life-span, a meaningful treatment of social security issues requires to specify a life-cycle pattern of labor earning. This is usually done by letting labor power deteriorate with age at a constant factor ζ

$$j(s,t) = j(t,t)\, e^{-\zeta(t-s)} \tag{13}$$

Define human capital h(s,t) to be the present value of gross labor income, discounted at the mortality-adjusted market rate [r (t) + λ]:

$$h(s,t) = \int_t^{\infty} x(s,v)\, e^{-\int_t^{v}[r(v)+\lambda]\,du}\, dv \tag{14}$$

where the individual discount rate is the sum of the market interest rate plus the rate of return on the insurance contract in the annuity market. Similarly, denote the present value of future net tax liability and future pension benefits with θ (s, t) and π (s, t), respectively. For a given allocation of labor between the two sectors, the solution to the individual consumption problem under rational expectations yields

$$c(s,v) = \eta\,[a(s,t) + h(s,t) - \theta(s,t) + \pi(s,t)] \tag{15}$$

where

$$\eta^{-1} \equiv \int_t^{\infty} e^{-\left[\frac{(R-1)}{R}\int_t^{v} r(u)\,du + \left(\lambda + \frac{\rho}{R}\right)(v-t)\right]}\, dv \tag{16}$$

Individual consumption is a fraction of individual wealth, which includes financial and human capital, plus pension wealth net of the present value of current and future tax liabilities. It is worth pointing out that, as in our endogenous growth model the marginal product of capital only varies with labor allocation, the rate of consumption growth

$$\frac{dc(s,t)}{c(s,t)\,dt} = \frac{r(\sigma_L) - \rho}{R} \tag{17}$$

is independent of the size of the capital stock.

Equilibrium in a competitive labor market implies that, at the current wages, individuals are indifferent about which sector they are employed in. For the sake of simplicity, the analysis will be developed under the assumption that each worker supplies a constant fraction of her labor-time to firms in the formal sector throughout her life. Under such an assumption, optimal individual labor supply will maximize individual wealth at birth

$$\frac{dh(s,s)}{d\sigma_L} - \frac{d\theta(s,s)}{d\sigma_L} + \frac{d\pi(s,s)}{d\sigma_L} = 0 \qquad (18)$$

2.2.2. *The aggregate economy*

In addition to a constant death rate λ, we consider a constant positive instantaneous birth rate β and normalize the population size by positing $N(0) = 1$.

As people ages at the constant age –and time– independent rate ζ, aggregate labor supply will only be a fraction of current population, as derived by aggregating individual labor force across generations

$$L(t) = \int_{-\infty}^{t} L(s,t)\,ds = \frac{\beta}{\beta+\zeta} e^{nt} \qquad (19)$$

For any individual variable, such as c(s,t), denote the corresponding aggregate variable by a capital letter, C(t), calculated by integrating c(s,t) over all currently alive generations

$$C(t) = \beta e^{-\lambda t} \int_{-\infty}^{t} c(s,t)\,e^{\beta s}\,ds \qquad (20)$$

Consumption can be expressed as a constant fraction of total aggregate wealth:

$$C(t) = \eta\,[A(t) + H(t) - \Theta(t) + \Pi(t)] \qquad (21)$$

Aggregate wealth includes financial and human capital, net of the present discounted value of tax liabilities $(A + H - \theta)$, plus the present discounted value of social security benefits (Π) that will accrue to all individuals alive at time t.

Disregarding public consumption, we assume that the government only administers a pay-as-you-go social security system, being subject to a balanced budget rule.

2.3. *Modelling a pay-as-you-go pension system*

Let pensions in our economy be financed by taxing wages in the formal sector at a flat rate t_1. Define γ as the fraction of social security wage tax (per efficiency unit of labor) that, *at the margin*, makes the present value of life-time taxes equal to the present value of pension payments paid conditional on past contributions. It is helpful to think of γ as the degree of future appropriability of an additional dollar of social

security contributions at the market capitalization rate, as it is determined by pension law, regulation and (explicit and implicit, legal and political) contracts. For instance, γ will be equal to one in a fully-funded system, while it will be zero in regimes where a social pension is granted to everybody regardless of past contributions. Equilibrium in the labor market thus implies:

$$w_1(1 - t_1 + \gamma t1) = [f(k_{i1}) - k_{i1} f'(k_{i1})](1 - t_1 + \gamma t1) = \alpha_2 = w_2 \qquad (22)$$

where w_1 and w_2 are wage rates per efficiency-unit of labor in the first and the second sector, respectively. Notice that the informal sector is (legally or illegally) sheltered from wage taxation and is not granting any sector-specific social security benefits. A positive γ inserts a wedge between cash wage rates net of taxes in the two sectors, the individual future appropriability of current contributions is crucial in assessing the magnitude of labor market distortions associated with alternative pension regimes.

