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SIMPLE ESTIMATES FOR THE US TERM PREMIUM*

ESTIMACION SENCILLA DE LA PRIMA POR PLAZO EN EE.UU.

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Abstract

In this paper, we propose a simple procedure to estimate term premium for US Treasury bond yields. The estimation procedure is based on the Dynamic-Nelson-Siegel model which has only one parameter under the risk-neutral measure, which we calibrate at 94% based on previous literature but also in empirical exercises using a monthly sample from 1980 to 2024. The calibration allows us to obtain latent factors by inverting a 3x3 matrix, and obtaining the term premium with the parameters of a VAR model estimated for the same sample. Results for the 5 and 10-year bond yields are broadly in line with figures published by the Federal Reserve of New York.

Keywords: *Yield curve, term premia, Nelson-Siegel model.*

JEL Classifications: *E43, E44, G12.*

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Resumen

En este artículo proponemos un procedimiento sencillo para estimar la prima por plazo de las tasas de los bonos del Tesoro de Estados Unidos. El procedimiento se basa en el modelo dinámico de Nelson-Siegel, que bajo la medida neutral al riesgo contiene un único parámetro. Este parámetro se calibra en 94%, de acuerdo con la literatura previa y con los resultados de una muestra de datos mensuales para el período 1980-2024. Esta calibración permite obtener los factores latentes y estimar la prima por plazo utilizando los parámetros de un modelo VAR. Los resultados obtenidos para los rendimientos de los bonos a 5 y 10 años son, en términos generales, consistentes con las estimaciones publicadas por la Reserva Federal de Nueva York.

Palabras clave: *Curva de rendimiento, prima por plazo, modelo de Nelson y Siegel.*

Clasificación JEL: *E43, E44, G12.*

I. INTRODUCTION

Financial theory argues that long-term rates contain markets' expectations of the future path of short-term interest rates and an additional component denominated term premium (TP). Estimates of TP require to impose a structural model, where explicitly the risk premium must be modeled (e.g., Wright, 2011). A standard framework for doing this implies that the short-term interest rate depends on a set of factors, which exhibit dynamics under two statistical measures: physical and risk-neutral (Singleton, 2006). The former could be related to the time-series of the yields, meanwhile the latter with the arbitrage restrictions across yields. Dai and Singleton (2000) introduce several continuous-time affine models, being the Gaussian-linear one the most popular. In such framework, the short-term interest rate depends linearly on a set of factors that have linear dynamics under both measures, which are described with VAR models (Nyholm, 2020).

The main empirical challenge of the Gaussian affine model is the identification of its parameters, for which several procedures have been suggested (Singleton, 2006; Nyholm, 2020). For instance, Hamilton and Wu (2012) formalize the use of the two set of yields: (i) with measurement error and (ii) without those, assuming that some of the parameters are identified only using one of the set of yields. This was introduced by Chen and Scott (1993), and it differs from the standard application of the Kalman filter, where all yields are assumed to be observed with measurement error (Diebold and Rudebusch, 2013). Finally, Adrian *et al.* (2013, ACM hereafter) propose the use

of sequential linear regressions, which are based on the principal components of all available yields.

Regarding TPs, Wright (2011) compares estimates for advanced economies, offering an interesting view of this variable and the inflation uncertainty, extensions to Latin America countries and to emerging market economies are presented in Blake *et al.* (2015), Ozbek and Talasli (2020), and van Schaik *et al.* (2024), respectively.[1] The purpose of this paper is to present a simple procedure to obtain TPs, based on the Dynamic-Nelson-Siegel (DNS) model which was introduced in Diebold and Li (2006, D&L hereafter) and further developed as affine model in Christensen et al (2011). The procedure is simple for two reasons: (i) the DNS model has only one parameter under the risk-neutral measure, which we calibrated according with the argument presented in D&L that states that the weight associated to the factor called "curvature" should be maximized at 30 months. In our discrete-time version of the model, proposed by Alfaro (2011), this implies a parameter of 94%. We confirm that this calibration is consistent with the estimates obtained by using Hamilton and Wu (2012)'s approach which relies on a set of yields observed without measurement errors. The empirical results obtained with monthly data from January 1980 to December 2024 suggest that values between 92% and 96% are in line with several sets of yields, and (ii) all the VAR models of the yields considered as observed without measurement errors have similar eigenvalues, being 99%, 96%, and 82-88%, respectively. This allows us to build a closed-form expression for the average value of the expected short-term interest rates, under the physical measure, and therefore estimates of the TP based on the DNS model (TP-DNS).

