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COPPER, FUTURES AND CODELCO*

ROBERT HUSSEY
ILADES/Georgetown University

JORGE QUIROZ
GERENS Ltda. and ILADES/Georgetown University

Abstract

Large financial losses associated with transactions in futures markets has become a common story in the news media. This paper studies the economic dynamics associated with the optimal use of these markets, using the case of the Chilean state copper company Codelco as an example. Between November 1993 and January 1994, Codelco lost approximately US$178 million in futures markets. The question arises of whether such occasional large losses are typical of transactions in futures markets or, in this case, due to error or inefficient management. This paper addresses the question by studying a maximization problem relevant for a firm such as Codelco. In the model, the firm chooses its operations in futures markets subject to stochastic processes estimated for spot and future prices for copper. Results indicate that the use of futures contracts does result in higher average income, but it occasionally generates significant losses over short periods of time. Nevertheless, the model does not generate large losses during the November 1993 to January 1994 period of the Codelco losses. The results also demonstrate that profits generated by using futures are a direct result of the intrinsically nonlinear nature of the stochastic processes of spot and future prices.

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I. Introduction

Large financial losses associated with transactions in futures markets has become a common story in the news media. Recent well known cases have involved large international firms such as Sumitomo of Japan, with losses on copper markets, Barings Investment Bank of England, with losses on currency markets, and Codelco, the Chilean state-owned copper company, also with losses in copper futures.[1] The record of financial disasters associated with futures transactions raises the question of whether occasional significant losses are typical of transactions in futures markets. Are the sort of cases mentioned above the result of poor trading strategies, or are they a necessary consequence of continuous active trading on futures markets?

This paper addresses that question by studying the economic dynamics associated with the optimal use of futures markets, using the case of Codelco as an example. Between November 1993 and January 1994, Codelco lost approximately US$ 178 million in copper futures. As a consequence, policies were implemented to limit the state-owned company's use of those markets. Although officials of Codelco are currently under investigation for criminal wrong doing, public discussion at the time lacked information as to whether losses generated by the transactions were typical for those types of operations, or whether they were due to management incompetence or fraud. Clearly, policy implications in each case are completely different.

In this paper, we develop an optimization problem relevant for a firm such as Codelco. The firm's objective is to maximize expected income while maintaining a certain aversion to risk. We also consider an alternative objective function that penalizes the occurrence of large losses in futures markets, which may be a relevant factor for a state-owned enterprise such as Codelco. Optimization of the model amounts to selecting a strategy for transactions in futures markets, subject to stochastic processes estimated for spot and future prices for copper. After estimating those processes, the model is resolved numerically for a decision rule that maps the spot prices, futures prices, and inventory levels into optimal positions on the futures market. Empirically evaluating that strategy allows one to estimate the dynamics of expected returns associated with the use of futures markets.

The primary results are as follows:

i) The use of futures contracts does achieve higher values of the objective function. With the limited use of futures considered in this paper, expected income increases by approximately 5.4% in the absence of transaction costs. If transaction costs are 2%, expected income increases by approximately 1.5%. However, in the case without transaction costs, the increase in income is accompanied by an increase in the variance of monthly revenues. When transaction costs are significant and optimal futures trading is more conservative, the lower increase in average income is accompanied by a smaller increase in volatility. In some additional cases with restricted trading, however, volatility is actually reduced at the same time that a small increase in expected income is achieved.

ii) Profits derived from using futures are a direct result of the intrinsically nonlinear nature governing the stochastic processes of prices. If the nonlinearity is ignored, potential gains are significantly reduced.

iii) Despite increases in average income associated with the use of futures markets, solutions to the model indicate that futures transactions generate significant losses from time to time. However, for the period November 1993 to January 1994 when Codelco experienced its large losses, the model generates profits from futures transactions. It is interesting to note that for the period immediately prior to this episode, the model shows significant losses under certain assumptions. The reason for those losses is that, even taking into account the nonlinearity of stochastic processes governing prices, there were two large negative shocks in the spot price of copper during that period. It is possible to generate losses over a 3-month interval during that period that approach the $ 178 million of Codelco, but only under the assumption that there are no transaction costs to futures trading, since much longer positions are taken in the market in that case. Even in that situation, however, the periods surrounding that particular interval show large profits from futures trading. In the presence of transaction costs, the losses during that 3-month period are much smaller.

The remainder of the paper is organized as follows. The following section formulates the maximization problem, and describes the strategy to solve the problem. Section 3 focuses on the empirical characterization of stochastic processes governing prices. Section 4 presents the main results derived from the model. The final section provides concluding remarks and briefly explores possibilities for extending this research.

II. An Optimization Model

2.1 The maximization problem

We consider a monthly decision model in which a firm is endowed in each month t with a production flow w_t that can be sold at a spot price s_t. The firm must decide how much of its product to sell in futures markets. In order to simplify the problem, we assume that the firm operates only in 3-month futures. We denote the number of futures contracts sold as f_t (negative values correspond to purchases), with a delivery price d_t, known at time t. Consequently, the firm's cash flow in the month t is given by:

$$x_t = s_t\,(w_t - f_{t-3}) + d_{t-3} f_{t-3} \qquad (1)$$

In each month, the firm chooses the number of futures contracts f_t to maximize a function of the expected cash flow three months hence when the contracts mature

$$\max_{f_t} \mathrm{E}_t[v(x_{t+3})] \qquad (2)$$

where $E_t[\cdot]$ denotes the conditional expectation in month t and $v(\cdot)$ is a function to be specified below. If the firm is risk neutral, $v(\cdot)$ is the identity function, and there will be no interior solution to the problem in the sense that maximization will imply an infinite sale (purchase) of futures whenever the futures price is above (below) the expected spot price in the future. The functional form chosen for $v(\cdot)$ will determine the willingness of the firm to trade increases in expected income for increases in risk.