The instantaneous flow of individual benefits is modelled after Saint-Paul [1992], in the following fashion:

$$p(s,v) = \Pi_1 e^{\Pi_2 (t-s)} \varepsilon(t) \qquad (23)$$

At each instant in time, individual benefits are scaled up to the size of economic activity $(\varepsilon(t))$. For a non zero Π_2, pension benefits also increase (or fall) with age. Setting public debt equal to zero, the government will be required to run a balanced (primary) budget. Aggregating contributions and transfer over all generations alive at time t and denoting net transfers by NT(t), we have

$$NT(t) \equiv P(t) - T(t) = \varepsilon(t)\,\beta\,e^{nt}\left[\frac{\Pi_1}{\beta - \Pi_2} - \frac{\sigma_L t_1 w_1}{(\zeta+\beta)}\right] = 0 \qquad (24)$$

where P(t) and T(t) are the economy-wide instantaneous flows of pension payments and tax revenue, respectively.

Notice that, as both net pension payments and wage income grow with the capital labor ratio, while agents choose their labor allocation at birth, in a long run equilibrium individuals will supply the same constant share of labor-time $\sigma_L(s,t) = \sigma_L(t) = \sigma_L$ to firms in the formal sector.

For the sake of simplicity, the degree of appropriability of current contributions by active workers (γ) will be determined as follows. Workers in the economy acquire the right to receive a full pension when their share of labor-time supplied to firms in the formal sector is higher than some threshold $\sigma_{L,\,min}$. Once crossed the implicit threshold of wage taxes, contributing to the system does not entitle the worker to any additional pension-related right. In other words, γ will be equal either to 0 or to the average (=marginal) degree of appropriability. Under these conditions, equilibrium in the labor market implies:

$$\begin{aligned} & w_1(1 - t_1) = w_2 && \text{if } \sigma_L > \sigma_{L,\,min} \\ & w_1\left(1 - t_1 + t_1 \frac{\beta - \Pi_2}{\beta+\zeta}\,\frac{r+\beta-g+\zeta}{r+\beta-g-\Pi_2}\right) = w_2 && \text{if } \sigma_L \le \sigma_{L,\,min} \end{aligned} \qquad (25)$$

where g is the rate of capital accumulation. Notice that, in our model, the average degree of appropriability γ may be higher than one *for all workers*.

3. *Pension, labor allocation and growth*

In order to make our model operational, we can reduce it to a system of four equations, including the equilibrium condition in the labor market plus three differential equations for consumption, capital and the present value of pension benefits, each one associated with the appropriate solvency and feasibility constraints (omitted from the text). The three differential equations in the system are presented hereafter.

$$\frac{dc(t)}{dt} = \mathbf{c}(t)^2 + \psi\, \mathbf{c}(t) - \eta(\beta + \zeta) - \eta\boldsymbol{\omega}(t) \tag{26}$$

$$\frac{dk(t)}{dt} = A - \mathbf{c}(t) - \frac{G}{K} - \delta \tag{27}$$

$$\frac{d\boldsymbol{\omega}}{dt} = [r + \beta - (A - \mathbf{c}(t) - \delta) - \Pi_2]\ \boldsymbol{\omega}(t) - \frac{\Pi_1\beta}{\beta - \Pi_2} \tag{28}$$

where

$$\psi \equiv \left(\frac{r-\rho}{R} + n + \zeta - A + \delta\right) \tag{29}$$

Bold variables denotes ratios to capital. The first two equations show the law of motion of consumption per unit of capital c (t) and the rate of growth (*i. e.*, the resource constraint of the economy). The third equation describes the evolution of the present value of the aggregate pension wealth to capital ratio (Π/K). Notice that, as steady state growth rates are positive in endogenous growth models, variables are appropriately expressed per units of capital, rather than per person.