We compare TP-DNS for the 5 and 10-year US Treasury bond yields with the ones published at the website of the Federal Reserve of New York which are based on ACM (TP-ACM). Empirical results support the use of the proposed procedure, implying that a simple average of a short-term (3 or 6-month), a medium-term (2 or 5 year), and the 10-year yield can be used to obtain estimates of the TPs for the US Treasury bond yields. We also provide 90% confidence interval, which is based on variance matrix of the weights for the long-term rate under the physical measure. Finally, we note that the identification of the term premia relies on long time-series of yields, an issue that is well-documented in the literature of the estimation of affine models (e.g., Bauer *et al.*, 2012), but has not been implemented in empirical applications for emerging markets (Blake *et al.*, 2015; Ozbek and Talasli, 2020).[2] In this respect, any

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[1] Notably all extensions use ACM approach which starts with getting the principal components of all observed yields, then a three-step procedure is applied which involves sequential linear regressions that combines time-series dynamics with cross-sectional restrictions.

[2] An exception is presented in van Schaik *et al.* (2024). The authors incorporated ridge regressions to address potential instability of the ACM procedure applied to the US Treasury yield's reduced sample January 1985-January 1995. Based on their results, they extended the procedure to the South Africa government bond market for which two samples are used: (i) January 2016-February 2024, and (ii) February 2005-December 2015.

bias-correction method (e.g., Analytical Approximation, Jackknife, Indirect Inference) could be easily applied to our procedure, as these methods were designed for VAR models. We leave this topic for future research.

Section II discusses the theoretical framework for yield curve modeling and its estimation. Section III provides empirical results, and it compares TP-DNS with TP-ACM for 5 and 10 years, given that these maturities are commonly used in practice. Section IV concludes.

II. TECHNICAL SECTION

In this section, we briefly discuss yield curve modeling and estimation. The material is mostly based on Diebold and Rudebusch (2013), and Nyholm (2020).

II.1. Yield Curve and Term Premium

Let z_{nt} be the zero-coupon n-maturity (continuous-compounded) yield at time t, then we consider the TP as the difference between the actual yield and the one obtained using the trajectories of the short-term interest rate under the physical measures, which is defined as:

$$x_{nt} \equiv \frac{1}{n}\sum_{k=0}^{n-1} E_t\left(r_{t+k}\right), \tag{1}$$

where r_t is the short-term interest rate and $E_t()$ is the expected value under the physical measure.

We assume that the yield curve is characterized by three latent factors: L_t, S_t, and C_t. D&L named as level, slope, and curvature, and are usually related to inflation, economic growth, and features of the financial market infrastructure. Thus, latent factors could be extracted from actual yields.

In the following, we assume that the short-term interest rate is defined as $r_t = L_t + S_t$, and the log-pure expectation hypothesis (LPEH) is valid. The latter states that Z_{nt} is an average of expected short-term interest rates, under the risk neutral measure (Campbell *et al.*, 1997):[3]

[3] Note that LPEH ignores the Jensen term, which under Gaussian disturbances depends only on elements of the variance-covariance matrix, being additive and time-invariant. Given that our focus is the TP, this omission should not be relevant but offers a useful simplification.

$$z_{nt} = \frac{1}{n}\sum_{k=0}^{n-1} E_t^*\left(r_{t+k}\right), \tag{2}$$

where $E_t^*()$ is the expected value under the risk-neutral measure.

We also assume that the dynamics of the latent factors are defined by $\Lambda_t = H_* \Lambda_{t-1} + \Sigma V_t$, where $\Lambda_t = \left(L_t, S_t, C_t\right)'$, V_t is standard multivariate Gaussian vector error under this measure and:[4]

$$H_* = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \phi & 1-\phi \\ 0 & 0 & \phi \end{pmatrix}. \tag{3}$$

We note that $E_t^*\left(\Lambda_{t+k}\right) = H_*^k \Lambda_t$, where the exponent of the transition matrix is:

$$H_*^k = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \phi^k & \pi_k \\ 0 & 0 & \phi^k \end{pmatrix}, \tag{4}$$

with $\pi_k = k(1-\phi)\phi^{k-1}$. The short-term interest rate can be defined as $r_t = \begin{pmatrix} 1 & 1 & 0 \end{pmatrix}\Lambda_t$, then we have $E_t^*\left(r_{t+k}\right) = \begin{pmatrix} 1 & 1 & 0 \end{pmatrix} E_t^*\left(\Lambda_{t+k}\right) = \begin{pmatrix} 1 & \phi^k & \pi_k \end{pmatrix}\Lambda_t$. Thus, the zero-coupon yield is:

$$z_{nt} = \frac{1}{n}\sum_{k=0}^{n-1} E_t^*\left(r_{t+k}\right) = L_t + \left(\sum_{k=0}^{n-1}\phi^k\right)\frac{S_t}{n} + \left(\frac{1-\phi}{\phi}\right)\left(\sum_{k=1}^{n-1} k\phi^k\right)\frac{C_t}{n}. \tag{5}$$