Equation (2) is maximized with respect to f_t subject to equation (1) and the laws of motion governing the exogenous variables w_t and s_t.[2] For simplicity we also assume that the two exogenous variables are independent. The stochastic law of motion for w_t is specified as a function of its own lags as

$$w_t = g(w_{t-1}, w_{t-2}, \ldots, w_{t-L_w}) + u_t \tag{3}$$

where u_t is a mean zero shock. The stochastic law of motion for the spot price is specified as a function of its own lags and lags of the futures price, d_t, and of world inventory stocks of the good, i_t

$$s_t = h(z_{t-1}, z_{t-2}, \ldots z_{t-L_z}) + \eta_t \tag{4}$$

where $z_t = (s_t, d_t, i_t)$ and η_t is a mean zero shock.

To solve the model, estimates are needed for distributions of w_{t+3} and s_{t+3} conditional on information in period t. In the case of w, it is straightforward to generate this distribution using equation (3) and the law of motion for u_t. The situation is more difficult for s since equation (4) implies that s_{t+3} depends on d_{t+2}, d_{t+1}, and i_{t+1}, and we do not develop models for the laws of motion of d and i. Instead of equation (4), we work with an alternative representation of the law of motion for s_t

$$s_t = H(z_{t-3}, z_{t-3}, \ldots, z_{t-3-L_z}) + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} \tag{5}$$

where ε_t is a mean zero shock. The MA(2) structure comes from the overlap in the data generated by using monthly data to predict the distribution of a variable three months in the future.

Conditional on estimations of equations (3) and (5) and on values of the explanatory variables in those equations, the maximization model will choose the amount of futures contracts, f_t, in each period that maximizes the objective function (2).

2.2 The solution strategy

Conceptually, the strategy for the solving the optimization problem is simple since there are no endogenous state variables. An optimal strategy must satisfy the Euler equation:

$$E_t\left[\frac{\partial v(x_{t+3})}{\partial f_t}\right] = E_t[v'(x_{t+3})\,(d_t - S_{t+3})] = 0 \tag{6}$$

Equation (6) is a fixed point equation in f_t. Solving for the optimal quantity of futures contracts amounts to finding the value of f_t that sets the conditional expectation in equation (6) equal to zero. The conditional expectation as a function of f_t can be expressed as a series of integrals with respect to the shocks u and ε in periods $t+1$, $t+2$, and $t+3$. Those integrals can be approximated numerically given equations (3) and (5) and the laws of motions of the shocks. That is, given estimates of equations (3) and (5), the solution to equation (6) gives an optimal decision rule for f_t, as a function of the state variables, which are w_t, and z_t and their lags. Thus the solution strategy consists in empirically estimating (3) and (5), and then numerically solving equation (6) for each month in which a decision is made.

2.3 The objective function

Two types of specifications of the function $v(\cdot)$ are considered. The first can be thought of as applying to a private firm. In that case, the firm desires to maximize the expected discounted sum of its profits, but it also has some aversion to risk. The second type of specification might correspond to a state-owned enterprise, such as Codelco. As in the first specification, the objective function of the firm reflects a desire to limit risk, but it also includes a penalty for cash flows below some pre-established level. The intuitive idea behind this penalty is that public opinion will tend to assess the *ex post* performance of a state-owned firm and punish losses more than it rewards gains, especially when they are associated with less conservative trading strategies.

Specifically, the objective function takes the form

$$v(x_t) = \frac{x_t^{1-\gamma}}{1-\gamma} - c(\max\{\hat{x}_t - x_t, 0\}) \tag{7}$$

The first term in the function corresponds to a standard constant relative risk aversion utility function. The objective function for the private firm would include only this term. For the state-owned enterprise, a second term is added that accounts for the idea of penalizing deviations below a preconceived level $\hat{x}_t$. The value of $\hat{x}_t$ may be constant or may vary over time. Given the institutional framework in Chile, it seems reasonable to let this value fluctuate with the copper price: public opinion is able to understand Codelco's low profits when copper prices are low, but it will penalize low profits in the presence of higher prices.

III. Estimating the Stochastic Processes

An original sample of monthly observations from 1981:01 through 1995:11 was constructed to estimate equations (3) and (5). Initial observations were excluded to be used as lagged variables in the estimation in such a way that the first selection of f_t by the model would correspond to 1982:01. This truncation implies that the equations were estimated with 164 observations on the dependent variable. The data and the estimation of the equations are described below.

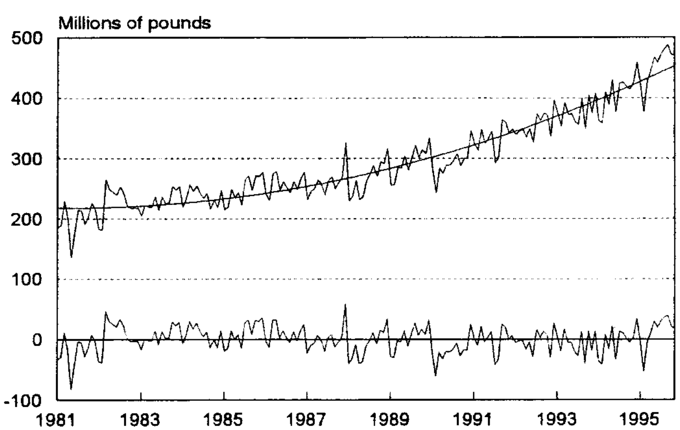
3.1 Production

Codelco's copper production has exhibited a clear increasing trend over time. An upward trend in the production data, however, is not desirable for estimating of the value of using futures since it would make it difficult to compare profits or losses from different periods. To correct for this problem, the series is adjusted by first extracting a quadratic trend and then recentering the data to have a mean of 400 million pounds, which corresponds approximately to Codelco's level of monthly production around the time of its losses at the end of 1993 (Figure 1). Thus we study the use of futures by a firm with an exogenous production stream with average monthly production of 400 million pounds of copper and some stochastic volatility around the mean. This scaling of production allows us both to compare gains and losses across time in the sample and to compare the results from the model with Codelco's performance on futures markets for the period around the end of 1993.[3]

After the detrending, equation (3) is estimated on the production data. The data is found to be well represented by a Gaussian autoregression with lags for months 1 through 6 and 12. Because of the exogenous nature of production in the model, including a large number of lags in the specification does not create additional computational expense in solving the model.