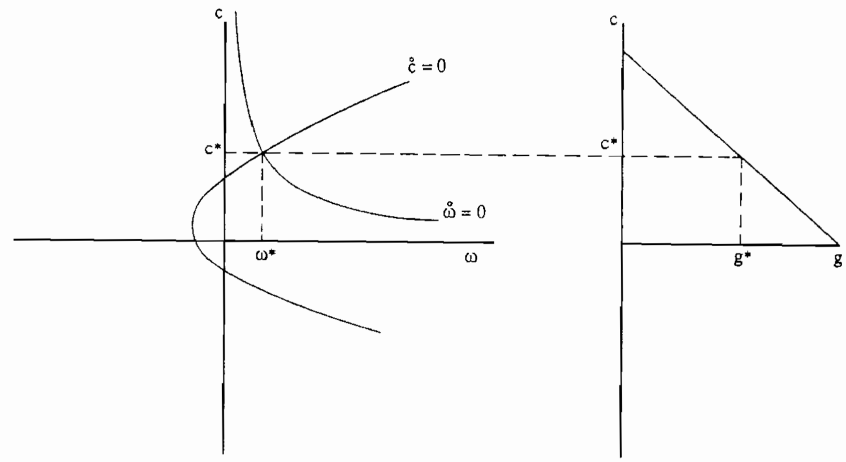
The graph to the left of Figure 1 draws the steady state loci of both the consumption to capital ratio and the pension –wealth to capital ratio in the space ($\boldsymbol{\omega}$, c), for some ψ sufficiently high. The first locus is a *consumption parabola* which crosses the horizontal axis at $\boldsymbol{\omega} = -(\beta + \zeta)$, and the vertical axis at

$$\mathbf{c} = \frac{-\psi \pm \sqrt{[(\psi^2 + 4\eta(\beta + \zeta)]}}{2} \tag{30}$$

where the first root is positive.

The second locus is a truncated rectangular hyperbola (hereafter the *pension hyperbola*) with a horizontal asymptote at $-(r + \beta - A + \delta - \Pi_2)$.

FIGURE 1



If a steady state equilibrium exists for positive values of $\boldsymbol{\omega}$ and c, it is identified by the coordinates of the point at which the consumption parabola intersects the pension hyperbola. The steady state growth rate can be read off the right hand side of Figure 1, which plots the rate of consumption against the growth rate of the economy. The downward sloping 45 degree line in the space (**c**, dk/kdt) crosses both axes at the social net productivity of capital ($A - \delta$).

Further information about the properties of this equilibrium can be derived by taking a linear approximation of the system around the steady state c*, $\boldsymbol{\omega}$*:

$$\begin{bmatrix} \dfrac{dc}{dt} \\[2ex] \dfrac{d\omega}{dt} \end{bmatrix} = \begin{bmatrix} 2c^* + \psi & -\eta \\[2ex] \omega^* & \dfrac{\Pi_1\Pi_2\beta}{\omega^*(\beta - \Pi_2)} \end{bmatrix} \begin{bmatrix} c - c^* \\[1ex] \omega - \omega^* \end{bmatrix} \tag{31}$$

Hereafter, the determinant of the constant coefficient matrix will be denoted by Δ:

$$\Delta = (2c^* + \psi)\,\frac{\beta\Pi_1\Pi_2}{\omega^*(\beta - \Pi_2)} + \eta\,\omega^* \tag{32}$$

Two types of equilibria are possible. One is a globally unstable equilibrium for positive values of Δ (a sufficient condition is $2c^* + \psi) > 0$); the second one is a saddlepoint equilibrium, if the determinant is negative. Figure 1 depicts an example of the first type of equilibrium, on which we will exclusively focus our analysis. Notice that, as neither variable is predetermined, **c** and $\boldsymbol{\omega}$ can both jump to the new stationary values in response to exogenous changes in parameters.

Having defined our apparatus, we now turn to the analysis of the macroeconomic long–run impact of different pension reforms. It is useful to take the case of a fully-funded actuarially-fair pension regime, where $\Pi_1 = \Pi_2 = 0$, as a benchmark allocation. Since social security contributions are part of individual voluntary saving, the variable ω (t) disappears from the system and the steady state consumption rate out of capital is identified by solving the consumption parabola for $\boldsymbol{\omega} = 0$. The parameter A [.] in the aggregate production function is determined by the labor equilibrium condition

$$w_1(\alpha_1, \alpha_2, \sigma_L) = w_2 \equiv \alpha_2 \tag{33}$$

which is independent from the tax rate t_1.