Following Alfaro (2011), we use standard results for series obtaining a discrete-time version of the well-known Dynamic-Nelson-Siegel (DNS) model:

$$z_{nt} = L_t + \left(\frac{1-\phi^n}{1-\phi}\right)\frac{S_t}{n} + \left[\left(\frac{1-\phi^n}{1-\phi}\right) - n\phi^{n-1}\right]\frac{C_t}{n}. \tag{6}$$

The DNS was introduced in D&L as a dynamic extension of the model proposed by Nelson and Siegel (1987), which is popular model for fitting yields. We note that the loading of the curvature is zero at both extremes of the curve (either when $n = 1$

[4] Nyholm (2020) provides alternatives specifications for matrix H_*, which involves rotation of the factors. One of the specifications implies that the short-term interest rate is a factor itself.

or when $n \to \infty$); moreover, that loading could be maximized at a given maturity choosing an appropriate value for ϕ. This recognizes the relevance of the curvature factor for some market participants. Indeed, D&L considered 30 months as a relevant maturity, therefore where curvature's loading is maximum. This implies, in discrete-time setting, that ϕ should be 94%.

Finally, we assume that the dynamics of the latent factors under the physical measure are defined by an empirical VAR model: $\Lambda_t = c + H\Lambda_{t-1} + \Sigma U_t$, where U_t is standard multivariate Gaussian vector error under this measure, and Σ lower triangular. Thus, there are 18 unknown parameters in this VAR model: 3 in c, 9 in H, and 6 in Σ. Iterating the VAR model we have the following expression: $E_t\left(\Lambda_{t+k}\right) = \left(I - H^k\right)(I - H)^{-1}c + H^k\Lambda_t$. Therefore, we compute the expected short-term rate as $E_t\left(r_{t+k}\right) = \begin{pmatrix} 1 & 1 & 0 \end{pmatrix} E_t\left(\Lambda_{t+k}\right)$, which implies:

$$x_{nt} = \frac{1}{n}\sum_{k=0}^{n-1} E_t\left(r_{t+k}\right) = \begin{pmatrix} 1 & 1 & 0 \end{pmatrix}\left[\left(I - \Gamma_n\right)(I - H)^{-1}c + \Gamma_n\Lambda_t\right], \tag{7}$$

where $\Gamma_n = (1/n)\left(I - H^n\right)(I - H)^{-1}$. We could express the yield as $x_{nt} = P_n + Q_n\Lambda_t$, where $P_n = \begin{pmatrix} 1 & 1 & 0 \end{pmatrix}\left(I - \Gamma_n\right)(I - H)^{-1}c$, and $Q_n = \begin{pmatrix} 1 & 1 & 0 \end{pmatrix}\Gamma_n$. Thus, both P_n and Q_n depend on the maturity of the yield and on the parameters of the VAR model.

For the purpose of building a confidence interval for x_{nt}, we consider the following two vectors: $R_n = \begin{pmatrix} P_n & Q_n \end{pmatrix}$ and $\tilde{\Lambda}_t = \begin{pmatrix} 1 & \Lambda_t' \end{pmatrix}'$, which implies that $x_{nt} = R_n\tilde{\Lambda}_t$. Therefore, the variance of x_{nt} is time-variant and depends on the maturity of the yield as follow: $\mathrm{Var}(x_{nt}) = \tilde{\Lambda}_t'\mathrm{Var}(R_n)\tilde{\Lambda}_t$. The center of the sandwich is the covariance matrix, which we estimate by relying on the asymptotic distribution of the parameters of the VAR and on 1000 Monte Carlo simulations.[5]

II.2. Estimation of the model

The maximal canonical Gaussian model implies that there is a maximum of 22 identifiable parameters when three factors are considered (Joslin *et al.*, 2011). In this case, the DNS model involves 19 parameters, due to the same constraint in the short-term equation and additionally one eigenvalue equals to one and a pair of repeated real eigenvalues (ϕ).

The estimation of model can be performed by several procedures: (i) one-step full system by means of Kalman filter, where a measurement error term is added to each yield defined as in (3), and the factors are considered with their dynamics

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[5] Our interest is to compute $\mathrm{Var}(R_n)$, which is a highly non-linear function of the parameters of the VAR model. We perform Monte Carlo simulations of these parameters, based on their asymptotic distribution. In each simulation we compute both P_n and Q_n, accordingly. Using 1000 replications, we compute the required variance-covariance matrix.

under the physical measure, (ii) partial system, where a measurement error term is added to some yields, meanwhile other are assumed to be observed without error; the estimation is performed by considering reduced-form outcomes (OLS and VAR), and (iii) two-step with calibration, where a fix value for ϕ is assumed and there is not measurement error, leading to identify the other parameters by exploiting the dynamics of the factors under the physical measure with a VAR.