FIGURE 1

MONTHLY PRODUCTION



3.2 Spot prices

The data for estimating equation (5) consist of observations on the spot price of copper, the 3-month futures price, and inventories. The price data correspond to observations from the last day of the month on the London Metals Exchange, deflated by the U.S. producer price index. The inventories series corresponds to the sum of stocks of copper in the inventories of the metals exchanges in London and New York, also on the last day of the month. Graphs of the data appear in Figures 2 through 4.

Two possible specifications are reported here for equation (5), the law of motion for the spot price. The first is a Gaussian ARMA model with 3 lags that takes the form

$$\ln s_t = \alpha_0 + \sum_{j=3}^{5} [\alpha_{1j} \ln s_{t-j} + \alpha_{2j}(\ln s_{t-j} - \ln d_{t-j}) + \alpha_{3j} i_{t-j}] + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} \qquad (8)$$
$$\varepsilon_t \sim N(0,\sigma^2) \quad E[\varepsilon_t \varepsilon_s] = 0,\ t \neq s$$

The variable $(\ln s_{t-j} - \ln d_{t-j})$ on the right hand side of equation (8) is the difference between the logs of the spot and future prices. We choose to use this variable instead of the log of the futures price because the spot and futures are highly correlated, as one can see in Figures 3 and 4. Econometrically, this choice of variables does not offer any advantage in the estimation of equation (8), but it does become useful when we consider the second specification described below. The estimate of the specification in equation (8) is reported in Table 1 with the label "ARMA model."[4]

