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VELOCITY AND MONEY DEMAND IN AN ECONOMY WITH CASH AND CREDIT GOODS

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Abstract

The traditional cash in advance macroeconomic models are characterized by a constant velocity of money. Based on the Lucas and Stokey (1987) model, this paper studies the behavior of velocity and money demand for the U.S., simulating an economy which includes stochastic monetary growth (monetary policy), and income taxes (fiscal policy). The results of the simulations are compared with the actual data using several methods. First, the classical metric of standard errors and correlations are evaluated using block-Wald testing procedures. Next, we implement a well specified VAR estimation to study the impulse response functions of interest rates, velocity and the deficit, among other variables. The impulse responses of the model with both policies (with and without fiscal sector) are compared with the corresponding impulse responses for the data. As a third distance evaluation method, based on Braun (1994), the money demand was studied under the Canonical Cointegrating Regression approach. As a conclusion, and based on the three metrics, the model is not rejected in its ability to reproduce an important proportion of the observed volatility in the U.S.

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I. Introduction

This paper focuses on the fact that the velocity of money is not constant over the business cycle; it features the calibration of an economic model which includes cash and credit goods along the lines of Lucas and Stokey (1987), and Cooley and Hansen (1992).[1] To improve the performance in the volatility of velocity and in the behavior of the money demand, including fiscal policy, which has been proven to improve the model's ability to reproduce some basic facts about business cycle fluctuations. Also, we allow the government to have debt (deficit) but requiring that this agent satisfies the standard transversality condition.[2] The model is solved and simulated after checking for sustainable fiscal policies, i.e., a government that satisfies its intertemporal and intratemporal budget constraint.

In current real business cycle research generally the aggregate demand or government policy variables are based on monetary or fiscal policy alone. From this "partial equilibrium perspective" it is very easy to arrive at theories like the "monetary theory of price level determination", or on the other side of the mirror to theories like "fiscal theory of the price level". This kind of feature is very common in the current real business cycle calibration literature. Examples of those who consider only a fiscal sector are Aiyagari (1991), Aiyagari et al. (1990), Braun (1992), Christiano and Eichenbaum (1992a), Greenwood et al. (1993), Judd (1989), McGrattan (1989, 1991, 1992, 1993), and McGrattan et al. (1993), among others. On the other side, we have those who consider only a monetary policy rule. Among them are Bansal and Coleman (1993), Christiano and Eichenbaum (1992b), Coleman (1993), Coleman et al. (1993), Cooley and Hansen (1989), Kydland (1989), Lucas (1987), and McGrattan (1991).

All these models have as mentor the work of Kydland and Prescott (1982). They calibrate a model that captures some of the facts of the U.S. economy, and, as they point out, this "real-side" model is able to reproduce the relative size of the fluctuations in output, consumption and investment. Stochastic shifts in the production function drove their economy. However, nominal variables play no role in the Kydland-Prescott model. This fact creates an incentive to modify this kind of "real" model to increase its performance with respect to the data, especially in order to account for interaction between real and nominal variables.

Consider, first, Christiano and Eichenbaum (1992a). They analyze an RBC model with and without government inside. They considered two specifications to introduce leisure in the utility function. One parameterization specifies that individual utility is linear in leisure (indivisible-labor model), while the second one states that leisure is a nonlinear argument in the utility function (divisible-labor model). Considering an AR(1) time series representation for both the logarithm of the productivity shock and the logarithm of government consumption, they concluded that, "when aggregate demand shocks arising from stochastic movements in government consumption are incorporated,...the model's empirical performance is substantially improved" (page 447).

In the same line, McGrattan (1989, 1991, 1992) incorporates taxes on factors of production and government spending as a proportion of the total output. Using a linear-leisure utility function (indivisible-labor model), and an AR(1) law of motion for the technology shock, McGrattan concludes that there is a great improvement when taxes are included in the model.[3]

Finally, and with a very similar structure, Aiyagari et al. (1990), addressed a different but related topic. Their paper investigates the effect on some aggregate indicators of changes in government spending, focusing particularly on transitory and permanent effects. They found that both temporary and persistent increases in government consumption increase the interest rate, and have important effects on employment and output.[4]

All of these articles suggest that incorporating a fiscal sector betters the performance of the standard Kydland and Prescott's growth RBC model, but all of these analyses are made without considering a monetary sector (nominal variables).

Almost simultaneously, and using a dynamic programming approach along the lines of Lucas (1990), Coleman et al. (1993) build and simulate a monetary model in which the monetary policy is identified as a stochastic supply schedule for reserves, and where the banks have a demand for reserves as a solution of the optimization process for the dynamic model. Closing the model with this monetary policy equation, they are able to reproduce most of the features of the U.S. data.[5]

The point is not to include all the variables of the real world in the analysis, but at least to consider the interaction between fiscal and monetary policy as basic components of the model, in line with Leeper (1993). One step ahead is to allow a maximizing behavior for these "new" agents, like Aiyagari (1991).

Leeper (1993)[6] points out that, given that there is a tendency among economic researchers to analyze the behavior of one policy authority at a time, the conclusions of these studies have hidden assumptions about the other policy not considered: assumptions that can yield misleading beliefs about policy effects. This problem arises from the lack of specification of interaction and connection with the other policy authority. In his paper, he considers a model with both monetary and fiscal policy, and he shows that, depending on the monetary and fiscal policy combinations, the economic implications are different from those that have to do with single policy models.

What is clear from these lines is that the traditional approach considers the monetary and fiscal policies separately. From previous research, we know that the performance of our models will improve if we consider either a fiscal sector or a monetary sector as a component of the models. It is a natural extension to consider monetary and fiscal policy jointly.

Based on the ideas of Chang (1991), Leeper (1991), Leeper (1993), Leeper and Sims (1993), Sims (1994), Smith (1994), Woodford (1994a), and Woodford (1994b), the basic framework of this study is a standard real business cycle model that includes a public sector that finances its budget with stochastic income and inflationary taxes, and debt. Government spending is a stochastic proportion of the output (mean around 20% based on the actual data). Households and firms behave in a competitive way.

The evaluation of the model is based on three metrics. First, we consider the classical standard deviation and correlation distance metric. For that we test the model implementing block tests on real and monetary variables, and on both of these together. The reported results indicate that the model is able to capture the main characteristics from the data. Second, focusing on the multiple time series properties of the model, we implement a statistically well specified vector autoregressive model to study the impulse response functions for selected real and nominal variables: interest rates, velocity, money, consumption, and fiscal deficit. The performance of the model in this dimension is very good, particularly with respect to interest rate, velocity, and deficit. Finally, the model's implications are evaluated using the novel Canonical Cointegrating Regression approach developed by Park (1992) and Ogaki and Park (1993). Following Braun (1994), we implement this CCR framework to the money demand relation, and contrast our estimations with those that come from the current literature. Our simulations generate CCR estimates that are in the range of the elasticities estimated using actual data with other approach e.g., such as Stock and Watson's (1993) dynamic ordinary least squares, Hoffman and Rasche's (1991) Johansen's ML estimate, and Taylor's (1994) Monte Carlo-GMM estimate.

In summary, the structure of this paper is as follows. Section II describes the model of the economy and solves it for an equilibrium. The next section presents the simulation results for three model parameterizations. Section IV evaluates the performance of the model from a Cointegrating approach. The paper ends with conclusions.

II. The Model

The economy to be studied is a version of the indivisible labor model of Hansen (1985) and the cash-in-advance model with cash and credit goods of Stokey and Lucas (1987). The economy has three agents: households that work, consume and invest; firms with constant returns to scale technology; and, the government which finances its spending with bonds, income and inflationary taxes. As we know, money is valued because it is required to purchase consumption goods.

I assume a continuum of identical infinitely lived households with preferences given by the time separable utility function,

$$E_0 \sum_{t=0}^{\infty} \beta^t \cdot U(c_{1t}, c_{2t}, l_t), \qquad 0 < \beta < 1$$

where c_{1t} is consumption in the cash good, c_{2t} is consumption in the credit good,[7] and l_t is leisure in time t. Each household is endowed with one unit of time each period ($0 \le l \le 1$), part of which ($h \equiv 1 - l \ge 0$) is supplied to a firm that produces the output.

I assume a constant marginal disutility of labor and a generalization of the logarithmic specification used by Cooley and Hansen (1992) that is separable in c_1 and c_2. It is represented by a CRRA utility function of the form:

$$U(c_1, c_2, h) = \begin{cases} \frac{1}{1-\gamma} \cdot \left[\alpha c_1^{(1-\gamma)} + (1-\alpha) c_2^{(1-\gamma)}\right] - \Gamma \cdot h & , \quad \gamma \neq 1 \\ \alpha \log c_1 + (1-\gamma) \log c_2 - \Gamma \cdot h & , \quad \gamma = 1 \end{cases} \tag{1}$$

where $1/\gamma$ is either the elasticity of substitution between cash and credit goods at any two points in time,[8] or the inverse of the coefficient of relative risk aversion. In Cooley and Hansen the CRRA parameter was equal to one,[9] which implies logarithmic preferences. The variable h is hours of work and is defined by 1-l.

At the beginning of the period, households have currency holdings that come from three sources: m_t is currency carried over from the previous period, interest and principal from government bonds, and a lump-sum monetary transfer given by the government. At this moment households acquire bonds, and then the asset market is closed. Thus, purchases of cash goods c_1 must satisfy the following cash-in-advance (CIA) inequality constraint,[10]

$$p_t c_{1t} \le m_t + (1+R_t) b_t - b_{t+1} + T_t \tag{2}$$

where p_t is the price level, and $(1+R_t)$ is the gross nominal rate of the one-period nominal government bonds b_t. The resource or flow constraint is given by:

$$\begin{aligned} c_{1t} + c_{2t} + i_t + \frac{m_{t+1}}{p_t} + \frac{b_{t+1}}{p_t} &\le (1-\tau_h) w_t h_t + \\ (1-\tau_k) r_t k_t + \tau_k \delta k_t + \frac{m_t}{p_t} &+ (1+R_t)\frac{b_t}{p_t} + \frac{T_t}{p_t} \end{aligned} \tag{3}$$

Hence, the household expenditures include purchases of the cash good (c_1), the credit goods (c_2) and investment (i), money to be carried into the next period (m_{t+1}), and one-period government bonds holdings (b_{t+1}). The sources of income include after-tax labor income (real), after-tax capital rental income (real), refunds from depreciation, currency carried from the previous period (m_t), the receipts from government bonds (capital and interest $(1+R_t)b_t$) and the lump-sum monetary transfer (T_t).