We note that the partial system, proposed by Hamilton and Wu (2012), can inform the calibration process and for that we briefly describe it. First, we need to define two sets of yields; in particular, let Z_t be a vector of yields observed without measurement error, and Y_t be the vector of yields observed with measurement errors. Thus, $Z_t = A_1 + B_1\Lambda_t$ and $Y_t = A_2 + B_2\Lambda_t + e_t$, where e_t is a measurement error with zero mean and volatility σ_e which is constant across maturities; also, A_1, A_2, B_1, and B_2 are matrices that depend on the parameters of the risk-neutral measure. Second, noting that $\Lambda_t = B_1^{-1}\left(Z_t - A_1\right)$ can be used in the second set of yields, having a linear relationship:

$$Y_t = \left(A_2 - B_2B_1^{-1}A_1\right) + B_2B_1^{-1}Z_t + e_t. \tag{8}$$

Thus, linear regressions of the yields with errors against the yields without them provide valuable information to identify the parameters ϕ and σ_e. Clearly, the size of the latter depends on the maturities considered in the vector of yield observed with measurement errors. Finally, parameters of the physical measure can be estimated using the set of yields observed without measurement error (set Z_t) and the VAR model ($\Lambda_t = c + H\Lambda_{t-1} + \Sigma U_t$), which can be expressed as follows:

$$Z_t = \left(I - B_1HB_1^{-1}\right)A_1 + B_1c + B_1HB_1^{-1}Z_{t-1} + B_1\Sigma U_t. \tag{9}$$

III. EMPIRICAL APPLICATION

In this section we consider monthly data from January 1980 to December 2024 (T=540) based on the dataset presented in Gürkaynak *et al.* (2007).[6] Using the information there, we extract the data of the last-of-month of the following seven maturities: 3-month, 6-month, 1-year, 2-year, 3-year, 5-year, and 10-year. Table 1 shows basic descriptive statistics for the sample period, for which is clear that the central moments are higher for long-term yields, meanwhile measures of dispersion tend to be higher for short- and medium-term yields.

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[6] The dataset includes the parameters of the Svensson (1994)'s model, which is a well-known extension of Nelson and Siegel (1987)'s model. It includes a second hump to improve fit and the daily information is available at https://www.federalreserve.gov/pubs/feds/2006/200628/200628abs.html.

TABLE 1

DESCRIPTIVE STATISTICS
(PERCENTAGES)

	Mean	Std. Dev.	Min	Max	P50	P75-P25	P90-P10
3m	4.2678	3.5772	0.0293	16.4949	4.3664	4.8822	8.4281
6m	4.3396	3.6046	0.0809	16.2192	4.4193	5.0978	8.7483
1y	4.4618	3.6270	0.0678	16.1100	4.4572	5.1558	9.0303
2y	4.6754	3.6074	0.1132	15.7815	4.4149	5.2223	9.3608
3y	4.8633	3.5523	0.1365	15.5746	4.4642	5.2625	9.3910
5y	5.1848	3.4283	0.2431	15.1776	4.5981	4.9979	9.1959
10y	5.7344	3.2138	0.5517	14.8922	4.9916	4.6961	8.7840

Note: End-of-month data (January 1980-December 2024). Source: Federal Reserve Board.

III.1. Number of factors

To better understand the factors contributing to the total variance of the monthly dataset, we conducted Principal Component Analysis (PCA) on this set of seven yields. PCA results are summarized in Table 2. It suggests that three factors account for almost all the variance, as indicated by the eigenvalues. The first factor explains 97.8% of the total variance, while the second and third factors contribute significantly less. Based on this result, we assume that three yields are assumed to be observed without measurement errors (yields labeled as Z_t). We consider four specifications: A, B, C, and D. Each specification includes, in the set Z_t, one short-term yield (3 or 6 month), one medium-term yield (2 or 5 year), and the 10-year yield as the long-term yield; meanwhile, the other four yields are included in set Y_t. Thus, our 4 specifications are: (A) 3-month, 2-year, and 10-year, (B) 3-month, 5-year, and 10-year, (C) 6-month,

TABLE 2

PRINCIPAL COMPONENT ANALYSIS

Component	Eigenvalue	Proportion	Cumulative
1	6.844	0.978	0.978
2	0.142	0.020	0.998
3	0.012	0.002	1.000
4	0.001	0.000	1.000
5	0.000	0.000	1.000
6	0.000	0.000	1.000
7	0.000	0.000	1.000

Note: Obtained using the correlation matrix of seven yields (3 and 6-month; 1, 2, 3, 5, and 10-year) for the sample January 1980 – December 2024. Source: Authors' calculations based on data from Federal Reserve Board.

2-year, and 10-year, and (D) 6-month, 5-year, and 10-year. These will be used to inform the parameters under the risk neutral measure (by considering linear regressions) and to estimate the parameters under the physical measure (by using VAR models).