FIGURE 2

INVENTORIES

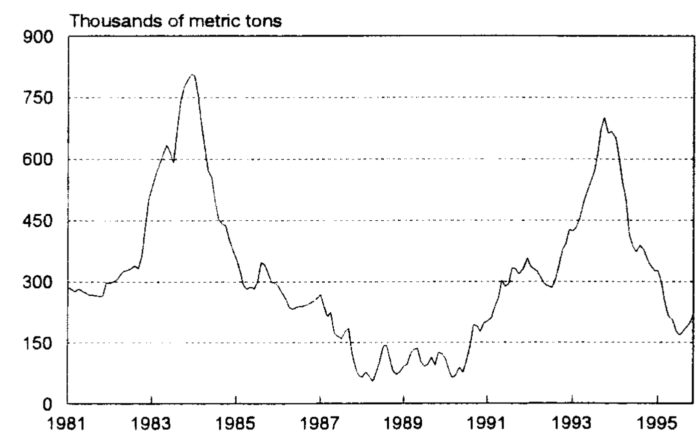


FIGURE 3

REAL SPOT PRICE

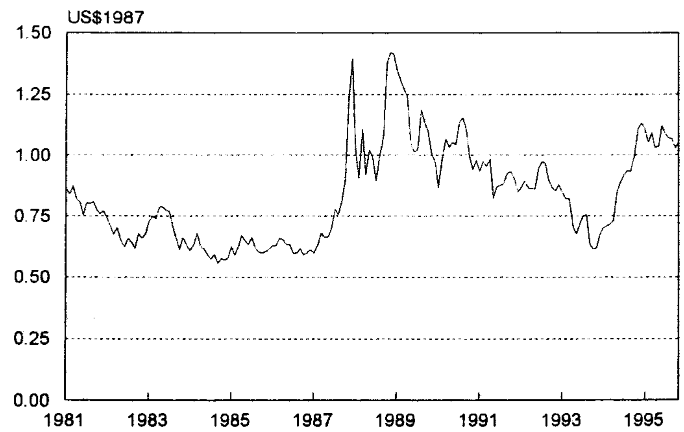


FIGURE 4

REAL FUTURES PRICE

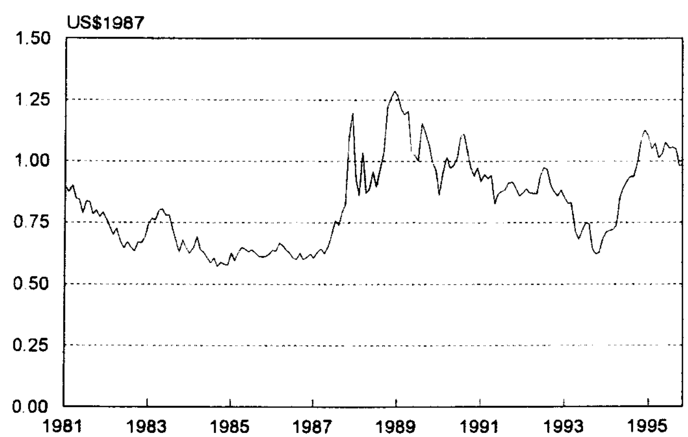


TABLE 1

ESTIMATES OF ARMA AND ARCH MODELS

Dependent variable: $\ln s_t$	ARMA model Conditional Mean	ARCH model Conditional Mean	ARCH model Conditional Std Dev
Intercept	1.2829 (1.9046)	1.6842 (4.9014)	0.1901 (1.3640)
$\ln s_{t-3}$	0.7754 (9.3101)	0.7377 (13.6951)	-0.0627 (-2.3924)
$\ln s_{t-3} - \ln d_{t-3}$	-0.6236 (-1.7964)	0.5446 (1.6810)	1.0349 (3.2061)
$\ln i_{t-3}$	0.0975 (0.0462)	0.1277 (4.3849)	0.0301 (3.9747)
$\ln s_{t-4}$	-1.0164 (-0.2295)	-0. 1670 (-3.4580)	
$\ln s_{t-4} - \ln d_{t-4}$	-0.3459 (-1.1169)	1.4232 (5.2255)	0.7129 (1.5941)
$\ln i_{t-4}$	-0.1368 (-4.1072)	-0.0732 (1.0394)	
$\ln s_{t-5}$	0.0161 (1.2423)	0.0496 (1.0394)	
$\ln s_{t-5} - \ln d_{t-5}$	0.7672 (2.8259)	1.3737 (6.2938)	0.6250 (2.3864)
$\ln i_{t-5}$	-0.0117 (-0.2274)	-0.0533 (-2.0168)	
ε_{t-1}	1.0687 (15.5578)	1.1783 (36.9506)	
ε_{t-2}	0.7750 (9.8744)	0.8790 (35.3875)	
$\lvert\varepsilon_{t-1}\rvert$			-0.3086 (-5.2398)
$\lvert\varepsilon_{t-2}\rvert$			-0.2120 (-2.8900)
σ^2	0.0681		
Log Likelihood	276.8078	290.5271	
Schwarz Information Criterion	-2.8251	-2.9211	
Akaike Information Criterion	-3.2294	-3.5430	

The adequacy of this specification is evaluated by examining properties of the estimated residuals. The statistics in Table 2 indicate that one cannot reject the null hypothesis, that the skewness of the unconditional distribution of the residuals is zero, but one would reject the null hypothesis that the kurtosis is three, the value of kurtosis of the normal distribution. Thus the residuals do not satisfy the assumption of normality in the specification. The deviation from normality can also affect the results of the model solution strategy since that strategy assumes normality in evaluating the conditional expectations in equation (6).

We also test for remaining dependence of the residuals on the past by regressing the estimated residuals and their squares on functions of lagged values of z_t. Table 2 reports the significance of the F-statistics from those regressions. The first statistic in the column labeled "residuals" corresponds to the significance level of a regression of the residuals on a constant and lags 3 through 5 of the variables ($\ln s_t$, $\ln s_t - \ln d_t$, $\ln i_t$). The second statistic in the column is from a regression on the same set of variables plus their squares. The values of the statistics indicate that in both cases one would reject the null hypothesis that the regression is significant. The column labeled "squared residuals" reports the statistics from similar regressions except in this case the dependent variable is the squared estimated residuals. The lower values of the statistics indicates that there is evidence of remaining dependence on the past in the variance of the residuals.

Given the problems with the ARMA specification, an alternative specification is considered that takes the form of an ARCH model (Engle, 1982). In this model, the specification of the conditional mean of $\ln s_t$ is equivalent to that in equation (8), but the heteroskedasticity in the variance is modeled as depending on the past:

$$\ln s_t = \alpha_0 + \sum_{j=3}^{5}[\alpha_{1j}\ln s_{t-j} + \alpha_{2j}(\ln s_{t-j} - \ln d_{t-j}) + \alpha_{3j}i_{t-j}] + \varepsilon_t + \theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2}$$

$$\varepsilon_t \sim N(0, r_t^2) \tag{9}$$

$$r_t = \delta_0 + \delta_1|\varepsilon_{t-1}| + \delta_2|\varepsilon_{t-2}| + \delta_3\ln s_{t-3} + \delta_4(\ln s_{t-3} - \ln d_{t-3}) + \delta_5 i_{t-3}$$

$$+ \delta_6(\ln s_{t-4} - \ln d_{t-4}) + \delta_7(\ln s_{t-5} - \ln d_{t-5})$$

In the most common specification of ARCH models, the variance of the residuals is specified as a function of lags of the residuals squared. However, the presence of conditional heteroskedasticity does not necessarily mean that the dependence of the variance on the past must assume that form (Gallant, Hsieh, and Tauchen, 1991). In the above model, the specification of the variance includes lags of the original series. This specification performed better than one with only lagged residuals.

The estimate of the model in equation (9) appears in Table 1. Both the Schwarz and Akaike model selection criteria prefer the ARCH model over the ARMA model. Analysis of the standardized residuals in Table 2 also indicates that their

TABLE 2

ANALYSIS OF RESIDUALS

	standard deviation	skewness[1]	kurtosis[1]	significance of F-statistic[2]	
				residuals	squared residuals
ARMA	0.0683	0.4008 (0.4417)	5.6923 (1.0620)	0.8685 / 0.6834	0.0575 / 0.1433
ARCH[3]	0.9941	0.1010 (0.1455)	2.8610 (0.2705)	0.2139 / 0.4675	0.6814 / 0.6090

[1] Consistent standard errors in parentheses.
[2] Significance of the F-statistic from regressions of the residuals or the squared residuals first on lags 3, 4, and 5 of $\ln s_t$, $\ln s_t - \ln d_t$, and $\ln i_t$ and, secondly on those variables and their squares.
[3] The statistics for the ARCH model are calculated using the standardized residuals.

behavior is consistent with the assumptions of the model. One would not reject null hypotheses that the unconditional distribution of the standardized residuals has a skewness of zero and a kurtosis of 3. Likewise, regressions with the residuals and squared residuals indicate that they retain little dependence on the past.

IV. Results From the Optimization Model

4.1 Benefits from optimal use of futures markets

The optimization model was solved using the actual data as the values of the state variables. In each period, a hypothetical decision maker chooses a quantity of 3-month futures contracts f_t given the estimate of equation (9) and the corresponding current and lagged values of production, spot price, futures price, and inventories. The returns to these choices relative to using only the spot market are evaluated in this section under various scenarios.