3.1. *Intergenerational transfers for a given aggregate pension flow with $\gamma = 0$*

In the first two cases under consideration, we assume that the relevant labor market equilibrium condition is

$$w_1(\alpha_1, \alpha_2, \sigma_L)(1 - t_1) = w_2 \equiv \alpha_2 \tag{34}$$

Both the sectoral allocation of labor and the productivity parameter are now functions of the (effectively distortionary) tax rates.

The first experiment consists of changing the intergenerational distribution of pension wealth keeping the instantaneous pension flow P(t) constant. In the second experiment, intergenerational redistribution is implemented through an increase in the pension flow P(t) financed by higher tax rates.

Consider an unexpected and immediately implemented reform of social security that alters the age-profile of lump-sum benefits for a given instantaneous flow of gross transfers P(t). Such a reform could be engineered as in Saint Paul [1992]:

$$-d\Pi_1 = \frac{\Pi_1}{\beta - \Pi_2}\, d\Pi_2 \tag{35}$$

An increase in Π_2 matched by an appropriate decrease in Π_1 entails an intergenerational transfer in favor of old generations. By construction, this policy does not alter the incentives to allocate labor between the two sector (contribution rates are the same), nor it changes the rate of instantaneous aggregate gross pension transfer per $\varepsilon(t)$.

In this case, it is easy to show that current aggregate consumption increases as, on impact, the reform raises the level of wealth of currently alive generations by the change in the present discounted value of social security benefits:

$$\left.\frac{d\Pi(t)}{d\Pi_1}\right|_{dNT=0} = -\beta e^{nt} \int_t^{\infty} \varepsilon(v)(v - t)\, e^{-\int_t^v [r(u) + \lambda - \Pi_2]\, du}\, dv < 0 \tag{36}$$

Since the rate of return is unaffected by the change in consumption patterns, there is no substitution effect at play. Individuals simply revise their consumption according

to the new level of perceived individual wealth. Notice that individual consumption keeps growing at the same rate (which is proportional to the difference between the rate of return and the rate of time preference) before *and* after the reform. Aggregate growth slows down because the *level* of individual consumption of each cohort is adjusted upwards.

The effect on the steady state allocation of such a reform can be studied by carrying out a standard comparative static exercise:

$$\left.\frac{d\boldsymbol{\omega}^*}{d\Pi_1}\right|_{dNT=0} = (2c^* + \psi)\,\frac{\beta + \dfrac{\omega^*(\beta - \Pi_2)}{\Pi_1}}{\Delta} \tag{37}$$

$$\left.\frac{dc^*}{d\Pi_1}\right|_{dNT=0} = \eta\,\frac{\beta + \dfrac{\omega^*(\beta - \Pi_2)}{\Pi_1}}{\Delta} \tag{38}$$

The denominator of these expressions coincides with the determinant of the linear approximation of the non-linear system of equations around the steady-state ($\mathbf{c}^*, \boldsymbol{\omega}^*$). If the equilibrium is globally unstable, both derivatives are positive: following the implementation of an unexpected but permanent reform, the consumption as well as the pension-wealth to capital ratios jump up to the new long-run values.

Figure 2 depicts the effect of the reform under consideration as a rightward shift of the pension hyperbola along the consumption parabola. Since the supply side in the system does not respond to the new regime, the growth rate decreases by the full amount of the change in consumption per unit of capital[7].

3.2 *Increasing aggregate pension expenditure for $\gamma = 0$*

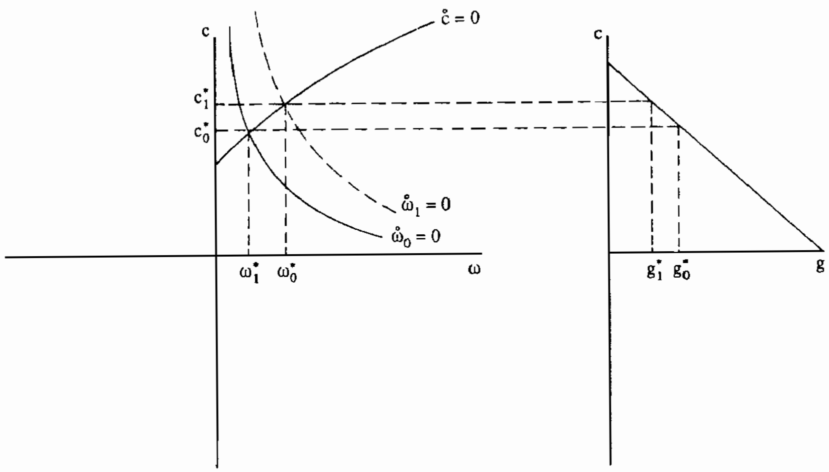
In our second experiment, an increase in intergenerational transfers via social security is financed by a higher distortionary wage tax. A balanced budget reform now implies a change in the tax base due to the re-allocation of labor between the two sectors. Instead of (35), we have:

$$\frac{\Pi_2}{(\beta - \Pi_2)}\, d\Pi_1 - \frac{\sigma_L w_1}{(\zeta + \beta)}\, dt_1 - \frac{w_1}{(\zeta + \beta)}\frac{d\sigma_L}{dt_1}\, dt_1 - \frac{\sigma_L}{(\zeta + \beta)}\frac{dw_1}{dt_1}\, dt_1 = 0 \tag{39}$$

As a result, the model generates a typical Laffer curve.

The analysis of the effects of a pension reforms is complicated by the fact that the re-allocation of labor reduces both the rate of return and the rate of overall productivity of capital. By denoting with w* the before-tax overall wage income from both sectors, the comparative static analysis of the tax reform yields:

FIGURE 2



$$\begin{bmatrix} 2c^* + \psi & -\eta \\ \omega^* & \dfrac{\Pi_1\Pi_2\beta}{\omega^*(\beta - \Pi_2)} \end{bmatrix} \begin{bmatrix} dc \\ d\omega \end{bmatrix} = \begin{bmatrix} c^* \left(\dfrac{dw^*}{dt_1} + \dfrac{R-1}{R} \; \dfrac{c^* + \omega + \beta + \zeta}{c^*} \; \dfrac{dr}{dt_1} \right) dt_1 \\ \omega \left(\dfrac{dw^*}{dt_1} \, dt_1 + \dfrac{P(t)}{\Pi_{(t)}} \; \dfrac{d\Pi_1}{\Pi_1} \right) \end{bmatrix} \tag{40}$$

where it can be easily shown that

$$\frac{dw^*}{dt_1} \equiv \frac{dA}{dt_1} - \frac{dr}{dt_1} < 0 \qquad \frac{dA}{dt_1} < 0 \qquad \frac{dr}{dt_1} < 0 \tag{41}$$

The latter inequalities highlight that, when the less productive informal sector expands, the private rate of return on capital decreases by less than the fall in output per unit of capital.

Consider an economy which is on the rising side of the Laffer curve –so that an increase in the pension payments flow requires a higher contribution rate– and focus on the vector on the right hand side of the above system. Around the equilibrium, the shift of the consumption parabola (first equation) depends on the combined effect of the drop in overall wage income and the change in the interest rate, the sign being determined by the coefficient of intertemporal substitution. If the desire to smooth consumption is relatively high (R > 1), the consumption locus moves downward around the equilibrium; it may move the other way around for a sufficiently low value of the coefficient R. That is to say, the sign of the first element in the vector is ambiguous, depending on the intertemporal elasticity of substitution. On the other hand, the shift of the pension hyperbola (second equation) responds to the drop in overall wage income as well as to the increasing flow of current pension benefits weighted by pension-wealth. The sign of the second element of the vector is also ambiguous.

Different results are possible, conditional on the specification of preferences and technology. For concreteness, posit R>1 and assume that the effect of the drop in wage-income is dominated by the effect of the increasing pension flow in the second equation. In this case, the consumption locus shifts downward, while the pension locus shifts upwards in the space (c, **ω**). As shown in Figure 3, pension wealth rises in steady state; consumption per unit of capital can either rise or fall. The lower productivity of the economy moves the 45 degree line down in the left graph of Figure 2. The new growth rate can read off this graph, as the difference between the new productivity level and the new consumption rate.

Notice that individual consumption growth slows down as a consequence of the reform. Nonetheless, it is difficult to predict what happens to the *level* of consumption of individual cohorts, which also adjusts to a revised level of wealth where all flows are capitalized at a new discount rate. The simple monotonic relationship between consumption and pension wealth which characterizes intergenerational transfers financed with lump-sum taxation disappears.

