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INPUT-OUTPUT DEEPENING AND EDUCATION IN AN AGGREGATIVE MODEL OF ECONOMIC GROWTH

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Abstract

This paper shows that a multisector model of economic growth where interindustry linkages are differentiated across sectors and technological diversification is characterized by a growing use of intermediate inputs, may be useful to analyze the process of development and industrial take-off. Education is assumed to be the driving force of technological diversification. The model yields that government subsidy to education is a necessary but not sufficient requirement for industrial take-off to occur. After aggregation, the optimal solution for the steady-state dynamic path yields that factor rewards increase with the quality of education. Local stability analysis yields that some minimum quality of education is required for long-run economic growth.

I. Introduction

Recent progress in the theory of economic growth has created new tools to understand the mechanics of sustained growth and the fundamental causes of industrial take-offs. However, some questions about economic development still puzzle the analysts: Why capital from developed economies does not flow to poor countries where the relative scarcity of capital should imply a high marginal productivity? Why industrialized economies enjoy higher wages than underdeveloped economies? Why newly industrialized countries experienced quick increases in real wage even though their labour supply was thought initially as "unlimited"?

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This paper shows that a model of economic growth embodying both education and an important feature of industrialization may be useful to answer, at least partially, the above questions. The referred feature is the following:

> "As countries industrialize, their productive structures become more "roundabout" in the sense that a higher proportion of output is sold to other producers rather than to final users" (Chenery and Syrquin, "Typical Patterns of Transformation", in Chenery, Syrquin and Robinson, 1986, p. 57).

Following these authors, we shall refer to the process leading to greater "roundaboutness" as input-output deepening.

Chenery and Syrquin distinguish two parts of input-output deepening:

> "(...) first, a shift in output mix toward manufacturing and other sectors that use more intermediate inputs; and second, technological changes within a sector that lead to a greater use of intermediate inputs" (idem.).

From this definition, it follows that an analysis of input-output deepening requires a multisector model where each economic activity demands a characteristic mix of intermediate goods. With this approach the model gains realism. An examination of any country's input-output matrix shows that there exists different degrees of backward technological integration: some economic activities require more inputs and some require less. Similarly there exists different degrees of forward technological integration: some sectors provide intermediate goods and/or final goods, and those sectors providing intermediate goods serve more or less economic activities. Hence, in this paper I build an economic model where interindustry linkages are differentiated across sectors. In addition, I assume that technological diversification comes with a growing degree of backward technological integration. These features are incorporated in the model by assuming that the technological structure of all past, present and future intermediate goods is described by a triangular superior input-output matrix.

At this level it is convenient to consider the relationship of this paper with the literature on technological diversification. Whilst input-output deepening implies technological diversification, the reverse is not necessarily true. In fact, models of endogenous growth based on technological diversification have assumed identical technologies for all inputs at any moment in time (Romer, 1987, 1990; Grossman and Helpman, 1991; Aghion and Howitt, 1992). Thus, the triangular structure of the input-output matrix is the main innovation of this paper. There is another important difference between this paper and the previous literature. Since the main interest of this paper is the economy of underdeveloped countries, I do not consider technological innovation as the source of technological diversification. I assume rather that education is the driving force of technological diversification by inducing technological transfers from foreign industrialized economies. This last point will be considered more carefully after examining some prominent concepts of Hirschman's and Leontief's visions of economic development.

Incorporating input-output deepening in growth models is not only important for realism, but also because it imposes some sequential process of technological diversification. If international trade is not a good substitute for the domestic production of intermediate inputs,[1] it follows that economic activities characterized by a low degree of backward technological integration (which implies a shallow use of intermediate inputs) should developed first than economic activities characterized by large and diversified demands for intermediate inputs. This vision of economic development as a sequential process of industrialization is not unrelated to Hirschman's theory of "unbalanced" economic growth (Hirschman, 1958). As is well-known, Hirschman placed in the center of his theory the differences among economic activities in interindustry technological linkages; in addition, he also considered differences in terms of consumption linkages, fiscal linkages, productive complementarities and external economies. Based on this conceptual system, Hirschman suggested that economic growth could be induced by investing in those sectors with strong forward and backward technological linkages; the steps towards the elimination of temporal disequilibria might bring about economic growth and technological diversification.