The parameters of the function v(·) in equation (7) were set as follows. The relative risk aversion coefficient γ was set equal to 1.5 based on previous method of moments estimates for Chile (Arrau, 1990). The penalty term of the function was parameterized as a constant multiplied by the deviation of x_t below the threshold value $\hat{x}_t$. We have no way of determining what the actual value of that constant might be for a state owned company, so we select a value that will affect the volume of futures trading, but that will not shut down trading completely. We then interpret our results as representative of the effects of this *type* of penalty in the utility function, with the caveat that the effects may be of larger or smaller magnitude depending upon whether the true penalty is more or less strict than the value we have chosen. If revenue is expressed in units of $10 million, a reasonable value for the constant is 0.025, so the penalty function part of equation (7) is parameterized as

$$c(\max\{\hat{x}_t - x_t, 0\}) = 0.025 \cdot \max\{\hat{x}_t - x_t, 0\} \tag{10}$$

Figure 5 provides a sense of the magnitude of the penalty implied by this specification. The figure shows an example of the utility function with and without the penalty for the case in which $\hat{x}_t = 40$.

The model was solved under two definitions of the threshold penalty value $\hat{x}_t$. In the first, $\hat{x}_t$ is equal to the revenue that would be obtained in period t if all of the production were sold on the spot market. In the second, $\hat{x}_t$ is equal to the revenue that would be obtained if all of the production were sold at a price of $.85 per pound.

The returns from using futures markets are summarized in Table 3 for a series of scenarios. In the first scenario futures markets are not used (i.e., the entire production is sold at the spot price); in the second scenario futures transactions are allowed and there is no penalty in the objective function; in the third and fourth scenarios futures contracts are used, but there is a penalty in the utility function corresponding respectively to the two specifications of equation (10) described above. In all cases, the returns reported are based on the constraint that the quantity of futures contracts transacted not exceed ± 3 times the current production level. This constraint prevents the firm from taking very long positions in futures markets. We discuss below the extent to which this constraint is binding.

The first section of Table 3 shows the results when there are no transaction costs. When the firm trades in futures without a penalty, income is 5.4% higher across the sample than when only the spot market is used (line 2 of Table 3). If there is a penalty in the utility function, the futures trades that are chosen result

FIGURE 5

UTILITY FUNCTIONS
example for threshold penalty value = 40

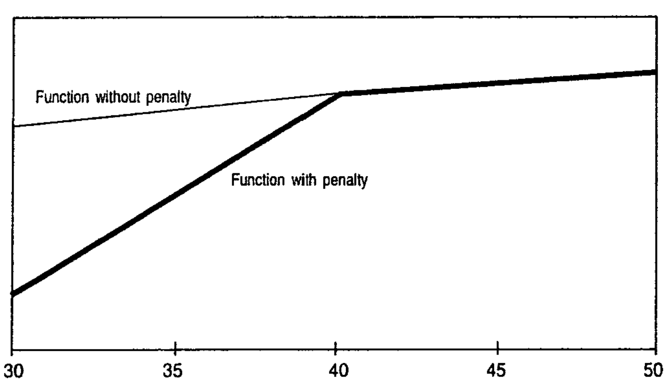


TABLE 3

RESULTS USING ARCH MODEL
RELATIVE RISK AVERSION COEFFICIENT 1.5

	Relative Total Income	Coefficient of Variation of Monthly Income	Skewness of Monthly Income	Minimum Relative Income	Maximum Relative Income
Without Transaction Costs					
1) Sales only in spot market	1.000	0.257	0.890	1.000	1.000
2) Without penalty	1.054	0.358	2.877	0.524	1.927
3) With penalty at the spot price	1.021	0.257	0.854	0.896	1.391
4) With penalty at a price of $.85	1.031	0.253	0.881	0.707	1.482
With Transaction Costs of 2%					
5) Sales only in spot market	1.000	0.257	0.890	1.000	1.000
6) Without penalty	1.015	0.309	2.389	0.728	1.583
7) With penalty at the spot price	1.007	0.255	0.826	0.944	1.286
8) With penalty at a price of $.85	1.000	0.243	0.589	0.788	1.302
Without Transaction Costs - Transactions only with Physical Backing					
9) Without penalty	1.004	0.240	0.810	0.788	1.259
10) With penalty at the spot price	1.002	0.255	0.886	0.987	1.113
11) With penalty at a price of $.85	1.000	0.236	0.758	0.788	1.259
With Transaction Costs of 2% - Transactions only with Physical Backing					
12) Without penalty	1.003	0.240	0.812	0.788	1.259
13) With penalty at the spot price	1.002	0.255	0.886	0.983	1.113
14) With penalty at a price of $.85	1.000	0.236	0.759	0.788	1.259

in smaller income gains. When the penalty is based on the spot price, average income is 2.1 % greater (line 3). When the penalty is based on a price of $.85, average income rises 3.1% with the use of futures (line 4). Note that in the case of no penalty, the rise in income is accompanied by an increase in the coefficient of variation of monthly earnings.[5]

Table 3 shows that the optimal trading strategy may sometimes generate large losses relative to trading entirely in the spot market. For the scenario without transaction costs and without a penalty in the objective function, the month with minimum relative income has a return of 52% of what would have been obtained if futures transactions had not been used (line 2, column 4). On the other hand, the maximum relative revenue achieved in the sample results in income 92.7% greater than would have been obtained in the absence of futures transactions (column 5). Figure 6a shows relative incomes for each period for the case without a penalty. (The spot price is also plotted in the figure on a different scale to give the reader an idea of what movements in the price correspond to the gains and losses).

When the objective function has a penalty based on the spot price, the optimum number of futures contracts is zero in many periods. This can be seen in Figure 6b where the *ex post* relative income is equal to one for many periods. Gains from using futures markets are still positive across the sample (2.1%), but less than in the case without the penalty. Even with the reduction in the quantity of futures transactions due to the penalty, there are still relative losses occasionally: relative income reaches a minimum value of 89.6% (line 3, column 4 of Table 3). When the objective function has a penalty based on a price of $.85, average benefits are higher (3.1%), but relative monthly losses also sometimes reach higher levels, with a minimum relative income of 70.7% (line 4 of Table 3 and Figure 6c).