III.2. Parameter ϕ (risk-neutral measure)

Table 3 shows, for each specification, the results of linear regressions of the other four yields, which we assumed to be observed with measurement errors (set Y_t), against the three yields described above that are assumed to be observed without measurement errors (set Z_t). First, we note that overall fits (R^2) are above 99% providing empirical support for the three-factor model. Second, each regression has an estimate for ϕ which is obtained by relative difference between the empirical coefficients and the theoretical weights; for instance, the one of the 3-year yield on the set C gives 94.23%, implying the following theoretical weights for the yields of the set: -0.1807 (6-month), 1.0161 (2-year), and 0.1645 (10-year), which are close to regression coefficients reported in the third column of Table 4 (named 3y).[7] We note that: (i) across all sets, minimum and maximum values are 91.61% and 95.87%, respectively, (ii) averages of each specification are 92.87%, 94.98%, 93.67%, and 95.52%, and (iii) overall average is 94.26%. Thus, the calibration introduced by D&L, which fixes ϕ at 94%, has empirical support.

TABLE 3

OLS RESULTS

	Specification A				Specification B			
	6m	1y	3y	5y	6m	1y	2y	3y
3m	0.6834	0.3150	-0.1091	-0.1316	0.8130	0.5595	0.2724	0.1212
	(0.0062)	(0.0064)	(0.0048)	(0.0076)	(0.0064)	(0.0096)	(0.0076)	(0.0043)
2y	0.4040	0.8064	0.9310	0.6225				
	(0.0094)	(0.0097)	(0.0072)	(0.0115)				
5y					0.4696	0.9947	1.3584	1.3590
					(0.0211)	(0.0317)	(0.0251)	(0.0141)
10y	-0.0936	-0.1292	0.1839	0.5183	-0.2850	-0.5576	-0.6322	-0.4809
	(0.0052)	(0.0053)	(0.0040)	(0.0063)	(0.0176)	(0.0264)	(0.0208)	(0.0117)
c	0.0708	0.0879	-0.0783	-0.1362	0.0697	0.1143	0.0951	0.0573
	(0.0100)	(0.0103)	(0.0077)	(0.0122)	(0.0165)	(0.0247)	(0.0196)	(0.0110)

[7] The estimate ϕ of was obtained using a very-fine grid. For each value of ϕ both matrices B_1 (related to yields without measurement errors) and B_2 (related to yields with measurement errors) were computed, and then $B_2 B_1^{-1}$ was compared with the regression coefficients of tables 3 and 4. A minimum distance criteria was used to get the optimal value. Standard errors were obtained using 1000 Monte Carlo simulations of the asymptotic distribution of OLS coefficients, and applying the procedure described above.

	Specification A				Specification B			
ϕ	0.9161	0.9223	0.9356	0.9406	0.9438	0.9479	0.9526	0.9548
	(0.0022)	(0.0018)	(0.0014)	(0.0013)	(0.0018)	(0.0014)	(0.0012)	(0.0010)
R^2	0.9995	0.9994	0.9997	0.9991	0.9987	0.9972	0.9982	0.9994
	Specification C				Specification D			
	3m	1y	3y	5y	3m	1y	2y	3y
6m	1.4009	0.4826	-0.1776	-0.2168	1.1904	0.7123	0.3571	0.1608
	(0.0128)	(0.0054)	(0.0058)	(0.0098)	(0.0094)	(0.0066)	(0.0070)	(0.0043)
2y	-0.5068	0.5909	1.0197	0.7331				
	(0.0181)	(0.0076)	(0.0082)	(0.0139)				
5y					-0.4727	0.6076	1.1429	1.2581
					(0.0290)	(0.0204)	(0.0217)	(0.0133)
10y	0.1103	-0.0768	0.1613	0.4899	0.2807	-0.3189	-0.4979	-0.4178
	(0.0081)	(0.0034)	(0.0037)	(0.0062)	(0.0230)	(0.0162)	(0.0172)	(0.0105)
c	-0.0751	0.0453	-0.0587	-0.1114	-0.0569	0.0487	0.0557	0.0384
	(0.0146)	(0.0061)	(0.0067)	(0.0113)	(0.0201)	(0.0142)	(0.0151)	(0.0092)
ϕ	0.9260	0.9324	0.9423	0.9462	0.9502	0.9545	0.9575	0.9587
	(0.0026)	(0.0018)	(0.0014)	(0.0012)	(0.0021)	(0.0014)	(0.0011)	(0.0010)
R^2	0.9989	0.9998	0.9998	0.9993	0.9981	0.9991	0.9990	0.9996

Note: Each column represents the result of a linear regression between a given yield and the ones expressed in each row. For instance, the first column titled "6m" under the block "Specification A" is the linear regression of yield 6-month against yields 3-month, 2-year, and 10-year plus a constant (named c). In line with Hamilton and Wu (2012), estimate ϕ are obtained by minimizing the distance between the regression coefficients and the theoretical weights. Standard errors are in parentheses and R^2 measures the overall fit.