The law of motion for the household's stock of capital evolves according to:

$$k_{t+1} = (1-\delta) k_t + i_t, \quad 0 < \delta < 1 \tag{4}$$

The second agent in the economy (firm) produces output y_t according to a constant returns to scale Cobb-Douglas technology[11] where the productivity shock is assumed to evolve as an AR(1) with root outside the unit circle:

$$y_t^f = e^{z_t}F(k_t^f, h_t^f) = e^{z_t}(k_t^f)^{\theta}(h_t^f)^{1-\theta}, \quad 0<\theta<1$$
$$z_{t+1} = \rho_1 z_t + \varepsilon_{t+1}, \quad E[\varepsilon] = 0, \; V[\varepsilon] = \sigma_\varepsilon^2 < \infty \tag{5}$$

Given constant returns to scale, we can rescale factor utilization by N, the number of firms, so $k_t^f = K_t \cdot N$, $h_t^f = H_t \cdot N$. The variables K and H represent the economy-wide per-capita stock of capital and labor demand, respectively. With this procedure in mind, we can redo the exercise with the marginal conditions for the representative firm (we will assume that N=1, so $y=y^f$). Then given that the firm seeks to maximize benefits, from the first order conditions for the firm's profit maximization problem we obtain that:

$$w_t = e^{z_t} \cdot F_2(K_t, H_t) = e^{z_t} \cdot (1-\theta) \cdot \frac{K_t^{\theta}}{H_t^{\theta}}$$
$$r_t = e^{z_t} \cdot F_1(K_t, H_t) = e^{z_t} \cdot \theta \cdot \frac{H_t^{(1-\theta)}}{K_t^{(1-\theta)}} \tag{6}$$

The role of the government in this economy is to collect taxes, and to issue debt and money to finance a sequence of stochastic government expenditures and the capital plus interest of the bond issued in last period. Finally, it returns the unspent income to the households via a lump-sum monetary transfer. B_t is the one-period nominal government debt, which has real value B_t/p_t and earns interest rate at the risk free gross nominal rate $(1+R_t)$. Hence the government's intratemporal budget constraint with the law of motion is given by:

$$G_t + \frac{T_t}{p_t} = \tau_{h,t} w_t H_t + \tau_{k,t}(r_t - \delta)K_t + \frac{M_{t+1} - M_t}{p_t} + \frac{B_{t+1} - (1+R_t)B_t}{p_t}$$
$$G_t = \zeta_t y_t$$
$$\zeta_{t+1} = \zeta_0 + \rho_3 \zeta_t + \eta_{\zeta,t+1} \tag{7}$$
$$\tau_{h,t+1} = \tau_{h,0} + \rho_{41}\tau_{h,t} + \rho_{42}\tau_{k,t} + \eta_{h,t+1}$$
$$\tau_{k,t+1} = \tau_{k,0} + \rho_{51}\tau_{k,t} + \rho_{52}\tau_{h,t} + \eta_{k,t+1}$$

where the stacked vector $\eta = [\eta_\zeta \eta_h \eta_k]'$ has $E[\eta] = 0$, with $V[\eta] = \sigma_\eta^2 < \infty$.

However, the government also must be solvent. Solvency requires that asymptotically the government cannot leave a debt that has a positive expected present value. This is the standard-transversality condition (TVC). We also, assume that it is not allowed to run Ponzi games against the government,[12] i.e., exclude is the possibility of government leaving a debt with negative expected value. This is the well-known non-Ponzi-game condition (NPGC).[13]

Imposing the transversality and non-Ponzi-game conditions (TVC and NPGC), we end up with the following intertemporal government budget constraint:

$$B_t = E_t \sum_{j=0}^{\infty} \frac{\left(\tau_{h,t+j} w_{t+j} H_{t+j} + \tau_{k,t+j}(r_{t+j} - \delta)K_{t-j} - G_{t+j} - \frac{T_{t+j}}{p_{t+j}}\right) \cdot p_{t+j}}{\prod_{i=0}^{j}(1+R_{t+i})} + E_t \sum_{j=0}^{\infty} \frac{(M_{t+1+j} - M_{t+j})}{\prod_{i=0}^{j}(1+R_{t+i})} \tag{7'}$$

When this condition holds, we can say that the expected sequences of taxes (including seigniorage) and government spending (including transfers) are sustainable.[14]

From this equation is clear that in considering future policy variables on the period t information set, it is important to recognize that the government budget constraint restricts the joint movements of fiscal variables (spending and taxes, including the inflationary tax). In other words, the evolution of the vector $(G, \tau_h, \tau_k, M, B, T)$ is subject to the above intertemporal budget constraint.[15] This is a very important fact, because depending on which policy (monetary or fiscal policy) is "active or passive" (Leeper, 1991), this approach indicates that fiscal policy will be a very important factor for price level determination, and also we can say that sometimes the "weight" of monetary policy in the economy is basically due to the fiscal policy. In this line of analysis, Woodford (1994) develops a theory of price level determination based on the intertemporal budget constraint. In his model, unexpected changes in government surpluses and deficits affect real activity and the price level, going against the "Ricardian equivalence" doctrine.

In summary, the law of motion for the government spending follows an AR(1) process with mean $\zeta_0/(1-\rho_3)$, while the marginal income taxes are assumed to follow a bivariate VAR with one lag.[16]

The monetary policy consists of issuing money to finance part of the stochastic government expenditures, and we assume the following AR(1) process for the monetary rule:

$$M_t = g_t \cdot M_{t-1}$$
$$\log(g_{t+1}) = (1-\rho_2)\cdot\log(\bar{g}) + \rho_2 \cdot \log(g_t) + \eta_{g,t+1} \tag{8}$$

where $E[\eta] = 0$, $V[\eta] = \sigma_{g\eta}^2 < \infty$, and $\bar{g}$ is the mean of the monetary growth rate.[17]

The usual change of variables is introduced to induce stationarity for the variables in the model. Let $\hat{m}_t = m_t/M_t$, $\hat{p}_t = p_t/M_{t+1}$, $\hat{b}_t = b_t/M_t$, $\hat{B}_t = B_t/M_t$ and in the following we denote variables in t+1 with a tilde (˜). In that case, given the initial equilibrium conditions $k_0 = K_0, m_0 = 1, b_0 = B_0 = 0$ and a sequence of $\{G, \tau_h, \tau_k, g', \hat{B}'\}_{t=0}^{\infty}$ that satisfies the government intratemporal and intertemporal budget constraint (7) and (7'), a competitive equilibrium is a set of sequences for the price level $\{\hat{p}_t\}_{t=0}^{\infty}$, factor prices $\{w_t, r_t\}_{t=0}^{\infty}$, interest rates $\{R_t\}_{t=0}^{\infty}$, household allocations $\{c_{1t}, c_{2t}, h_t, i_t, \hat{m}_{t+1}, \hat{b}_{t+1}, k_{t+1}\}_{t=0}^{\infty}$ and per capita quantities $\{H_t, I_t, K_t\}_{t=0}^{\infty}$ such that the following conditions hold:

(i) Households and firms solve their maximization problem.

(ii) $\hat{m}_{t+1} = 1$, $\hat{b}_{t+1} = \hat{B}_{t+1}$, $h_t = H_t$, $i_t = I_t$, $k_{t+1} = K_{t+1}$, for all t, and,

(iii) Market clearing condition: $c_{1t} + c_{2t} + i_t + G_t = y_t$.

Hence, the dynamic programming problem solved by the household can be written as in the system (9), where I, H, P are functions that express the relationship between investment, labor, and price level, with the state of the economy $(z, g, K, \lambda, \tau_h, \tau_k)$.

$$
\begin{aligned}
v(z,\zeta,\tau_h,\tau_k,g,K,k,\hat{m},\hat{b}) &= \max\{u(c_1,c_2,1-h)+\beta\cdot Ev(z',\zeta',\tau_{h'},\tau_{k'},g',K',k',\hat{m}',\hat{b}')| \\
&\quad z,\zeta,\tau_h,\tau_k,g,K,k,\hat{m},\hat{b}\} \quad s.t. \quad z' = \rho_1 z + \varepsilon' \\
\log(g') &= (1-\rho_2)\cdot\log(\bar{g}) + \rho_2\cdot\log(g) + \eta_{g'} \\
\zeta' &= \zeta_0 + \rho_3\cdot\zeta + \eta_{\zeta'} \\
\tau_{h'} &= \tau_{h,0} + \rho_{41}\cdot\tau_h + \rho_{42}\cdot\tau_k + \eta_{h'} \\
\tau_{k'} &= \tau_{h,0} + \rho_{51}\cdot\tau_k + \rho_{52}\cdot\tau_h + \eta_{k'} \\
K' &= (1-\delta)\cdot K + I \\
k' &= (1-\delta)\cdot k + i \\
c_1 + c_2 + i + \frac{\hat{m}'}{\hat{p}} + \frac{\hat{b}'}{\hat{p}} &= (1-\tau_h)\cdot w\cdot h + (1-\tau_k)\cdot r\cdot k + \tau_k\cdot\delta\cdot k_t + \frac{\hat{m}}{g'\hat{p}} + (1+R)\cdot\frac{\hat{b}}{g'\hat{p}} \\
c_1 &\le \frac{\hat{m}}{g'\hat{p}} + (1+R)\cdot\frac{\hat{b}}{g'\hat{p}} - \frac{\hat{b}'}{\hat{p}} \\
\zeta\cdot y &= \frac{g'-1}{g'} + \frac{\hat{b}'}{\hat{p}} - (1+R)\cdot\frac{\hat{b}}{g'\hat{p}} + \tau_h\cdot w\cdot H + \tau_k(r-\delta)\cdot K \\
I &= I(z,\zeta,\tau_h,\tau_k,g,K) \\
H &= H(z,\zeta,\tau_h,\tau_k,g,K) \\
\hat{p} &= P(z,\zeta,\tau_h,\tau_k,g,K)
\end{aligned} \tag{9}
$$

The method to find these three functions is developed in the paper by Cooley and Hansen (1989) for a monetary economy and, in Cooley and Hansen (1992), for an economy with distortions. It assumes that the cash in advance constraint is always binding.[18] Because it is not possible to find equilibrium allocations by solving a planning problem, it is necessary to find those allocations by solving a fixed point problem.[19] Using this method we wish to find a recursive competitive equilibrium, which consists of decision rules for the households. We seek a decision rule determining the amount of money the household carries into the next period, $\hat{m}' = m(z,\zeta,\tau_h,\tau_k,g,K,k,\hat{m})$; a set of aggregate decision rules I, H; a function determining the aggregate price level P; and a value function $v(z,\zeta,\tau_h,\tau_k,g,K,k)$, such that: (i) Given the aggregate decision rules and the price level function, the value function satisfies equation (9) and $h(z,\zeta,\tau_h,\tau_k,g,K,k)$, $i(z,\zeta,\tau_h,\tau_k,g,K,k)$, and $\hat{m}'$ are the associated decision rules. (ii) Given the pricing function P, individual decisions are consistent with aggregate outcomes: $H = h$, $I = i$, $1 = \hat{m}(z,\zeta,\tau_h,\tau_k,g,K,K,1)$.