The second section in Table 3 shows the benefits from using futures when there are transaction costs of 2%. The level of transaction costs is chosen somewhat arbitrarily, since brokers generally do not specify transaction costs separately from the cost of the good, but quote a contract price including these costs (Del Solar, 1994). When the model is solved with transaction costs, it is assumed that the costs apply to both spot and futures contracts.[6]

Including transaction costs reduces the return to using futures markets to 1.5% for the case without penalty (line 6), to 0.7% for the case with the penalty based on the spot price (line 7), and to zero for the case with the penalty based on a price of $.85 (line 8). The extremes of relative monthly income gains and losses are also reduced (columns 4 and 5) since transaction costs in general result in taking fewer long positions in futures markets.

Several comments are necessary for interpreting the magnitudes of gains and losses reported in Table 3.

a) One should not expect large profits in the long run from using futures markets since this would indicate a major imperfection in those markets.
b) In the optimization model developed here, the ability of a firm to use futures markets has been drastically restricted. This fact suggests that incorporating other possibilities for operation in futures would reinforce the conclusion that futures transactions can increase a firm's income. For instance, the model does not consider the possibility of a firm using simultaneously with the 3-month futures either options or futures contracts with different maturities. The trading strategy developed here also does not allow the firm to cancel a futures contract prior to its maturity. Each of these alternatives would be an important means to limit the losses associated with the use of futures markets. Given that the firm in the model has been restricted in this way, the benefits reported

FIGURE 6a

RELATIVE INCOME (WITH/WITHOUT FUTURES)
without penalty & without transaction costs

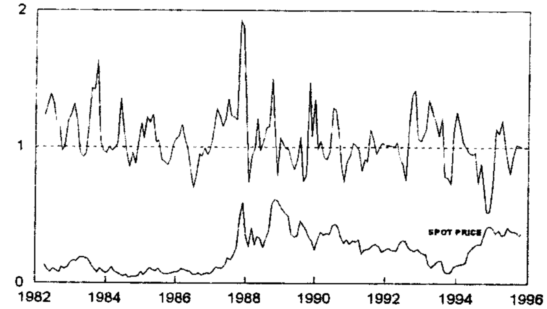


FIGURE 6b

RELATIVE INCOME (WITH/WITHOUT FUTURES)
without penalty at spot price & without transaction costs

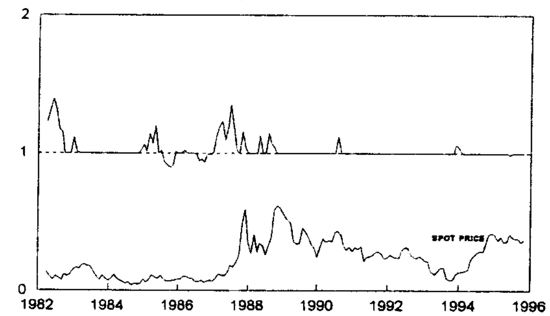
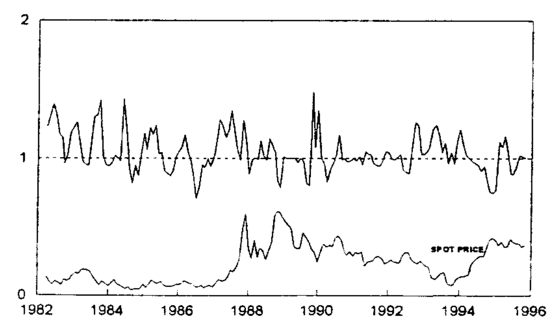


FIGURE 6c

RELATIVE INCOME (WITH/WITHOUT FUTURES)
with penalty at price $.85 & without transaction costs



here may be considered lower limits with respect to those that could be obtained on these markets.

c) Increases in income are reported here as a percentage of the spot sales, which results in lower values than if they were measured as a percentage of company profits.

The reported increase in the variance of income resulting from using futures markets should also be interpreted with caution. First, the increase in variance is accompanied by a rise in the value of the right skewness of the distribution of monthly income. Thus a large part of this increase in variance is due to a larger right tail in the income distribution, that is, to more observations of larger monthly incomes. Secondly, the decision maker chooses to accept this increase in variability in return for increased average earnings, under the assumption of a relative risk aversion factor of 1.5. Thus, to a certain extent the results reflect a standard tradeoff between risk and return. However, it is important to note that the model does not force this kind of tradeoff. First, the model does not assume that futures markets are efficient but rather takes the stochastic processes governing prices as given empirically. If there is imperfection in those markets, it is conceivable that a trader could achieve an increase in return simultaneously with a reduction in risk. Secondly, because the agent in our model enters the market with an endowment of the good to be traded, not all of her trading on futures markets takes the form of speculation. Any volume of futures contracts sold between zero and the quantity of the endowment does not amount assuming speculative risk, but rather to trading one kind of risk for another kind-risk of fluctuation of the spot price in the future for basis risk (the risk of fluctuation is the difference between the current futures price and the future spot price). In this exchange of one kind of risk for another, it is possible that one would achieve both an increase in average income and a reduction in the variance of income. In some of the examples reported in the tables, one observes an increase in average return along with a reduction in the coefficient of variation of returns.

The experiment of including a penalty in the objective function shows that an aversion to deviation of the cash flows below a threshold level results in lower increases in income from using futures markets. In every case, the return with the penalty is less than without the penalty. To the extent that a state-owned enterprise is subject to this type of penalty due to a public scrutiny that is inherent to its status, the results suggest that public ownership limits the firm's ability to benefit from futures markets.