III.3. Latent Factors and their dynamics

Table 4 shows the estimates from the VAR model for the four specifications. The overall fit for all factors exceeds 83%, with curvature having the lowest fit percentage. Additionally, all VAR models are stable, as indicated by the eigenvalues for the full sample, which are all real with values between 82% and 99%.[8]

[8] Appendix A describes the Jackknife estimator for the VAR model which addresses potential small-sample biases by combining the full-sample estimator with the average estimators derived from the first and second halves of the sample. Empirical results suggest that, in general, the small-sample bias-corrected estimators do not differ, in general, from those shown in Table 4.

TABLE 4

VAR ESTIMATES

	Specification A			Specification B		
	L(t)	S(t)	C(t)	L(t)	S(t)	C(t)
L(t-1)	0.9929	-0.0273	0.0418	0.9913	-0.0250	0.0344
	(0.0052)	(0.0084)	(0.0154)	(0.0056)	(0.0090)	(0.0149)
S(t-1)	0.0204	0.9127	0.0701	0.0194	0.9155	0.0517
	(0.0087)	(0.0140)	(0.0255)	(0.0091)	(0.0147)	(0.0244)
C(t-1)	-0.0010	0.0695	0.8671	0.0018	0.0522	0.9115
	(0.0074)	(0.0120)	(0.0219)	(0.0069)	(0.0111)	(0.0184)
c	0.0724	0.1077	-0.3408	0.0894	0.1128	-0.3346
	(0.0396)	(0.0640)	(0.1168)	(0.0444)	(0.0718)	(0.1193)
λ	0.991	0.962	0.820	0.991	0.962	0.866
R^2	0.9881	0.9221	0.8358	0.9870	0.9189	0.8918

	Specification C			Specification D		
	L(t)	S(t)	C(t)	L(t)	S(t)	C(t)
L(t-1)	0.9916	-0.0265	0.0341	0.9896	-0.0230	0.0321
	(0.0054)	(0.0083)	(0.0140)	(0.0056)	(0.0085)	(0.0148)
S(t-1)	0.0169	0.9149	0.0500	0.0138	0.9238	0.0420
	(0.0090)	(0.0140)	(0.0235)	(0.0092)	(0.0140)	(0.0243)
C(t-1)	0.0027	0.0673	0.8899	0.0061	0.0448	0.9200
	(0.0080)	(0.0124)	(0.0208)	(0.0067)	(0.0103)	(0.0178)
c	0.0790	0.1139	-0.3076	0.0988	0.1135	-0.3347
	(0.0408)	(0.0633)	(0.1067)	(0.0450)	(0.0688)	(0.1193)
λ	0.991	0.960	0.846	0.991	0.962	0.880
R^2	0.9878	0.9264	0.8610	0.9869	0.9250	0.8974

Note: Each block represents the result of a VAR model for the 3 factors: level (L), slope (S), and curvature (C). The latter are obtained using different sets of yields. For instance, under the one titled "Specification B" the following yields are used: 3-month, 5-year, and 10-year. Each row denotes the corresponding lag, and c stands for constant parameter. Standard errors are in parentheses and R^2 measures the overall fit.

III.4. Yields under the Physical Measure and TPs

For a given set of yields observed without measurement error (set Z_t) we have the following expression for yields under the physical measure: $x_{nt} = P_n + Q_n \Lambda_t = \tilde{P}_n + \tilde{Q}_n Z_t$, where the constant is $\tilde{P}_n = \begin{pmatrix} 1 & 1 & 0 \end{pmatrix}\left[\left(I - WR_n W^{-1}\right)(I-H)^{-1} c - WR_n W^{-1} B_1^{-1} A_1\right]$ and the weights for the yields observed without measurement error are $\tilde{Q}_n = \begin{pmatrix} 1 & 1 & 0 \end{pmatrix} WR_n W^{-1} B_1^{-1}$.

Thus, a closed-form expression is available based on eigenvalues and eigenvectors; however, that depends on the set of Z_t (specifications A, B, C or D). For illustration purposes, Table 5 shows the weights for 5 and 10-year yields. For instance, the estimate for the 10-year yield using the specification C is $x_{120t} = 1.445 - 0.09 z_{6t} + 0.739 z_{24t} - 0.122 z_{120t}$

TABLE 5

WEIGHTS TO COMPUTE LONG-TERM YIELD UNDER PHYSICAL MEASURE

	Specification A		Specification B		Specification C		Specification D	
	5y	10y	5y	10y	5y	10y	5y	10y
c	1.046	1.407	1.346	1.674	1.111	1.445	1.458	1.771
3m	0.053	0.025	0.187	0.113				
6m					-0.087	-0.090	0.189	0.098
2y	0.849	0.622			1.006	0.739		
5y			1.532	1.187			1.468	1.157
10y	-0.227	-0.119	-1.061	-0.783	-0.250	-0.122	-1.005	-0.741

Note: Each column has the coefficients associated with each yield, and c is the constant. See text for an example.