3.3. *Increasing the perceived degree of future appropriability of current contributions*

Suppose now that the relevant labor market equilibrium condition includes the annualized flow of future pension payments. In principle, it could be possible to develop the analysis as in the previous section, except that it would be necessary to check the consistency of the steady state equilibrium with the labor market equilibrium condition. *Ceteris paribus*, the supply-side response to the social security reform should be smaller, *i. e.*, we should have a lower degree of informalization associated with each particular contribution rate.

4. *Pension, informalization and growth: numerical examples*

In the traditional OLG literature, an increase in intergenerational transfers, financed with lump-sum taxation, reduces the steady state capital labor ratio (if technology is neoclassical) or the growth rate (in endogenous growth models with constant output rate). Nonetheless, simulation exercises show that the magnitude of these effects is rather small (see for example the survey in Corsetti and Schmidt-Hebbel 1994).

The question is therefore how much the long-run macroeconomic impact of pension reforms can vary, once other factors (in addition to the change in pension wealth) are taken into account. By way of example, this section provides a numerical simulation of our model, based on standard values for both preferences and technology. Endogenous growth models are of course biased analytical tools *vis-à-vis* our goal: growth rates are permanently affected by any change of parameters. Our exercise is therefore only aimed at capturing qualitative features of the response of economic systems to changing pension regimes, rather than providing a quantitative assessment of the effects under consideration. Results are reported in Table I.

Vis-à-vis the benchmark allocation under a fully-funded pension regime, our simulations study the growth and consumption effects of an increase in pensions financed through wage taxation, while varying the magnitude of labor market

TABLE 1

I.0. A fully-funded system.

t_1	σ_L	Capital growth	100* C/K	Π_1	Labor income share
Any	1	3.7%	14.38	0.11	75%

I.1. Effects of an increase in pensions financed with lump-sum taxation.

t_1	σ_L	Capital growth	100* C/K	Π_1	Labor income share
6%	1	3.5%	14.46	.11	75%
10%	1	3.4%	14.56	.17	75%
15%	1	3.3%	14.68	.25	75%
20%	1	3.1%	14.81	.34	75%

I.2. Effects of an increase in pensions financed with distortionary taxation: labor moves from the formal to the informal sector in response to higher tax rates.

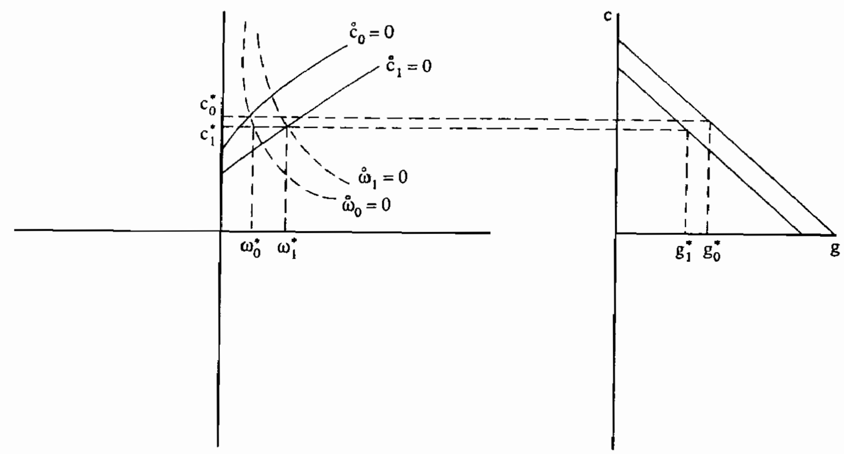
t_1	σ_L	Capital growth	100* C/K	Π_1	Labor income share
6%	1	3.5%	14.46	.11	75%
10%	.85	3.0%	14.76	.15	77%
15%	.675	2.4%	15.01	.19	80%
20%	.525	1.8%	15.10	.21	83%

I.3. Same as 2.2, but at the margin, the degree of appropriability of future pension payments at the current capitalization rate is assumed to be equal to 20%.

t_1	σ_L	Capital growth	100* C/K	Π_1	Labor income share
–	–	–	–	–	–
10%	1	3.4%	14.55	.16	77%
15%	.825	2.9%	14.89	.22	80%
20%	.70	2.3%	15.11	.24	83%

Values of parameters in the simulation:
1. Preferences: $R = 1.1$, $\rho = .02$.
2. Technology: $\alpha_1 = .2$, $\alpha_2 = .14$, the production function in the formal sector is a Cobb-Douglas, capital share = .25, $\delta = .02$.
3. Demography: $\beta = .06$, $\lambda = .03$, $\zeta = .03$.
4. $\Pi_2 = 0$.