The Euler equations to the dynamic programming problem are represented by the following system of eight equations, in addition to the restrictions.

$$
\begin{aligned}
c_1 &: \; U_{c_1'} - \lambda\cdot\hat{p} - \varphi\cdot\hat{p} = 0 \\
c_2 &: \; U_{c_2'} - \lambda\cdot\hat{p} = 0 \\
h &: \; U_{h'} + \lambda\cdot\hat{p}\cdot(1-\tau_h)\cdot w = 0 \\
i &: \; \beta\cdot E\left[\frac{\partial v'}{\partial k'}\cdot\frac{\partial k'}{\partial i}\right] - \lambda\cdot\hat{p} = 0 \\
\hat{m}' &: \; \beta\cdot E\left[\frac{\partial v'}{\partial \hat{m}'}\right] - \lambda = 0 \\
\hat{b}' &: \; U_{c_1'}\cdot\left(\frac{1}{\hat{p}}\right) + \beta\cdot U_{c_1'}\cdot\frac{(1+R')}{g''\hat{p}'} = 0
\end{aligned}
$$

$$
\lambda: \; \frac{\hat{m}'}{\hat{p}} = (1-\tau_h)\cdot w\cdot H + (1-\tau_k)\cdot r\cdot k + \tau_k\delta k + \hat{m}'/g'\hat{p} + \tau_h\cdot w\cdot H + \tau_k\cdot(r-\delta)\cdot K + \left(\frac{g-1}{g\hat{p}}\right) - \zeta\cdot y - c_1 - c_2 - i
$$

$$
\varphi: \; c_1 \begin{cases} \le \dfrac{\hat{m}+g'-1}{g'\hat{p}} + \tau_h\cdot w\cdot H + \tau_k(r-\delta)K - \zeta\cdot y = \dfrac{1}{\hat{p}} + \tau_h\cdot w\cdot H + \tau_k(r-\delta)K - \zeta\cdot y, & if \;\varphi = 0 \\ = \dfrac{\hat{m}+g'-1}{g'\hat{p}} + \tau_h\cdot w\cdot H + \tau_k(r-\delta)K - \zeta\cdot y = \dfrac{1}{\hat{p}} + \tau_h\cdot w\cdot H + \tau_k(r-\delta)K - \zeta\cdot y, & if \;\varphi > 0 \end{cases}
$$

In equilibrium $\hat{m} = \hat{m}' = 1$, $K = k$, $I = i$, $H = h$. The functions λ, φ are the multipliers for the budget constraint and the cash in advance constraint, respectively. Additionally, the envelope conditions are:

$$\frac{\partial v}{\partial k} = \lambda \hat{p}\left[(1-\tau_k)r + \tau_k \delta + (1-\delta)\right]$$
$$\frac{\partial v}{\partial \hat{m}} = \frac{\lambda}{g} + \frac{\varphi}{g}$$

Using the two given marginal conditions for the firm, the law of motion for the stock of capital, the laws of motion for income taxes and proportion of government spending, the production function, and the first order conditions for the household together with the envelope conditions, we can solve for all the variables in the economy $\{c_1, c_2, h, i, k, y, G, \hat{p}, r, w, b, z, \zeta, \tau_h, \tau_k, \lambda, \varphi\}$. The usual procedures are used to solve the problem through the following system:

$$\lambda = \beta E\left[\frac{Max\{\lambda', c'_1 \cdot U_{c'_1}\}}{g'}\right]$$
$$\hat{p} = Min\left\{\frac{U_{c'_1}}{\lambda}, \frac{1}{c_1}\right\}$$
$$\varphi = Max\{0, c_1 \cdot U_{c'_1} - \lambda\}$$

Under perfect foresight and steady state, and with the particular preferences and technology given above, it is very easy to solve this system. Once we solved for λ, we can see that the solution for the price level implies that the cash in advance constraint is always binding. To satisfy monotonicity and discounting properties in the fixed point equation, we require the usual restriction over the expected monetary growth, i.e., that $\beta \cdot E[1/g'] \leq 1$. This restriction guarantees a non-negative nominal interest rate.

III. Simulating the Model

3.1 Calibration and characterization of the data

For the model described above, hours of work, investment, and the price equation are linear functions of the states. Hence, the equilibrium expressions for H, I, and $\hat{p}$ (the inverse of consumption in cash good) are:

$$H = \phi_0 + \phi_1 \cdot z + \phi_2 \cdot \log(g) + \phi_3 \cdot K + \phi_4 \cdot \tau_h + \phi_5 \cdot \tau_k$$
$$I = \varphi_0 + \varphi_1 \cdot z + \varphi_2 \cdot \log(g) + \varphi_3 \cdot K + \varphi_4 \cdot \tau_h + \varphi_5 \cdot \tau_k$$
$$\hat{p} = \psi_0 + \psi_1 \cdot z + \psi_2 \cdot \log(g) + \psi_3 \cdot K + \psi_4 \cdot \tau_h + \psi_5 \cdot \tau_k$$

For the purpose of the simulations the length of the period is one quarter, and in order to make comparisons with previous studies,[20] the parameter values used were $\beta = 0.997$, to match the average real interest rate (1.3%), and $\theta = 0.36$, $\delta = 0.025$, $\Gamma = 2.86$, $\rho_1 = 0.95$

From the monetary policy rule, the values ρ_2, σ_η^2 were estimated for the period 1972:3 to 1993:1 following the same procedure used by Cooley and Hansen; we fit the following AR(1) equation for the money growth:

$$\Delta \log(m_{t+1}) = \xi_0 + \xi_1 \Delta \log(m_t) + \eta_t$$

The estimated parameters were 0.464 (with a standard deviation of 0.0985) for the AR(1) coefficient, and 0.0096 for the standard error of the estimate. These values are similar to the Cooley and Hansen estimates (0.48 and 0.009, respectively) with sample 1955:3-1984:1. Earlier versions of this paper (Johnson, 1994) show that in bootstrapping experiments,[21] the values obtained using the OLS estimates are similar under simple random sampling.

The values used for α, γ were obtained comparing Cooley and Hansen (1992) with Chari et al. (1991). For the α parameter, Chari et al. (1991) estimate the marginal intratemporal Euler equation that comes from the ratio of the first order conditions for c_1 and c_2. Using quarterly data for the period 1959-1989, and measuring real money balances by monetary base, and consumption by consumption expenditures (including durables), they estimate the intratemporal previous equation by OLS getting an α equal to 0.43 and γ equal to 0.17. The risk aversion parameter obtained by Chari et al. was substantially low. Using another procedure (panel data), Cooley and Hansen (1992) estimate α. They found that this parameter is around 0.8-0.84, deciding to use 0.84. Given these facts, in our simulation we fix these parameters at $\beta = 0.997$(to match annual average real interest rate), $\alpha = 0.7$ (something in the middle of Chari's and Cooley and Hansen's results), and we simulate with three values for the CRRA parameter: 0.5, 1, and 1.5.[22]

For each economy (with different CRRA parameters) the model was simulated with and without a fiscal sector. So in total we have six representations to compare with the actual realization of the economy. As a metric we used several procedures. First, we compare the classical contemporaneous first and second moments that comes from several variables from the model with the respective from the data. Next, using selected variables we implement a vector autoregressive (VAR) estimation and compare the impulse response functions that come from the model with those from the data.[23] Finally, we analyze the money demand using Ogaki and Park's Canonical Cointegrating Regression (CCR) approach.

With the first metric, the standard deviations for the error terms are calibrated for each of the simulated economies in order to match the standard error of the output in the artificial series with the value in the actual data (see next table for the calibrated values). Each simulation has the same number of periods as the data sample (85), and it used a burning period of 6% of the sample. All the reported statistics are sample means of statistics computed for each of the 500 simulations. Each simulated time-series and U.S. data sample were logged and detrended using the Hodrick-Prescott filter,[24] following many studies in the real business cycle research area. As we already mention it, even when the data generated from the model is stationary, we decide to filter using the Hodrick-Prescott filter mainly in order to implement fair comparisons with previous studies. However, the general results did not change if the filtering is not executed.

Calibrated Standard Deviation for the Technology Shock				
Model		No Fiscal Policy	Deterministic Policy	Stochastic Policy
CRRA (γ)	0.5	0.00582	0.00598	0.00620
	1.0	0.00717	0.00692	0.00710
	1.5	0.00785	0.00766	0.00783

Using data from McGrattan (1992), we interpolated the series on capital and labor income taxes to obtain quarterly observations (see Figure 1 in Appendix I). The data comes from 1947 to 1987. Using this interpolated data, the law of motion for the taxes was estimated as a bivariate VAR(1):

$$\hat{\tau}'_h = -0.015357 + 1.016375\cdot\tau_h + 0.023584\cdot\tau_k\,, \quad SEE = 0.002281,\ \bar{\tau}_h = 0.228071,\ \bar{R}^2 = 0.9932$$
$$(3.47) \quad (123.83) \quad (3.98)$$

$$\hat{\tau}'_k = 0.042780 - 0.084422\cdot\tau_h + 0.952894\cdot\tau_k\,, \quad SEE = 0.005707,\ \bar{\tau}_h = 0.507503,\ \bar{R}^2 = 0.9781$$
$$(3.87) \quad (4.11) \quad (64.25)$$

where the values in parentheses are the absolute t-statistics under the null that the parameter is zero and SEE represents the standard error of the estimates. This bi-VAR(1) is going to represent the law of motion for the marginal income taxes. In relation to the stationarity of the VAR, it is easy to check that the implied VAR-polinomial has its roots outside the complex unit circle. Consequently, the corresponding process is stationary.[25]

Table 1 shows some of the basic statistics for the U.S. sample data, and Figures 2a, 2b, and 2c in Appendix I present the most relevant of these time series.