Based on the weights presented above, we compute the long-term yields for 5 and 10-years over the sample period (1980-2024); however, these are estimates and therefore proper confidence intervals should be considered.

Considering the specification A, we compute the variance of x_{nt} by using 1000 Monte Carlo simulations as we discussed above. This implies that the variances for the 5-year and 10-year yields are, on average, 45 and 111 basis points, respectively. These figures translate to the TPs estimates, acknowledging that there is uncertainty behind this unobservable concept.

Figures 1 and 2 show TP-ACM and TP-DNS along with 90% confidence intervals under the specification A. There are several elements to conclude from these figures. First, we confirm that TP-DNS tracks the movements exhibited by TP-ACM, which is supported with a high degree of correlation (97.4% and 98.7% for the 5-year and 10-year yields, respectively). Thus, the direction obtained from TP-DNS is consistent with the one observed from TP-ACM. Second, estimates of TP-DNS tend to be slightly higher than TP-ACM, which could be attributed to the missing Jensen term, ignored for simplicity in (1). Additional exercises show that previous two elements are consistent across specifications.[9] Third, confidence intervals for TP-DNS: (i) acknowledge the level of uncertainty of the estimates, notably at the beginning of the sample which could be attributed to the volatility of the short-term part of the yield curve, and (ii) broadly include TP-ACM, noting that the latter is also an estimate.

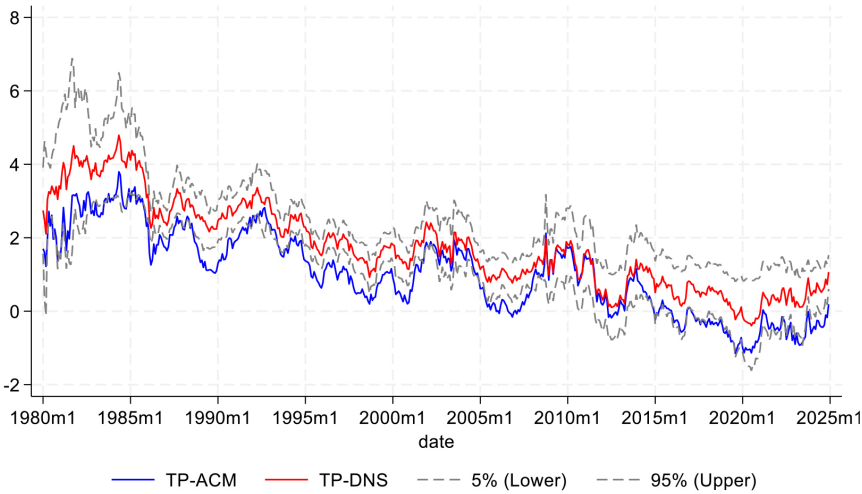
[9] For other specifications correlations are also relevant. For instance, correlations for 5-year yield are 99.1%, 97.4%, and 98.7% for specifications B, C, and D, respectively; meanwhile for 10-year yield are 99.6%, 98.7%, and 99.3%. Respect to levels, similar conclusion is obtained for other specifications. TP-ACM has an average of 1.03 for 5-year and 1.61 for 10-year. Averages for TP-DNS (5-year) are 1.18, 1.18, and 1.06 for specifications B, C, and D, respectively; meanwhile for TP-DNS (10-year) are 1.91, 1.92, and 1.79.

IV. CONCLUSIONS

In this note we propose a simple procedure to estimate the Term Premium for US Treasury bond yields. The procedure relies on the DNS model, which is estimated by a two-step algorithm, and it requires to calibrate just one parameter under the risk-neutral measure. The one proposed by Diebold and Li (2006) is chosen and it is also empirically supported with the estimates of the linear regressions, following Hamilton and Wu's (2012) procedure. Our empirical results show that, for the sample 1980-2024, both Term Premia (TP-DNS and TP-ACM) are relatively similar, although TP-DNS has a slightly higher average. This could be attributed to the Jensen term, which is ignored in our analysis for simplicity, but should be considered for policy discussion whenever that is relevant (e.g., neutral rate).[10] Nevertheless, the procedure is simple, and it can be applied to any other market; however, we note that potential biases can arise from small-sample for which bias-correction methods can be applied directly to the VAR model. Finally, we exploit the linearity of the proposed TP and provide confidence intervals, which are not available for other estimates.