FIGURE 3



distortions. Contribution rates are raised from 6% to 20%. In section I.1 of the table, there is no informal sector. In section I.2, the informal sector emerges as a consequence of the pension reform, absorbing up to 50% of the labor force. In section I.3, the individually perceived marginal appropriability of future social security benefits is raised from 0 to 20%.

A striking piece of information stems from comparing the third columns of sections I.1 and I.2 in the table. In section I.1, the intergenerational transfer-effect of an increase in pensions reduces growth from 3.53% to 3.18%; long-run growth drops to 1.8% in section I.2. Nonetheless, a stronger link between workers' current contributions and future pension benefits can substantially lower the labor market distortions induced by a higher tax rate. In section I.3, as a larger share of the labor force keep working in the formal sector, growth only decreases down to 2.3%.

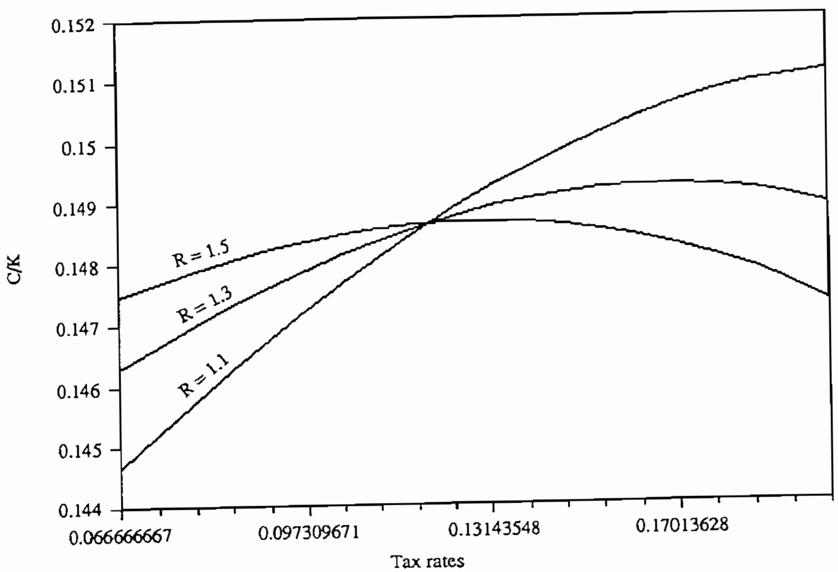
In assessing the reported numerical results, it is important to keep in mind that, in our model, the long run macroeconomic impact of a pension reform cannot be determined unambiguously because of the reform-related effects on the degree of labor market efficiency. For instance, Figure 4 plots consumption per unit of capital as the intertemporal elasticity of substitution is lowered in the simulation. Consumption per capital increases monotonically with pension wealth only for a low R; it is non monotonic as the parameter R goes above the value of 1.3.

5. *Conclusion: can labor market distortions be avoided by adopting a fully-funded system?*

Pension reforms are increasingly advocated as growth-promoting policy in many developing countries. While the effects on saving of intergenerational transfers are well understood, it is likely that relevant growth implications of a switch from a pay-as-you-

FIGURE 4

CONSUMPTION AND CONTRIBUTION RATES



go to a fully-funded system will stem from induced changes in both the capital market and the labor market.

This paper focuses on the role of social security in the informalization of production from a macro-economic perspective, by providing a discussion of the impact of labor market distortions on growth in an economy where people can avoid taxes by switching from a formal to an informal, labor intensive sector.

The allocation of labor between sectors in our model depends on the perceived marginal degree of appropriability of social security saving capitalized at the market interest rate. *Vis-à-vis* the benchmark allocation with full appropriability in a fully-funded system, the text has discussed different degrees of appropriability in a pay-as-you-go system. In a world without uncertainty and without borrowing constraint, social security contributions are a component of private saving in our benchmark case, they may be perceived as pure tax (at the margin) in the other cases.