Some of the U.S. statistics reported in Table 1 differ from those reported in Kydland and Prescott (1982), Cooley and Hansen (1989) and McGrattan (1993) because of data choices. As we can see, the correlation between output and price level is negative (-0.55) while the autocorrelations of almost all the series considered are still high until three quarters, showing high persistence. Money, velocity, consumption and the price level are among the variables with more persistence (the autocorrelation parameter for the fourth lag is greater that 0.30),

TABLE 1

U.S. SAMPLE STATISTICS: 1972:1 TO 1993:1

Series	( )	Standard Deviation	1	2	3	4	5	6
GNP	y	1.80	0.86	0.65	0.42	0.21	0.05	-0.10
Inflation	p	0.58	0.53	0.30	0.38	0.15	-0.03	-0.01
T-Bill Rate	r	0.36	0.79	0.53	0.42	0.28	0.12	-0.07
M1	m	2.29	0.89	0.70	0.51	0.30	0.12	-0.05
Velocity	v	2.97	0.91	0.75	0.57	0.35	0.14	-0.05
Consumption	c	0.91	0.87	0.70	0.53	0.32	0.14	0.01
Investment	i	6.15	0.90	0.72	0.50	0.27	0.07	-0.11
Hours	h	2.39	0.90	0.70	0.46	0.23	0.03	-0.15
Productivity	y/h	0.96	0.81	0.59	0.29	0.05	-0.11	-0.25
CPI	cpi	1.70	0.93	0.81	0.66	0.49	0.30	0.13
Deficit	def	1.80	0.75	0.54	0.28	0.07	-0.10	-0.23

Correlations

	p	r	m	v	c	i	h	y/h	cpi	def
y	0.46	0.36	0.23	-0.23	0.86	0.94	0.93	-0.45	-0.55	0.78
p	1.00	0.62	0.13	0.04	0.25	0.38	0.59	-0.61	0.12	0.44
r		1.00	-0.20	0.42	0.08	0.24	0.53	-0.65	0.43	0.37
m			1.00	-0.92	0.33	0.29	0.13	0.12	-0.44	0.18
v				1.00	-0.35	-0.32	-0.09	-0.20	0.70	-0.20
c					1.00	0.83	0.74	-0.22	-0.70	0.59
i						1.00	0.90	-0.48	-0.62	0.79
h							1.00	-0.74	-0.39	0.84
y/h								1.00	-0.06	-0.63
cpi									1.00	-0.42

while inflation and the deficit (measured as the ratio between total government deficit and nominal GNP[26]) have low persistence. Even when we did not test for heteroscedasticity, just looking at the Figures 2c still there exists evidence for the presence of an ARCH-like process in the deficit process, particularly because of the period spanning 1974-1976. The standard deviation of deficit, due in part to these years, is 1.80, with an AR(1) coefficient of 0.75. As we expect, the volatility of investment is relatively high (6.15) with respect to consumption (0.91) and output (1.80). The behavior of money and velocity are also very volatile (2.29 and 2.97, respectively), especially because of the 1980's.

The correlation between money and velocity is negative (-0.92), while the correlation between output and interest rates is positive (0.36). The correlation between velocity and the interest rate, and the correlation between consumption and output, have the signs that we can expect from the theory (0.42 and 0.86). Interesting are the resulting correlation between deficit and inflation (0.44) and deficit with interest rate (0.37). It seems that there exists a sort of endogeneity in the fiscal deficit, due in part to the level of the debt, implying high payments in terms of interests.

Some of these facts are not consistent through the sample. If we split the sample in 1982:4,[27] perhaps the most notorious change is in the behavior of velocity and money. Before the 1980's, the velocity presented a marked trend, while since early last decade this variable begins to fluctuate with a decreasing trend. During the first sub-sample the correlations between velocity and either inflation or output were positive (0.17 and 0.20, respectively), while in the second sub-sample the same correlations were negative (-0.12 and -0.76). Another important fact is the change in the sign of the correlation between interest rate and money. In the first sub-sample, this correlation is negative (-0.56) while in the second sub-sample, it is positive (0.57). The same happened with the correlation of the innovations. The correlation between inflation and money turns from negative (-0.29) to positive (0.38), while considering all the sample this statistic is around zero (0.01). Something similar happens with velocity and interest rate. Their correlation change from 0.30 to -0.20 (or from 0.62 to -0.12, if we consider the simple correlation sample instead of the innovations).

From studying Figures 2a-2c in Appendix I, we can see some of the usual properties in the real aggregates variables.[28] The low volatility of consumption, in comparison with the higher volatility of GNP, and the high negative correlation between velocity and money. In particular, note the high (low) volatility of velocity (GNP) and money since 1982. Before this date there was a high correlation between money and both inflation and GNP. All these procyclical movements disappear after 1982-1983.

In exploratory analysis using a five variable VAR(1) (with order y-p-r-m-v), the impulse response functions of the interest rate on output is negative indicating that if we shock the interest rate upward, the activity level is going to decrease (with a lag of two to eight quarters), while the inflation is going to increase in the short run (two quarters). The effect of the monetary expansion on the interest rate does not show a liquidity effect.[29]

As it was mentioned, the exercise consists in simulating two economies with different structures for the states variables, each under three specifications of the objective function. The first set of two economies (with and without fiscal sector) assumes a logarithmic utility function without credit goods ($\gamma = 1$) as in Cooley and Hansen (1989), but with cash and credit goods, while the second set (with and without fiscal sector) assumes a non-logarithmic utility ($\alpha = 0.7, \gamma = [0.5, 1, 1.5]$, see text above). For all the simulations, an autoregressive process for the money growth rate (AR(1)) with mean $\bar{g} = 1.015$ is assumed. This implies an average inflation of six percent (based on data from 1972:1 to 1993:1). The government spending is a stochastic proportion of the output, with an average of 20%, the actual average figure found in our sample.

3.2 Contemporaneous moments and testing the model

In this section we present the main results of the experiments in terms of means and χ^2 of the standard errors and correlations of several simulations (for this purpose we consider 500 repetitions). In Tables 2a, 2b and 2c are represented the standard deviation for artificial economies, considering three CRRA coefficients (0.5, 1.0, and, 1.5), while in Tables 3a, 3b and 3c we present the correlations for the same economies showed in Tables 2a-2c. The shaded column (on the left hand side of each table) shows the standard deviation and correlation of the actual U.S. data, for a period spanning from 1972:1 to 1993:1 (quarterly basis).

The first simulated economy with no fiscal policy and CRRA $\gamma = 0.5$, is represented in the second column with its respective chi-squared test (with their probability or p-values in parentheses) next to the right. These tests evaluate the null hypothesis that the statistic predicted by the model equals the corresponding value for the data in the first shaded column. All these statistics are asymptotically χ^2 (df) distributed with df degrees of freedom. The next two columns present the simulation results for the same parametric model but now including the fiscal policy. The same structure is represented in the following wide-columns in each table, with the CRRA parameter changed from 0.5 to 1.0 and from 1.0 to 1.5. Being a little more sophisticated, we implemented a Wald test to evaluate the overall performance of the model, using either all the variables as a metric or just a subset of these. These tests (with respective p-values in parentheses) are presented in the last three rows of Tables 2a-2b. The first one is a χ^2 (4), and tests the joint hypothesis that inflation, velocity, interest rate, and deficit implied from the particular model equals the corresponding from the shaded column. This test was implemented to test the model performance only considering the nominal variables. The next reported test (χ^2 (6)) considers the real variables, i.e., output, consumption, investment, capital stock, hours, and productivity. This test, in particular, will show the predictions of models which include a fiscal sector outperform the predictions of models that do not include it. The last test (χ^2 (10)), evaluates the overall performance of the model in terms of all the variables generated by it. This tests the joint hypothesis with the variables included in the previous two tests.

Looking first at the χ^2 (1) tests, we can see that the performance of the model improves in term of consumption, hours, and productivity, while it decreases in terms of investment. The volatility in taxes makes a good contribution with respect to the variable hours. With stochastic income taxes we capture the volatility in the elasticities of the labor supply, increasing the standard deviation of productivity and hours worked. Also, in particular, the model's volatility of inflation and interest rates exceeds that of the data (by a factor of two), while the volatility of velocity is lower than the actual value (the model explains almost 50% of the actual volatility). For most of the real variables it is hard to reject the null hypothesis that the model predicts individual accurate moments.

In implementing the three joint tests we can easily see that the model which includes the fiscal sector performs better than the model with no fiscal sector.

The evaluation of the overall model with the χ^2 (10) joint test, indicates that, including all real and nominal variables from the table (excluding CPI), the model has good performance and it is not rejected with respect to the actual U.S. values. When we consider only the real variables, the model without the fiscal sector is not rejected with a 5% significance level, however the performance of the model is notably improved once we include the fiscal policy. Now the model presents a p-value range from 0.15 to 0.20. From the point of view of this paper, the most important test is the first joint test χ^2 (4). It represents the joint test considering the last four variables from Tables 2a-2c. This test presents the stronger favorable results with respect to the model with taxes, with p-values around 0.50. When the CRRA parameter is 0.5 the model with or without fiscal sector predicts almost the same results in terms of the tests: the model is not rejected at 5%. Once the CRRA moves to 1.0, the p-value reaches 0.546 for the model without taxes. Including taxes improves the model performance; now the p-value is 0.556. Finally, in the last simulation, considering a CRRA parameter of 1.5, the same modification increases the p-value from 0.529 to 0.543. In summary, the performance of the model with respect to inflation, velocity, interest rate, and deficit improves once we consider a stochastic fiscal policy.

Now, we analyze the performance of the model with respect to the cross correlations. In Tables 3a-3c in Appendix II are presented the simulated moment with their respective χ^2 statistics. The correlation of consumption and output is well captured by almost all the simulated economies. For the first economy with CRRA of 0.5, the p-value is equal to 0.00 for the economy without taxes; once we include taxes, this value increase to 0.27. In the second economy, with a 1.0 CRRA parameter, the p-values change from 0.06 to 0.21, again showing the good performance of the model. This results is repeated when $\gamma = 1.5$. The same analysis follows for the price level, except that the inclusion of the stochastic taxes changes the sign of the correlation coefficient when the CRRA is greater that 1.0. The level of the correlation between output, and either consumption, investment, or hours is well captured by the model.

Even when the individual tests do not show a good performance for the model, this is because the variability of the correlations is very low. However, considering only the level of the correlations we can see that in general, the model explains very well the actual correlations.

In summary, the percentage of explanation in volatility of consumption is almost 90%, investment 80%, capital stock 82%, hours 72%, productivity 60%, price level 120%, velocity 50%, interest rate 300%, and deficit 75%. Hence, looking at the joint tests the model with stochastic fiscal policy show a marked improvement in terms of the nominal variables and in term of the real variables.

3.3 VAR analysis: Impulse Response functions

In this section we estimate a statistically well specified vector autoregressive (VAR) model, considering a subset of five real and nominal variables: interest rate, money, consumption, velocity and deficit. The first subsection examines the stationarity of the data, using the classical unit roots literature. Following the implementation of the unit root tests, we analyze the existence of cointegration among the variables, to establish the necessity of error correction mechanisms inside the estimated VAR. Finally we implement Granger-Causality tests to establish the order of the VAR.

3.3.1. *Unit Roots and Cointegration*

The first step in estimating a VAR is to determine the existence of an integrated process in the variables of interest.[30] Tests of unit roots are designed to establish such possibility. In Table 4 in Appendix III we present the Augmented Dickey-Fuller (ADF) unit root tests for a subset of real and nominal variables. The ADF equation was specified considering one, two and four lags, to ensure that the error term is a white noise. For each of the specifications was considered the existence of a constant and a trend variable in the equations. Columns (1) denoted the pure ADF test, without trend and constant term; columns (2) include the constant term, while columns (3) is a complete ADF with a constant and a trend term. The shaded cell means that the null hypothesis of no unit root is rejected with 5% of significance level.