Next we consider the use of the futures under three alternative scenarios. For the first, futures transactions were limited to the operations with physical backing. More specifically, the model was solved to determine the optimal number of futures contracts in each month, subject to the constraint that futures sales be nonnegative (the speculative *purchase* of futures was forbidden) and less than the expected level of production three months in the future. The results appear in the last two sections of Table 3.[7] The use of futures results in a small increase in average income (0.4% in the case without a penalty) and a reduction in the variance of monthly income.

For the second alternative scenario, we examine the sensitivity of the results to the value of the relative risk aversion coefficient. Table 4 reports the results from futures trading when there is significantly greater aversion to risk in the objective function ($\gamma = 3.0$). As one would expect relative to Table 3 with $\gamma = 1.5$, the results show lower increases in income from futures trading, lower coefficients of variation of income, and less extreme values for minimum and maximum relative income.

For the last scenario, Table 5 shows the results that would be obtained if the specification problems of the ARMA model for the spot price are ignored and it

TABLE 4

RESULTS USING ARCH MODEL
RELATIVE RISK AVERSION COEFFICIENT = 3.0

	Relative Total Income	Coefficient of Variation of Monthly Income	Skewness of Monthly Income	Minimum Relative Income	Maximum Relative Income
Without Transaction Costs					
1) Sales only in spot market	1.000	0.257	0.890	1.000	1.000
2) Without penalty	1.029	0.269	1.539	0.645	1.482
3) With penalty at the spot price	1.003	0.255	0.872	1.000	1.318
4) With penalty at a price of $.85	1.014	0.218	0.190	0.695	1.454
With Transaction Costs of 2%					
5) Sales only in spot market	1.000	0.257	0.890	1.000	1.000
6) Without penalty	1.005	0.247	0.979	0.769	1.302
7) With penalty at the spot price	1.002	0.256	0.881	1.000	1.237
8) With penalty at a price of $.85	0.991	0.227	0.282	0.695	1.302
Without Transaction Costs - Transactions only with Physical Backing					
9) Without penalty	1.001	0.237	0.762	0.767	1.259
10) With penalty at the spot price	1.001	0.256	0.889	1.000	1.113
11) With penalty at a price of $.85	0.992	0.221	0.454	0.695	1.259
With Transaction Costs of 2% - Transactions only with Physical Backing					
12) Without penalty	1.001	0.237	0.764	0.769	1.259
13) With penalty at the spot price	1.001	0.256	0.889	1.000	1.105
14) With penalty at a price of $.85	0.992	0.221	0.458	0.695	1.259

TABLE 5

RESULTS USING ARMA MODEL
RELATIVE RISK AVERSION COEFFICIENT = 1.5

	Relative Total Income	Coefficient of Variation of Monthly Income	Skewness of Monthly Income	Minimum Relative Income	Maximum Relative Income
Without Transaction Costs					
1) Sales only in spot market	1.000	0.257	0.890	1.000	1.000
2) Without penalty	1.038	0.329	2.445	0.449	1.617
3) With penalty at the spot price	1.011	0.286	1.801	0.934	1.617
4) With penalty at a price of $.85	1.018	0.280	1.927	0.561	1.617
With Transaction Costs of 2%					
5) Sales only in spot market	1.000	0.257	0.890	1.000	1.000
6) Without penalty	1.010	0.275	1.666	0.725	1.506
7) With penalty at the spot price	1.008	0.279	1.568	1.000	1.506
8) With penalty at a price of $.85	1.004	0.263	1.586	0.725	1.506
Without Transaction Costs - Transactions only with Physical Backing					
9) Without penalty	1.003	0.240	0.793	0.743	1.259
10) With penalty at the spot price	1.000	0.257	0.890	1.000	1.000
11) With penalty at a price of $.85	0.998	0.237	0.729	0.743	1.259
With Transaction Costs of 2% - Transactions only with Physical Backing					
12) Without penalty	1.003	0.240	0.794	0.743	1.259
13) With penalty at the spot price	1.000	0.257	0.890	1.000	1.000
14) With penalty at a price of $.85	0.998	0.237	0.749	0.743	1.259

is used to represent the law of motion for s_t in solving the model. (Table 5 is comparable with Table 3 since γ has been reset to 1.5). The results indicate that the gains from using futures markets are significantly reduced when one does not account for the deviations from conditional homogeneity and normality in the stochastic process for prices.

As was mentioned above, all the results reported are based on the constraint that the quantity of futures contracts transacted not exceed $\pm$ 3 times the current production level. This constraint prevents the firm from taking very long positions in futures markets. When there are no transaction costs and only a modest amount of risk aversion ($\gamma = 1.5$) the solution hits the upper bound 26 times and the lower

bound 25 times, out of 164 possible occurrences.[8] Figure 7a graphs the number of futures purchased. One can see that most of the periods in which the solution hits the bounds, the spot price is going through a stable phase and is usually at a low level. Both of these factors mean that the uncertainty in income created by buying or selling futures contracts is very low. Thus it is not surprising that the solution opts for long positions on futures markets at these times. The one exception to the pattern of hitting the boundry when the price is stable is during 1994 when the price was steadily rising. When transaction costs are introduced, the solution hits the upper bound 7 times and the lower bound 4 times (Figure 7b). This reduction in the importance of the constraint reflects the additional cost of taking long positions that is implied by the existence of transaction costs.