In our analysis, the indicator of labor market distortions is the size of the socially less productive informal sector. Are fully-funded systems the least distortionary of labor allocation decisions? In an endogenous growth model with an external effect of capital on labor, a distributional issue arises from the fact that a share of the return on investment goes to capital embodied in labor. Future wages increase with capital accumulation at some rate which may be higher than the market rate without any adverse consequence for the dynamic efficiency of the economy. Therefore, it is possible that, once law, regulation and contracts appropriately link contributions to benefits, income incentives to work in the formal sector be higher in a pay-as-you-go than in a fully-funded system. Of course, this need not be the case, and it should be stressed that, while the link between current contributions and future benefits is clear in a fully-funded actuarially-fair system, it must be carefully built in the design of the social security institutions of a pay-as-you-go system.

It is also important to keep in mind that the relationship between the size of the formal sector and the growth rate is non- monotonic. First, the growth performance of an economy is negatively affected by the intergenerational-transfers of a pay-as-you-go system. Second, income and substitution effects at play make very hard to determine the consumption reaction to long run structural changes in production due to social security.

As a final remark, consider the consequences of allowing for borrowing constraints, for uncertainty, as well as for the transition from a pay-as-you-go to a fully funded system, in which past pension obligations must be honored by the government. Borrowing constraints give workers an incentive to resort to the informal sector in both (fully-funded and pay-as-you-go) systems. The labor market equilibrium condition includes an extra-term where each additional unit of currently disposable income is weighted by the appropriate shadow price.

Uncertainty highlights one of the main weaknesses of actuarially-fair schemes, that is, the low provision of individual insurance. If workers cannot diversify their portfolio optimally because of missing markets or other inefficiencies, people may expect the government to provide some insurance (either explicitly or implicitly), within the framework of a fully-funded system. To the extent that public insurance weakens the link between contributions and pensions, the pay-as-you-go (inter or intra-generational) component of the scheme shows up in the labor market equilibrium conditions. Some public insurance is of course desirable, to an extent which depends on the social welfare criterion relevant to policy-makers. Nonetheless, the moral-hazard problem implicit in such schemes calls for a careful assessment of their institutional design.

Finally, in the process of switching from a pay-as-you-go to a fully-funded system, the public nature of the pension debt tends to be clearly perceived by currently active workers. As overall fiscal pressure increases, because of the need to build pension funds while paying-off existing obligations, the incentive to tax evasion may be high. Even taking into account the set of fiscal incentives which may differentiate a pension fund from, say, a mutual fund or an insurance contract, employment in the informal sector can still bring about substantial tax savings. Fiscal visibility of wages and incomes generally implied by fully-funded publically guaranteed pension systems may reduce the re-allocation of labor in response to a pension reform.

Notes

1 The privatization of social security is often aimed at developing a fully-funded system. A well functioning market requires a sufficiently large number of competing funds, otherwise then administrative costs tend to be higher than socially desirable.
Publicly managed funds or appropriately regulated large private funds could be a viable alternative, but public management of a fully funded social security system is generally considered too weak of a shelter against current and future political decision making.

[2] In some countries, a related issue is the degree to which the well functioning of national financial market depends on foreign capital. Pension funds are expected to increase the thickness of domestic markets.

[3] Stating that informality depends on regulations and that abolishing rules makes the informal sector disappear either is a tautology or requires a theoretical model explaining both regulation and its violation in an integrated way. Most of the theoretical model in the literature base their analytical structure on the presence of public goods and externality, coupled with policy mismanagement.

[4] In the case of a Cobb-Douglas production function, we have

$$Y = AK^qJ^{1-q} = AK^qK^{1-q}\left(\frac{L_1}{L}\right)^{1-q} = A\sigma_1^{1-q}K$$

[5] In our specification, output from the two sectors is a perfect substitute in both investment and consumption. Easterly [1993] discusses a representative agent growth model where two kinds of capital enter production with a constant parametrical elasticity of substitution.

[6] In the absence of fiscal distortions, if it is the second sector to be more productive, arbitrage in the labor market would make the wage rate so high that the gross return to capital would be negative. The first production process would not be economically viable and the demand for capital would be zero. A competitive equilibrium could still be viable if we introduced a third technology, using capital only, where the rate of return to capital is the negative of the depreciation rate. In other words, the first production process would be replaced by an imperfect storage technology, linear in K.

[7] Notice that in a saddlepoint equilibrium (a negative Δ), consumption per unit of capital would decrease with a rising present value of pension per unit of capital.

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