The results are pretty standard and are consistent with the literature. Treham and Walsh (1990) evaluate the existence of nonstationarity in government expenditure, tax rate, inflation and velocity, while Stock and Watson (1993) have presented evidence that output, real balances, and nominal interest rates are integrated of order one and cointegrated.[31] Price level, interest rate, real money, inflation and velocity are among the candidates for high probability of not rejecting the unit root null. For money, output, consumption and deficit, the tests still do not reject the null of a unit root, but not as strongly as the other variables just mentioned. The results are consistent when we consider three lags and, at the same time, for the different specifications of the ADF tests.

Stock and Watson (1993) reported some degree of cointegration among output, real balances, and interest rates. Using annual data from 1900-1989, and looking for stability in the money demand equation, they found that the residuals constructed using either the full-sample or first-half point (1900-1945) estimates are consistent with cointegration, while the residuals based on the postwar estimates are not. Our results are consistent with their results, not rejecting the null of no cointegration. Table 5 in Appendix III reports these results. The entries in this table report the test of cointegration between any pair of variables listed in the first column and the first row. Using a ADF test with constant, trend and four lags,[32] we never reject with a 5% of significance level the null of unit root in the residuals of the cointegrated equation. Particularly important for the public finance literature are the results with respect to deficit. The last second row of Table 5 (before the comments) reports the cointegration between deficit and the rest of the variables under consideration. It seems that the data report some degree of long term relationship between deficit and either interest rate or inflation since the p-values are around 0.10. Even when we did find strong evidence to reject the null of no cointegration (with a 10% and with ADF tests), the entries give us some idea that some pairs are actually cointegrated, i.e., that there is a good possibility that in the long run some of the process variables are really linked. This is the case for the pairs money-inflation, velocity-inflation, and, real balances-output, among many others.

3.3.2. *Dimension of the Model: AIC, HQ and BIC Tests*

Given that we did not reject statistically the null of no cointegration and unit root in the residuals (as Stock and Watson (1993)), it is not necessary to revise the specification of the VAR to introduce any error correction terms in the data generation process. The only thing left is the specification of the order for the variables in the Choleski decomposition. Because we compare the simulated HP-filtered data, it was necessary to recalculate the unit root tests for the actual data after being filtered by the Hodrick-Prescott filter. The results are a quite different in comparison with the unit root tests for the data without detrending. In Table 6 in Appendix III we report the ADF tests for the elected subset of variables (this table follows the same structure than Table 4). Here, and for most of the specifications, either interest rate or deficit do not present evidence of having a unit root in the process, changing the results found in Table 4. However, for the other three variables (money, consumption, and velocity) the unit root tests confirm the null of a unit root. Having these results in mind, the VAR will include two variables in levels (interest rate, and deficit) and three in first differences (money, consumption, and velocity).[33]

Once we determine the included variables, it is necessary to specify the order to do the decomposition for the impulse response analysis.[34] To study causality among the variables, we implement the Granger-causation test (Granger, 1969); the results are reported in Table 7 in Appendix III. Entries off the diagonal indicate the value of the test under the null H_0: X1 is not Granger-caused by X2, with its respective p-values in parentheses below the test. This test can be done with different specifications of the causality equation, basically to ensure white noise in the residuals. We implement the F tests with different number of lags, all with the same causality results. Table 7 reproduces the results with 1 lag for the Granger equation.

The test values, altogether with the p-values, indicates that there is a causality going from interest rate to money growth (we reject the null that money is not caused by interest rate with test equal to 16.5, and a p-value 0.0001). The same is true for interest rates with consumption growth and velocity growth. Both hypotheses are rejected with a p-value lower that 0.01 (1%). Other important results are that velocity growth causes money growth (p-value 0.0016) and consumption (p-value 0.07), and that consumption growth causes the deficit (p-value 0.0006). From these results, we conclude that the final ordering will be: interest rate, velocity growth, money growth, consumption growth, and, deficit (r-dv-dm-dc-def).

Finally, it is necessary to define the correct dimension of the model. There

are many tests that address that question. However, based on Lutkepohl 1985's Monte Carlo simulations, among the most robust tests are the Akaike Information Criterion (Akaike, 1974), the Hannan and Quinn Criterion (Hannan and Quinn, 1979), and the Bayesian or Schwarz Information Criterion (Schwarz, 1978).[35] What these criteria do is minimize the following function,

$$\lambda_{(k)} = \ln\left|\hat{\Sigma}_k\right| + \left(\frac{k \cdot d^2}{T}\right) \cdot \Delta$$

where $\ln\left|\hat{\Sigma}_k\right|$ represent the logarithm of the determinant of the variance-covariance matrix for the residuals in the equation with "k" lags (one up to six). The number of equations in the VAR is represented by "d" (five in our problem), and the total number of observations is denoted by "T".

The specific representation for Δ depends on the criteria used. For the Akaike criterion $\Delta = 2$, Hannan and Quinn use $\Delta = 2 \cdot \ln(\ln(T))$, while in the Schwarz criterion $\Delta = \ln(T)$. Table 8 in Appendix III reproduces these results for a span of lags from one up to six.

As we expect, when the number of lags increase, the value of the logarithm of the determinant decreases, which means that we are always going to choose the maximum number of lags based on traditional Tiao-Box-Sims criterion (Sims, 1980 and Tiao and Box, 1981). The existence of the AIC, HQ and BIC criteria solve this problem, and more importantly, based on Monte Carlo simulations, we know the relative power of the tests. Once we consider different criteria (AIC, HQ,BIC) the $\lambda_{(k)}$ function chooses three lags or one lag (AIC, and HQ and BIC, respectively), given the same results when the specification of the VAR is made in terms of levels instead of first differences for money, consumption and velocity. This gave us confidence in choosing 1 as the optimal number of lags to define the dimension of the model. Now we go to analysis of the impulse response functions.

3.3.3. *Impulse Response Functions*

Based on the VAR previously specified, we estimate the impulse response functions for interest rate, velocity and deficit. First, we see in Figure 3 in Appendix III the impulse response functions for the actual data considering a montecarlo with 100 simulations and a band of two standard deviations. Figures 4, 5 and 6 in Appendix III present the montecarlo simulations for the impulse response functions for the simulated economies considering an average of 500 realizations for each model. A common feature of the calibrated models is the lack of persistence, however this is not the case in our model. From the impulse response functions showed in Figures 3 up to 6 we can see that both the actual and simulated data, show a persistence that lasts for about seven quarters. Moreover, the sequence of deficits seems to be well represented by the model. In the actual data, the shock to interest rates increase the level of deficit for about three quarters, and there is an increase in velocity that lasts 12 quarters. The responses of deficit and velocity (in less percentage) are captured by the model.

Looking at the response of interest rate to a one standard deviation shock in all the VAR variables, we can see that the model losses a little persistence relative to the actual data impulse response functions. While most of the variables induce a 7 to 8 response in the interest rate, the model only generates 5 quarters of response, half a year less. However, the signs of the interest rate process are very similar in both the actual and simulated models. A one standard deviation shock in deficit induces an increase in interest rate that lasts 2 years in the actual data, and one year in the simulated model. The shock in velocity growth generates a 6 quarters negative response in the actual data and 3 quarters for the model. The model seems to reproduce with success the behavior of interest rates as the economy encounters an unanticipated shock. With respect to velocity, the analysis of the actual data and the model, tell us that velocity responds positively to a shock to interest rates, although the magnitudes and lasting periods are different. For the first case (actual data), the positive impact lasts almost 12 quarters, while in the simulated economies the positive effects are present only 4 quarters, and then there is a negative compensation that lasts almost one year. Hence the response in time looks very similar.

The last impulse response analysis corresponds to the deficit. The response of velocity to a shock in deficit are very similar between the model and the data. It seems to be more dynamic in the model with higher CRRA parameters. The response of the deficit to a one standard deviation shock in interest rate seems to be captured by the model. For the actual data the positive impact lasts for about one year, while for the simulated economies, the impact stay for a little more than a year. The comparison is really good once we consider the model with a CRRA parameter of 1.5. The small response of deficit to a money shock is well captured by the model, although in general the actual data induces smoothed paths for the deficit levels, in comparison with the model. As a summary, the model captures most of the paths shown for the actual data, specially, the sequences of deficit and velocity (positive response to a shock in interest rate), and interest rate (positive response to a shock in deficit). The model captures correctly the signs of the responses and also presents good persistence in the variables.

IV. Money Demand and Canonical Cointegrating Regressions

This section presents the estimation results for a money demand equation, based on the canonical cointegrating regressions (CCR) approach developed by Ogaki and Park (1991) and Park (1992).[36] Starting from the conclusions of cointegration tests developed in section 3.3.2. and the results obtained by Stock and Watson (1993), I assume that there is some degree of cointegration among output, interest rate, and real balances.[37] We know that the series generated from the model are stationary, which implies that is not necessary to worry about cointegration. However, this result of stationarity comes from the main characteristic

of the model. It is possible to model an economy where the series generated from it are non-stationary (classical growth model with technological progress for instance) and then to study the cointegration problem. But the approach of this model is that even when the data generated from the model is stationary because we did not built any stochastic or deterministic trend in it, we know that they are cointegrated so for this reason we implement this cointegrated approach.

As we know, the OLS estimates from a cointegrated regression are consistent but asymptotically biased (their standard errors are meaningless), and also they have a very unusual distributions, making inference of statistics very hard to implement. To solve this problem, and obtain efficient estimates, we follow Park (1992). The idea is to utilize a nonparametric estimate of the long-run covariance parameters. Among the advantages of CCR is that we can make inferences using just standard distributions like asymptotic t tests, and it imposes the restriction that the cointegrating vector removes all deterministic and stochastic trends. Also, we can test very easily deterministic and stochastic cointegration using asymptotic chi-squared standard distribution.

This section follows Braun's (1994) procedure and produces similar estimations for the money demand in the U.S. using the CCR approach. Table 9 reports the estimation based on OLS and CCR of the following money demand function:

$$\ln\left(\frac{M_t}{P_t}\right) = \varphi_0 - \varphi_1 \ln(R_t) + \varphi_2 \ln(Y_t) + v_t \qquad \text{(Money Demand)}$$

where M/P, R and Y, are real balances, gross nominal interest rate, and activity level (measured by output or consumption), respectively.

Table 9 in Appendix IV contains the estimated parameters values from OLS and CCR, using as a measure of activity level, consumption and output. This table also reports two asymptotic χ^2 cointegration tests. The H(p,q) tests[38] were obtained by estimating the CCR on the money demand equation. Under the alternative of no cointegration, this statistic goes to infinity, so these tests are consistent. In particular H(0,1) tests for deterministic cointegration, while H(1,5) statistics tests the null of stochastic cointegration. Technically speaking, Park et al. (1991) find that when the Andrews and Monahan (1992)'s VAR pre whitening method is used to obtain the long-run variance-covariance matrix of the residuals, the CCR estimate has smaller mean square errors than other estimates (like Johansen (1991)'s ML estimate) , and also they find that Park (1990)'s H(p,q) tests have good small sample properties. The estimation process begins with the OLS initial estimate for the CCR parameters. This is called the first stage CCR. The second stage CCR is obtained from the long-run covariance estimates from the first stage. This procedure is repeated in the third stage CCR, but in the fourth stage CCR it is not, because it implies larger mean square errors. However, it is in this fourth stage when we obtain consistent H(p,q) tests. Hence, as suggested by Ogaki (1993), this table reports the third stage CCR parameter values and the fourth stage H(0,1) and H(1,5) cointegration tests.