FIGURE 1

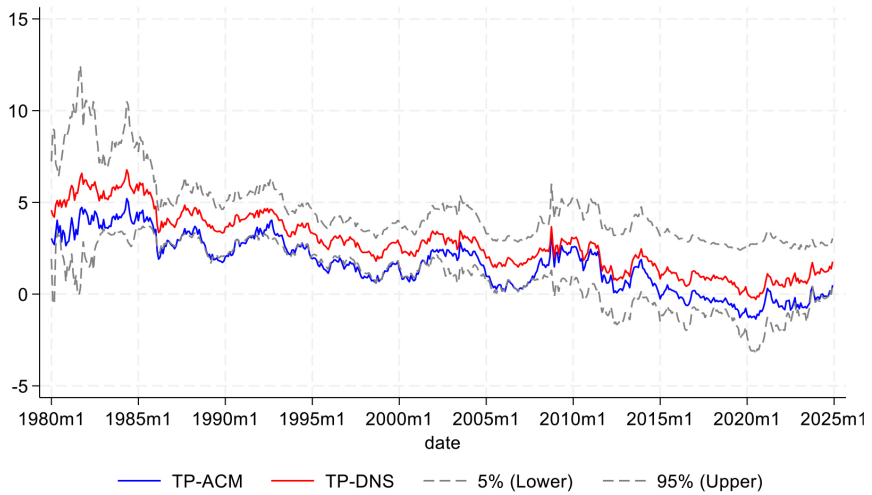
TERM PREMIUM 5-YEAR
(PERCENTAGE)



[10] As it is discussed in Diebold and Rudebusch (2013), the Jensen term in the DNS model depends on variance and covariance parameters. Those will affect the level of the estimates, which in turn also affect our estimates of TPs, but become crucial for the estimates of the neutral rates.

FIGURE 2

TERM PREMIUM 10-YEAR
(PERCENTAGE)



Note: TP-ACM and TP-DNS stand for TP for ACM and DNS models. Lower and Upper lines are obtained using the covariance of the long-term yield under the physical measure. See text for details.

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APPENDIX A

In finite-samples, the parameters of autoregressive models are downward biased, leading to biased term-premia (Bauer *et al.* 2012). The Jackknife estimator can be implemented to reduce that bias, and it is obtained by applying the following procedure: (i) split the sample of T observations in M sub-samples each with L observations, such that $T=ML$, (ii) compute $\widehat{\theta}_T$ as the estimator based on the whole sample and $\widehat{\theta}_{L(i)}$ are the ones based on sub-samples i's, and (iii) compute the Jackknife estimator as:

$$\widehat{\theta}_J = \frac{M}{M-1}\widehat{\theta}_T - \frac{1}{M-1}\left(\frac{1}{M}\sum_{i=1}^{M}\widehat{\theta}_{L(i)}\right). \tag{A.1}$$

For the case of $M=2$, (A.1) collapses to $\widehat{\theta}_J = 2\widehat{\theta}_T - \left(\widehat{\theta}_1 + \widehat{\theta}_2\right)/2$, where $\widehat{\theta}_1$ and $\widehat{\theta}_2$ are the estimators for the first and second half of the sample; meanwhile, for $M=3$ the formula implies $\widehat{\theta}_J = 1.5\widehat{\theta}_T - \left(\widehat{\theta}_1 + \widehat{\theta}_2 + \widehat{\theta}_3\right)/6$.

Table A.1 provides estimates for the case of $M=2$, being broadly similar to the ones presented in Table 4. This is consistent with the fact that the sample size in this case is relatively large; thus, we should expect small changes from the biased-corrected estimator.

TABLE A.1

VAR JACKKNIFE ESTIMATES

	Specification A				Specification B		
	L(t)	S(t)	C(t)		L(t)	S(t)	C(t)
	Set A				Set B		
L(t-1)	1.0015	-0.0586	0.0763		0.9999	-0.0549	0.0595
S(t-1)	0.0293	0.9134	0.0808		0.0218	0.9152	0.0635
C(t-1)	0.0007	0.0512	0.9098		0.0052	0.0325	0.9652
c	0.0390	0.2077	-0.3661		0.0530	0.1841	-0.2296
	Specification C				Specification D		
L(t-1)	0.9995	-0.0567	0.0629		0.9984	-0.0520	0.0547
S(t-1)	0.0220	0.9129	0.0636		0.0135	0.9228	0.0535
C(t-1)	0.0074	0.0470	0.9345		0.0087	0.0254	0.9759
c	0.0499	0.2075	-0.3107		0.0578	0.1796	-0.2104

Note: Each block represents the result of averaging the parameters of VAR models, obtained using different sets of yields. For instance, under "Specification B" the following yields are used: 3-month, 5-year, and 10-year in three VAR models. One for the first half of the sample, one for the second half, and another as reported in Table 4.