FIGURE 7a

FUTURES CONTRACTS AND PRODUCTION
without penalty & without transaction costs

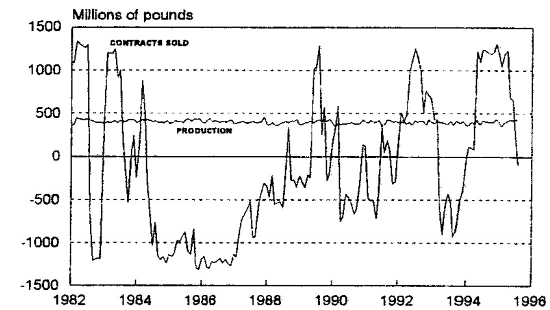
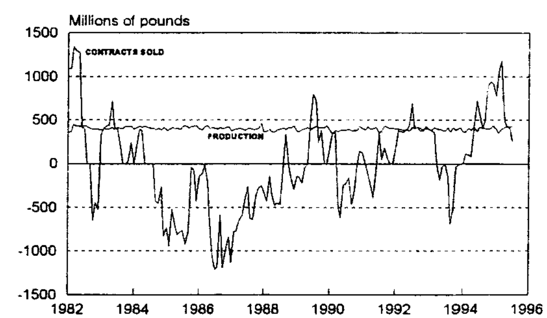


FIGURE 7b

FUTURES CONTRACTS AND PRODUCTION
without penalty & with transaction costs



4.2 The "Codelco Affair"

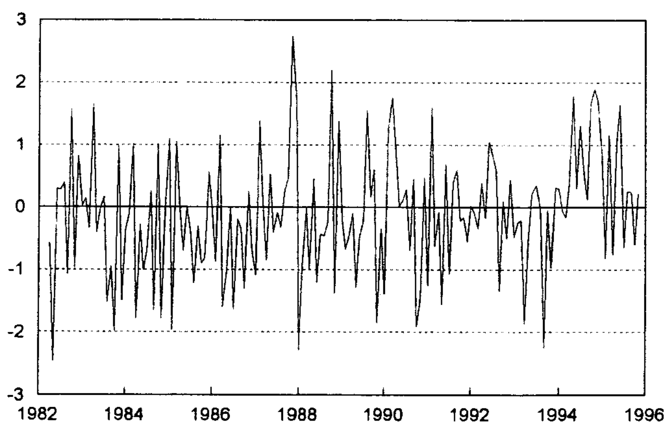
Around the end of 1993, Codelco incurred significant losses resulting from transactions in futures markets. Estimates show that losses for the period between November 1993 and January 21, 1994 were US$178 millions.[9] Given that an optimization model such as the one proposed here also generates significant losses from time to time, the question arises of whether the losses generated by Codelco are within the margin forecast by the model.

For the 3-month period corresponding to the Codelco losses the model actually generates profits from futures transactions, as can be seen in Figure 6a. However, in the months immediately prior, the model does show significant losses under one of the scenarios considered here. The source of those losses is two large negative shocks in the spot price of copper during that period. Those shocks can be seen in Figure 8, which is a plot of the standardized residuals from the estimated ARCH model for spot prices. Only under the scenario of no transaction costs does the model generate losses over that 3-month interval that approach the $178 million of Codelco. Even in that situation, however, the months surrounding that particular interval show large profits from futures trading.

In the presence of transaction costs, the losses during that 3-month period are much smaller. The difference between the results with and without transaction costs is due to very long positions in the futures market chosen by the model when there are no transaction costs. In the presence of such costs, the model opts for a much more conservative trading strategy. In fact, the only possible way to generate large losses in the last half of 1993 is to have very long positions in the market because the very low prices at that period of time require a large volume of transactions to make large income gains or losses possible.[10]

FIGURE 8

STANDARDIZED RESIDUALS



V. Conclusions and Possible Extensions

In this paper, a model for taking positions in copper futures was formulated and solved. The results indicate that even though there is a possibility of generating profits in futures transactions, the chance of incurring significant economic losses from time to time cannot be eliminated. Such losses seem to be large enough to limit participation in these markets, both on behalf of state-owned and private enterprises. The occasional occurrence of large losses is consistent with the financial disasters that have made news headlines, although it is important to stress that the empirical result for copper obtained here may not be extendable to other cases. In the specific case of the losses incurred by Codelco, the model actually predicts gains from futures operations during the corresponding period. It does, however, predict large losses in the prior period, but only under the assumption of no transaction costs.

There are several possibilities for extending this investigation. First, it would be interesting to increase the range of financial instruments available within the model. This could include adding futures contracts of different maturities, which would allow the model to accommodate the termination of contracts prior to their maturity. Secondly, the methodology proposed here for decision making in futures markets could easily be adapted to valuing options. Finally, the model could be applied to futures markets for goods other than copper to study the dynamics of gains and losses associated with operations in those markets.

Notes

1 Large losses from using futures contracts have not been limited to metals and currency markets, nor has Codelco's recent crisis been the only such episode that Chile has suffered. In the early 1980's, the Chilean sugar refining company CRAV was forced to declare bankruptcy as a result of speculative transactions in sugar futures. The size of their losses were large enough to have adverse effects on the stability of Chile's capital flows at the time.

2 To simplify the problem, we have assumed that the production of the firm w_t is an exogenous endowment and that the firm does not have the possibility of adjusting its production level in response to changes in prices. To the extent that this assumption is not correct for the example considered below, the gains reported from the use of futures markets will be understated.

3 The method of detrending does not have a significant effect on the results reported below since the majority of the gains and losses from using futures markets comes from uncertainty in the prices, not in the level of production. We could have alternatively specified an experiment with a constant level of production ($w_t = \bar{w}$). Instead, by using deviations around a trend, we introduce some volatility in production and thereby create a more realistic experiment.

4 Even though tests indicated that one could not reject the presence of a unit root in the spot price, we chose to model the level of the series because of the usual reasons in the debate over whether or not to difference variables and because economic theory would suggest that the process governing the real price of copper is not explosive. We also did not want to lose any information that might be present in the levels of the series, which could be particularly relevant in modeling the variance of the residuals in the ARCH model considered below.

5 Basch and Engel (1993) consider a rollover strategy for trading copper futures which results in a five percent decrease in average earnings, but a 64% reduction in the variance of earnings.

6 When calculating monthly incomes here, transaction costs are assigned to the month in which the contract matures, not to the month when the contract was made.