The parameter estimates for the income elasticity φ_2 of 0.320, with a standard deviation of 0.075 (see shaded area), are comparable to the one found by Braun (1994) using postwar monthly data (0.238 and 0.076, respectively). This elasticity increases to 0.559 (with a standard deviation of 0.064) when we use output as a measure of activity level, instead of consumption. Braun reports an elasticity of 0.162 (0.070 standard deviation), while using a dynamic ordinary least squares estimation strategy proposed by Stock and Watson (1993) the reported value was 0.364 with a standard deviation of 0.056. Using 20th century annual data (1900-1985), Braun reports an elasticity of 0.929 with a standard deviation of 0.035. Hence our estimates using quarterly data can be seen as an interpolation of Braun's estimations using monthly and annual data.

The following three subsections of the table contains the CCR estimation for three parameterizations of the utility function (the levels of CRRA parameters used were 0.5, 1.0, and 1.5). For each CRRA parameter value, we estimate the money demand expression using simulated data from the model with and without fiscal policy, and considering consumption and output as proxy of activity level (Y_t). So in total we estimate twelve CCRs. We also report the OLS estimates as a contrast.

The results from the simulated data with fiscal policy are very good. For the case of 0.5 CRRA parameter value, and using consumption, the model without fiscal sector gives us an elasticity of 1.049 (with a standard deviation of 0.011), while in the economy with stochastic taxes the elasticity is 0.864 (0.084 standard deviation). However, when using output instead of consumption, these figures decrease to 0.258 (0.034)[39] and 0.298 (0.056), respectively. These values are not so far from the actual data reported in the shaded area of the table.

With a logarithmic utility function ($\gamma = 1.0$), the income elasticity of the model without taxes is 1.038 (0.022) or 0.270 (0.027), once we use consumption or output as a regressor, respectively. Considering stochastic taxes these figures changes to 0.944 (0.140), when using consumption, and 0.214 (0.073) when using output. Again the results suggests that with respect to the CCR money demand estimation, the model with stochastic fiscal policy performs better than a model without it. These results are confirmed with the analysis for $\gamma = 1.5$. In general our estimates are comparable to Baba, Hendry and Starr's (1992) single-equation nonlinear least squares estimate of 0.5 (based on quarterly data for a sample period of 1960-1988), Hoffman and Rasche's (1991) VAR error correction model's ML estimate 0.78 (monthly data from 1953 to 1988), Taylor's (1994) Monte Carlo GMM estimate of 0.481 (with standard deviation 0.031), and Stock and Watson's (1993) dynamic OLS estimate of 0.46 (monthly data from 1960.1 to 1988.6).

Taylor (1994) also reports an interest elasticity of 0.149 (0.015), while Stock and Watson (1993) report an interest semielasticity of 0.1 (with 95% confidence interval of 0.075, 0.127) using their dynamic ordinary least squares estimation procedure. Using CCR and based on the actual data, our elasticities are in the range of the figures reported by Braun, however, with the simulated data from the models our CCR estimates are comparable to the ones found by Taylor (1994), and Stock and Watson (1993). With a CRRA parameter value of 0.5, the CCR

estimates were 0.021 (0.007) and 0.193 (0.054), considering consumption and output respectively. The inclusion of stochastic taxes changes the CCR estimates to 0.111 (0.070) and 0.170 (0.090), respectively. In the logarithmic utility case, the estimates without taxes were 0.034 (0.012) and 0.146 (0.046), with consumption or output, and with fiscal policy these were 0.118 (0.086) and 0.150 (0.108). For $\gamma = 1.5$, the same figures were 0.028 (0.013) and 0.110 (0.040), in a world without taxes and using consumption or output as proxies, while in the other environment with taxes were 0.114 (0.093) and 0.127 (0.115). In general, these results confirm the simulation-estimations made by Taylor (1994) and the estimations made by Stock and Watson (1993).

As a summary, we can see that the CCR estimation gave us good results in comparing the data with the simulated economies, through the filter of the money demand equation. Based on our model, our estimates explain a large percentage of the actual estimated parameters, and also they are according to the latest findings in the money demand studies.

V. Conclusions

The purpose of this study is to extend the standard cash in advance models to include cash and credit goods and stochastic monetary and fiscal instruments to generate realistic predictions with respect to several metrics. Specifically, we structured a cash-credit good economy which includes seigniorage, income taxes, and debt, to mimic some of the standard features of the U.S. data. I use a variety of diagnostics to evaluate the performance of the models. These diagnostics suggest that the second moments of the monetary economy with stochastic taxes compare favorably with the performance of the simple cash and credit goods economy (without a fiscal sector). Also, Wald tests validate the model in terms of monetary and real variables, specially inflation, interest rate, velocity and deficit. With this testing procedure we were unable to reject the model, even in blocks of real variables or in blocks with nominal variables, or a complete joint test considering both blocks of variables.

The second metric used in this paper was to estimate a VAR for a sub set of variables (interest rate, velocity, money, consumption, and deficit). The impulse response functions based on montecarlo simulations were studied and the model reflects a good performance, especially with respect to interest rate, velocity and deficit.

The monetary implications of the model under various parameterizations were analyzed under the canonical cointegrating regression approach developed by Park (1992) and Ogaki and Park (1993). Following Stock and Watson (1993), and Braun (1994), this framework was applied to the traditional money demand equation. Once again, the model presents good predictions and is able to capture very well the income and interest elasticities previously estimated in the current literature.

The inclusion of stochastic money growth and taxes improves the ability of the model to produce a kind of behavior similar to that of the actual data. The effect on the unexpected inflation is not neutral when we link the monetary and fiscal policy, through the budget constraint. Moreover, the sensitivity of the monetary policy to the productivity shock makes the economic activity more inelastic to the productivity shock. In summary, the percentage of explanation of the volatility of consumption is almost 90%, investment 80%, capital stock 82%, hours 72%, productivity 60%, price level 120%, velocity 50%, interest rate 300%, and deficit 75%. Hence, looking at the joint tests, the model with stochastic fiscal policy shows a marked improvement, not only in terms of the nominal variables, but also in terms of the real variables.

APPENDIX I

Characterization of the U.S. Economy

FIGURE 1

MARGINAL INCOME TAXES 1947:1-1987:4.

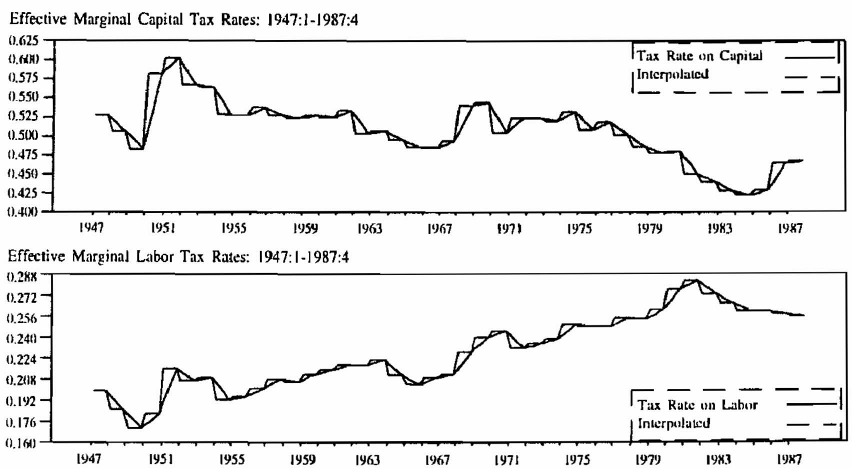


FIGURE 2a

CYCLICAL BEHAVIOR OF U.S. TIME SERIES: DEVIATION FROM TREND

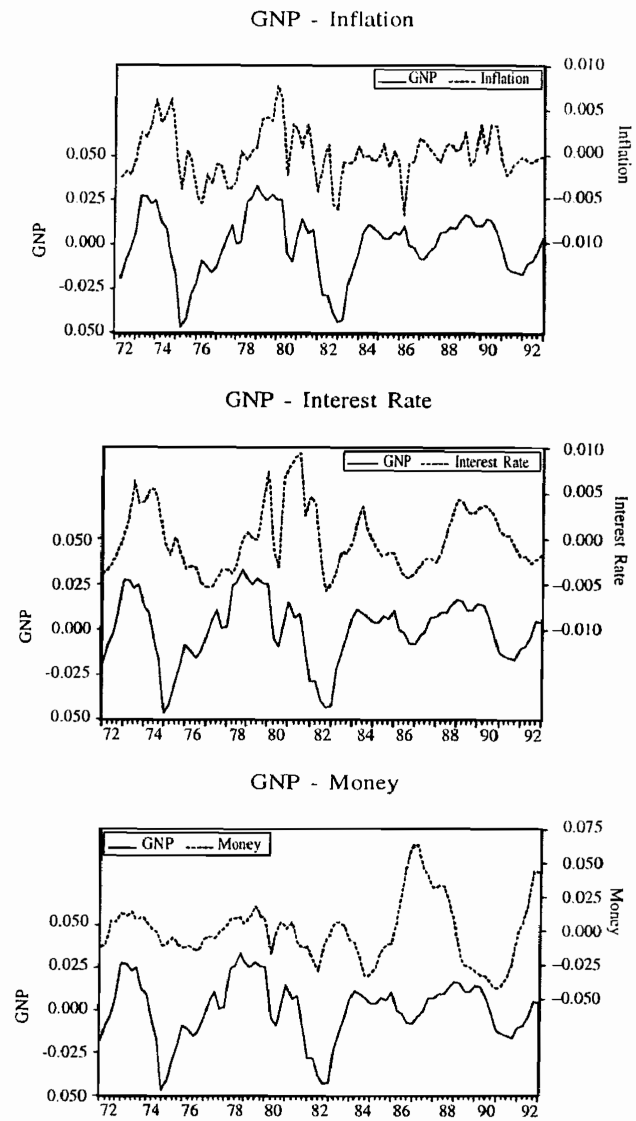


FIGURE 2b

CYCLICAL BEHAVIOR OF U.S. TIME SERIES: DEVIATION FROM TREND

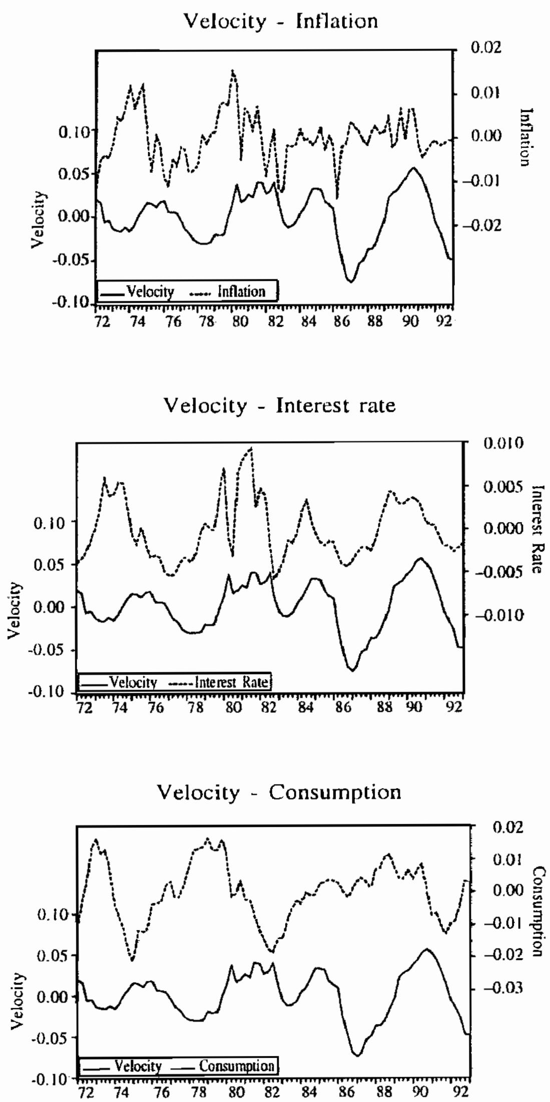
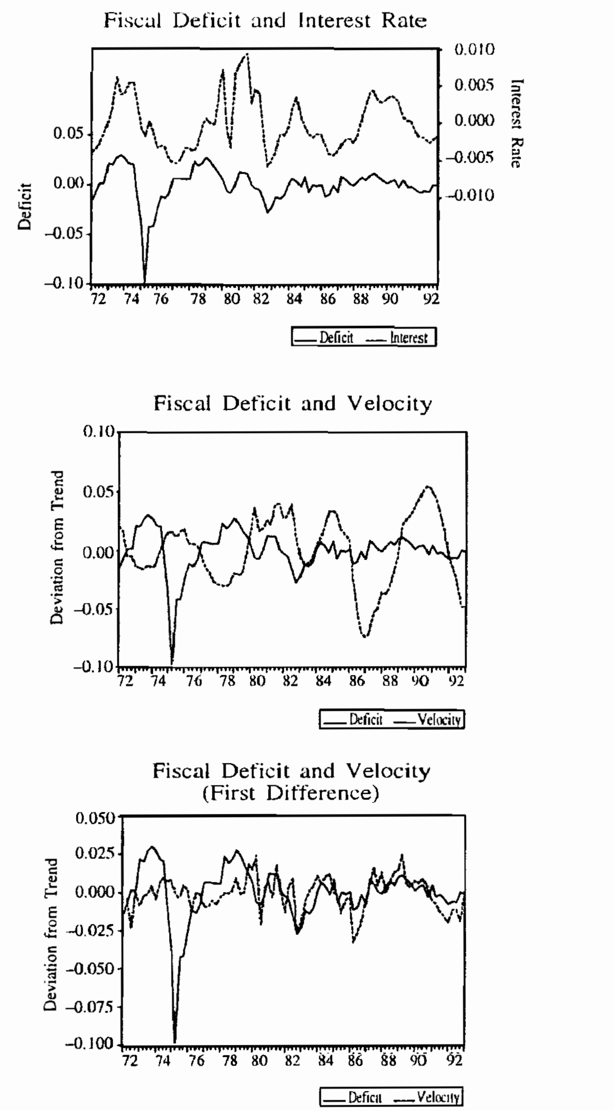


FIGURE 2c

CYCLICAL BEHAVIOR OF U.S. TIME SERIES: DEVIATION FROM TREND



APPENDIX II

Simulations Results

TABLE 2a

Standard Deviations for Artificial Economies

Series	USA:72-93	Simulated Economy with CRRA $\gamma = 0.5$					
		No Fiscal Policy		Deterministic Policy		Stochastic Taxes	
	Std Dev.	Moment	$\chi^2(1)$	Moment	$\chi^2(1)$	Moment	$\chi^2(1)$
y	1.80	1.83	0.01 (0.91)	1.83	0.01 (0.93)	1.83	0.01 (0.92)
c	0.91	0.66	7.52 (0.01)	0.54	18.30 (0.00)	0.45	32.09 (0.00)
in	6.15	5.17	1.40 (0.24)	4.98	2.16 (0.14)	5.06	1.77 (0.18)
k	0.45	0.42	0.08 (0.78)	0.41	0.18 (0.67)	0.41	0.11 (0.74)
h	2.39	1.70	6.75 (0.01)	1.44	18.37 (0.00)	1.32	27.70 (0.00)
y/h	0.96	0.22	211.1 (0.00)	0.45	35.69 (0.00)	0.58	13.08 (0.00)
cpi	1.70	2.19	2.12 (0.15)	2.12	1.56 (0.21)	2.07	1.22 (0.27)
p	0.58	1.35	38.96 (0.00)	1.26	34.77 (0.00)	1.20	31.64 (0.00)
v	2.97	1.01	413.1 (0.00)	1.03	376.6 (0.00)	1.03	379.02 (0.00)
r	0.36	1.13	53.88 (0.00)	0.99	51.14 (0.00)	0.98	50.97 (0.00)
def	1.80	1.50	1.11 (0.29)	1.14	10.71 (0.00)	1.02	18.80 (0.00)
$\chi^2(4)$		3.078 (0.545)		3.267 (0.514)		3.355 (0.500)	
$\chi^2(6)$		11.711 (0.069)		11.562 (0.072)		11.364 (0.078)	
$\chi^2(10)$		10.698 (0.382)		11.833 (0.296)		12.100 (0.278)	

The $\chi^2(1)$ statistics (and their p-values in parentheses) are for testing the null hypothesis that the statistic predicted by the model equals the corresponding value for the data in the first column. The $\chi^2(4)$ statistic tests the joint hypothesis that inflation, velocity, interest rate, and deficit moments equal the corresponding value for the data in the shaded column. The $\chi^2(6)$ tests the joint with output, consumption, investment, capital stock, labor and productivity. The $\chi^2(10)$ tests the joint that all the above moments equal the corresponding values for the data in the shaded column.

TABLE 2b

Standard Deviations for Artificial Economies

Series	USA:72-93	Simulated Economy with CRRA $\gamma = 1.00$					
		No Fiscal Policy		Deterministic Policy		Stochastic Taxes	
	Std Dev.	Moment	$\chi^2(1)$	Moment	$\chi^2(1)$	Moment	$\chi^2(1)$
y	1.80	1.84	0.02 (0.89)	1.83	0.01 (0.91)	1.83	0.01 (0.91)
c	0.91	0.80	1.01 (0.31)	0.59	12.40 (0.00)	0.50	21.86 (0.00)
in	6.15	4.27	7.10 (0.01)	4.44	5.17 (0.02)	4.50	4.77 (0.03)
k	0.45	0.36	1.08 (0.30)	0.37	0.68 (0.41)	0.38	0.58 (0.45)
h	2.39	1.75	5.50 (0.02)	1.55	12.05 (0.00)	1.41	20.00 (0.00)
y/h	0.96	0.34	120.59 (0.00)	0.44	48.21 (0.00)	0.56	17.53 (0.00)
cpi	1.70	2.25	2.68 (0.10)	2.21	2.28 (0.13)	2.19	2.11 (0.00)
p	0.58	1.41	42.93 (0.00)	1.36	41.72 (0.00)	1.33	41.58 (0.00)
v	2.97	1.21	166.9 (0.00)	1.32	108.2 (0.00)	1.38	88.79 (0.00)
r	0.36	1.21	57.34 (0.00)	1.19	52.83 (0.00)	1.27	42.06 (0.00)
def	1.80	1.20	1.10 (0.29)	1.29	0.69 (0.41)	1.47	0.23 (0.63)
$\chi^2(4)$		3.073 (0.546)		3.054 (0.549)		3.012 (0.556)	
$\chi^2(6)$		8.593 (0.198)		9.819 (0.132)		10.051 (0.123)	
$\chi^2(10)$		10.873 (0.368)		11.142 (0.347)		10.890 (0.366)	

See comments on Table 2a.

TABLE 2c

Standard Deviations for Artificial Economies							
Series	USA:72-93	Simulated Economy with CRRA $\gamma = 1.5$					
		No Fiscal Policy		Deterministic Policy		Stochastic Taxes	
	Std Dev.	Moment	$\chi^2(1)$	Moment	$\chi^2(1)$	Moment	$\chi^2(1)$
y	1.80	1.84	0.02 (0.90)	1.84	0.02 (0.90)	1.83	0.01 (0.91)
c	0.91	0.73	3.38 (0.07)	0.53	17.86 (0.07)	0.46	25.72 (0.00)
in	6.15	4.33	6.65 (0.01)	4.46	5.15 (0.02)	4.51	4.83 (0.03)
k	0.45	0.36	1.02 (0.31)	0.37	0.69 (0.41)	0.37	0.61 (0.44)
h	2.39	1.64	8.40 (0.00)	1.47	16.26 (0.00)	1.33	26.79 (0.00)
y/h	0.96	0.23	276.2 (0.00)	0.39	68.62 (0.00)	0.54	21.51 (0.00)
cpi	1.70	2.16	1.99 (0.16)	2.10	1.50 (0.22)	2.07	1.27 (0.26)
p	0.58	1.32	37.79 (0.00)	1.24	33.96 (0.00)	1.30	32.40 (0.00)
v	2.97	1.14	215.95 (0.00)	1.22	152.7 (0.00)	1.26	133.57 (0.00)
r	0.36	1.14	54.82 (0.00)	1.12	51.19 (0.00)	1.21	39.69 (0.00)
def	1.80	1.05	11.07 (0.00)	1.15	12.71 (0.00)	1.33	5.27 (0.02)
$\chi^2(4)$		3.173 (0.529)		3.167 (0.530)		3.091 (0.543)	
$\chi^2(6)$		9.382 (0.153)		10.340 (0.111)		10.368 (0.110)	
$\chi^2(10)$		11.550 (0.316)		11.856 (0.295)		11.553 (0.316)	

See comments on Table 2a.

TABLE 3a

Standard Deviations for Artificial Economies							
Correl. y With	USA:72-93	Simulated Economy with CRRA $\gamma = 0.5$					
		No Fiscal Policy		Deterministic Policy		Stochastic Taxes	
	Correl.	Moment	$\chi^2(1)$	Moment	$\chi^2(1)$	Moment	$\chi^2(1)$
c	0.86	0.52	8.70 (0.00)	0.72	2.42 (0.12)	0.76	1.24 (0.27)
in	0.94	0.97	14.51 (0.00)	0.99	138.13 (0.00)	0.99	329.4 (0.00)
k	0.65	0.06	0.59 (0.44)	0.02	0.07 (0.79)	0.00	0.00 (0.99)
h	0.93	1.00	1806.3 (0.00)	0.99	282.97 (0.00)	0.98	93.69 (0.00)
y/h	-0.45	0.63	290.5 (0.00)	0.88	2739.2 (0.00)	0.91	4061.3 (0.00)
cpi	-0.55	-0.16	3.68 (0.06)	-0.18	3.22 (0.07)	-0.16	3.45 (0.06)
p	0.46	-0.16	3.68 (0.06)	-0.13	21.21 (0.00)	-0.12	19.20 (0.00)
Correlation of Velocity with:							
y	-0.23	-0.02	1.99 (0.16)	0.00	2.19 (0.14)	0.00	2.23 (0.13)
p	0.04	0.88	31.10 (0.00)	0.91	31.30 (0.00)	0.93	31.93 (0.00)
def	-0.20	-0.90	684.08 (0.00)	-0.92	953.5 (0.00)	-0.90	686.7 (0.00)

The $\chi^2(1)$ statistics (and their p-values in parentheses) are for testing the null hypothesis that the statistic predicted by the model equals the corresponding value for the data in the first shaded column.

TABLE 3b

Correlations for Artificial Economies							
		Simulated Economy with CRRA γ = 1.00					
Correl. y With	USA:72-93	No Fiscal Policy		Deterministic Policy		Stochastic Taxes	
	Correl.	Moment	$\chi^2(1)$	Moment	$\chi^2(1)$	Moment	$\chi^2(1)$
c	0.86	0.67	3.64 (0.06)	0.68	2.31 (0.13)	0.71	1.54 (0.21)
in	0.94	0.97	7.55 (0.01)	0.98	58.39 (0.00)	0.99	123.25 (0.00)
k	0.65	0.02	0.04 (0.85)	-0.02	0.05 (0.82)	-0.03	0.13 (0.72)
h	0.93	0.98	62.5 (0.00)	0.98	35.99 (0.00)	0.97	13.77 (0.00)
y/h	-0.45	0.34	15.54 (0.00)	0.70	112.1 (0.00)	0.80	318.24 (0.00)
cpi	-0.55	-0.05	5.86 (0.02)	0.06	8.67 (0.00)	0.11	10.35 (0.00)
p	0.46	-0.07	18.86 (0.00)	-0.01	14.05 (0.00)	0.02	12.28 (0.00)
Correlation of Velocity with:							
y	-0.23	0.33	14.01 (0.00)	0.40	17.71 (0.00)	0.43	20.68 (0.00)
p	0.04	0.72	20.83 (0.00)	0.72	21.18 (0.00)	0.72	21.56 (0.00)
def	-0.20	0.22	1.22 (0.27)	0.27	1.96 (0.16)	0.26	2.20 (0.14)

The $\chi^2(1)$ statistics (and their p-values in parentheses) are for testing the null hypothesis that the statistic predicted by the model equals the corresponding value for the data in the first shaded column.

TABLE 3b

Correlations for Artificial Economies							
		Simulated Economy with CRRA γ = 1.5					
Correl. y With	USA:72-93	No Fiscal Policy		Deterministic Policy		Stochastic Taxes	
	Correl.	Moment	$\chi^2(1)$	Moment	$\chi^2(1)$	Moment	$\chi^2(1)$
c	0.86	0.69	3.20 (0.07)	0.75	1.18 (0.28)	0.80	0.45 (0.50)
in	0.94	0.97	17.99 (0.00)	0.99	189.9 (0.00)	0.99	432.9 (0.00)
k	0.65	0.02	0.014 (0.84)	-0.02	0.05 (0.83)	-0.03	0.13 (0.72)
h	0.93	1.00	9231.7 (0.00)	1.00	2497.4 (0.00)	0.99	760.0 (0.00)
y/h	-0.45	0.87	1207.6 (0.00)	0.94	6922.2 (0.00)	0.96	13319.2 (0.00)
cpi	-0.55	0.01	7.24 (0.01)	0.11	9.86 (0.00)	0.16	11.37 (0.00)
p	0.46	-0.05	16.00 (0.00)	0.01	11.90 (0.00)	0.04	10.31 (0.00)
Correlation of Velocity with:							
y	-0.23	0.45	27.94 (0.00)	0.51	35.26 (0.00)	0.55	41.93 (0.00)
p	0.04	0.76	31.75 (0.00)	0.79	35.61 (0.00)	0.79	38.93 (0.00)
def	-0.20	0.39	16.61 (0.00)	0.44	29.08 (0.00)	0.42	33.09 (0.00)

The $\chi^2(1)$ statistics (and their p-values in parentheses) are for testing the null hypothesis that the statistic predicted by the model equals the corresponding value for the data in the first shaded column.

APPENDIX III

Unit Roots, Cointegration, Causality and Dimension of the Model

TABLE 4

AUGMENTED DICKEY-FULLER UNIT ROOT TESTS: U.S. DATA 1972.1-1993.1

Variable	1 Lag			2 Lags			4 Lags		
	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
CPI	2.33	-2.70	-1.12	1.86	-2.67	-1.36	1.44	-2.57	-1.62
Money	4.68	0.35	-2.17	3.43	0.59	-2.89	3.31	0.46	-2.73
GNP	3.12	-0.64	-2.72	2.80	-0.60	-2.93	2.77	-0.13	-2.96
Interest rate	-0.70	-2.02	-2.10	-0.56	-1.43	-1.53	-0.76	-1.69	-1.75
Real Money	0.77	-0.30	-1.55	0.54	-0.62	-2.10	0.59	-0.64	-2.21
Inflation	-1.20	-2.68	-3.59	-0.79	-1.79	-2.82	-1.05	-2.12	-3.08
Velocity	0.66	-2.34	-1.15	0.50	-2.88	-1.79	0.33	-2.73	-1.84
Consumption	4.70	-1.05	-1.60	3.97	-0.93	-1.70	3.12	-0.62	-2.22
Deficit	-1.65	-2.77	-2.80	-1.95	-3.39	-3.40	-1.69	-3.50	-3.46

All the variables but inflation are in logs. Column (1) tests unit root with ADF, column (2) tests unit root with ADF and constant, while column (3) tests unit root with ADF, constant and trend. Critical Values are: col (1): -2.59 (1%), -1.94 (5%), -1.62 (10%); col (2): -3.51 (1%), -2.90 (5%), -2.59 (10%); col (3): -4.07 (1%), -3.46 (5%), -3.16 (10%). A shadowed cell means reject with 5%.

TABLE 5

COINTEGRATION TESTS: U.S. DATA 1972.1-1993.1

X\Y	cpi	m	y	r	m/p	p	v	def	c
cpi		-1.34	-0.73	-2.22	-1.97	-1.79	-2.69	-1.13	-1.72
m	-2.48		-2.63	-2.50	-1.97	-2.57	-1.94	-2.57	-2.84
y	-2.73	-2.97		-2.98	-2.74	-3.16	-2.86	-2.59	-2.26
r	-2.86	-1.66	-1.83		-2.71	-1.25	-2.78	-1.84	-2.01
m/p	-2.48	-1.34	-1.44	-2.56		-1.86	-2.40	-1.71	-1.90
p	-2.89	-3.11	-3.16	-2.46	-2.92		-2.90	-3.34	-3.02
v	-2.94	-0.79	-1.39	-2.43	-2.17	-1.58		-1.55	-1.90
def	-3.36	-3.41	-3.27	-3.45	-3.34	-3.53	-3.40		-3.29
c	-2.44	-2.58	-1.11	-2.58	-2.16	-2.88	-2.40	-1.88	

H_0: No Cointegration and Unit Root in the residuals (ADF test with constant, trend, and 4 lags). Critical Values are -4.53 (1%), -3.90 (5%), and -3.59 (10%).

TABLE 6

AUGMENTED DICKEY-FULLER UNIT ROOT TESTS: U.S. DATA DETRENDED BY HP-FILTER 1972.1-1993.1

Serie	1 Lag			2 Lags			4 Lags		
	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
r	-3.72	-3.70	-3.68	-2.65	-2.64	-2.63	-3.00	-2.98	-2.96
m	-2.86	-2.83	-2.79	-3.42	-3.39	-3.31	-3.23	-3.20	-3.14
c	-3.03	-3.01	-2.99	-2.98	-2.96	-2.94	-3.59	-3.57	-3.62
v	-2.86	-2.84	-2.80	-3.35	-3.33	-3.26	-3.88	-3.86	-3.79
def	-3.37	-3.35	-3.33	-4.00	-3.98	-3.95	-3.91	-3.88	-3.86

All series are logged, and detrended by the Hodrick-Prescott filter. For critical values see footnote to Table 4. Shaded cells means reject at 5%.

TABLE 7

GRANGER CAUSALITY TESTS FOR U.S. DETRENDED DATA

X1\X2	r	dm	dc	dv	def
r		3.6278 (0.0604)	3.9183 (0.0512)	1.6341 (0.2048)	2.8765 (0.0937)
dm	16.4732 (0.0001)		0.1813 (0.6714)	10.6133 (0.0016)	2.9691 (0.0887)
dc	30.2945 (0.0000)	0.2584 (0.6126)		3.3092 (0.0726)	7.4993 (0.0076)
dv	9.0985 (0.0034)	7.9585 (0.0060)	0.3064 (0.5814)		3.8063 (0.0546)
def	2.0778 (0.1533)	1.6315 (0.2052)	12.7110 (0.0006)	1.2132 (0.2740)	

H_0: X1 is not Granger caused by X2. The F-tests were done with 1 and 4 lags not reporting significant differences. This table reproduces the results with 1 lag. P-values reported in parentheses.

TABLE 8

ESTIMATING THE DIMENSION OF THE MODEL: AKAIKE, HANNAN-QUINN AND SCHWARZ CRITERIA

Number of Lags	$\ln\lvert\hat{\Sigma}\rvert$	Akaike Information Criterion (AIC)	Hannan and Quinn Criterion (HQ)	Bayesian Information Criterion (BIC)
1	-53.95898	-53.3261	-53.0257	-52.5762
2	-54.76144	-53.4956	-52.8948	-51.9960
3	-55.43006	-53.5313	-52.6301	-51.2819
4	-56.01248	-53.4808	-52.2792	-50.4815
5	-56.50851	-53.3440	-51.8419	-49.5948
6	-56.94324	-53.1458	-51.3434	-48.6468

See text for a description of the tests. The VAR is form by r-dv-dm-dc-def, ordered following the results of the Granger-causality test (see Table 7). We must choose the minimum value.

FIGURE 3

IMPULSE RESPONSE FOR THE U.S. ECONOMY: ACTUAL DATA