This sequential vision of economic development is also consistent with Leontief's comparison of underdeveloped and developed economies:

> "Displayed in the input-output table, the pattern of transactions between industries and other major sectors of the system shows that the more developed the economy, the more its internal structure resembles that of other developed economies.(...).
>
> Recent advances in input-output analysis and in the book-keeping of underdeveloped countries have made it possible to apply the technique to a number of these economies. Their input-output tables show that in addition to being smaller and poorer they have internal structures that are different, because they are incomplete, compared with the developed economies. From such comparative studies a fundamental analytical approach to the structure of economic development is now emerging" (Leontief, 1963; reprinted in Leontief, 1986, p. 163).

Leontief shows that underdeveloped economic structures are characterized by a relative lack of technological linkages; developed economies provide then the technological horizon for underdeveloped economies to follow. Hirschman indicates that underdeveloped economies tend to follow development paths according to the order imposed by the degree of backward technological integration: inputs are developed first.[2] Thus, as followers, developing economies depend mainly on technological transfers from advanced industrialized economies in order to "complete" their internal economic structures. Technological innovation may have a role in the industrialization process; but it does not make sense to design goods if they are already invented. I do not mean to deny the possibility of important technological breakthroughs in developing countries -especially if they are forced to develop their own solutions for specific technological problems. However, it is clear that the non-rival nature of technological information and also the limited possibility of excluding developing countries from using the technologies previously discovered in industrialized countries, make it advantageous for developing countries to become specialized in copying and adapting existing technologies.

Although technological information about established industries is usually a public good, its use does not come without cost. Technological transfers require educated agents, and education implies, in turn, a previous effort of investment in human capital. Thus, the second innovation of the model follows: education is a necessary condition for inducing economic diversification in underdeveloped economies. This assumption is quite relevant for newly industrialized countries; the experience of Japan, Korea and Taiwan shows that investment in education and consequently the learning process of foreign technologies, are fundamental elements of late industrialization successes (Amsden, 1989).

By assuming that technological transfers occur as a by-product of education -people once educated are better able to freely copy foreign technologies- our model does not need to depart from a competitive environment. This makes another difference between our model and models of technological change where the existence of R&D activity is justified by assuming defined property rights on innovation and a non-competitive environment. These latter assumptions support the existence of monopolistic rents on patents.

The mathematical tractability of the model is allowed by the reduction of the competitive economy to the (instantaneous) aggregate production function. This function nets intermediate goods and maps primary factors into maximum final-good output (Samuelson, 1966). Although intermediate inputs are excluded from the aggregate technology, it embodies the degree of technological diversification as an externality enhancing the productivity of primary factors. Thus, as in Romer's 1987 paper, the multisector model becomes an aggregative model with externalities.

Since education is the fundamental source of technological diversification in the model, the optimal solution for the steady-state dynamic path shows that economic growth depends on the quality of education. However, even with government intervention, poor quality education might imply a poverty trap. In addition, the competitive solution for this economy does not allocate resources to education and yields no long-run economic growth. Hence, government intervention through education subsidies might be Pareto improving. These results are consistent with the great effort of newly industrialized countries in education and also with one of the more robust findings of cross-country regression analysis of economic growth: growth rates are positively correlated with initial rates of schooling (Levine and Renelt, 1992).

The rest of our paper is organized as follows. The second section defines the basic features of the growth model. We solve for the instantaneous competitive equilibrium and deduce the aggregate production function in the third section. In the fourth section we find for the steady-state dynamic paths of the aggregative model. Finally, the fifth section includes a brief summary and some conclusions.

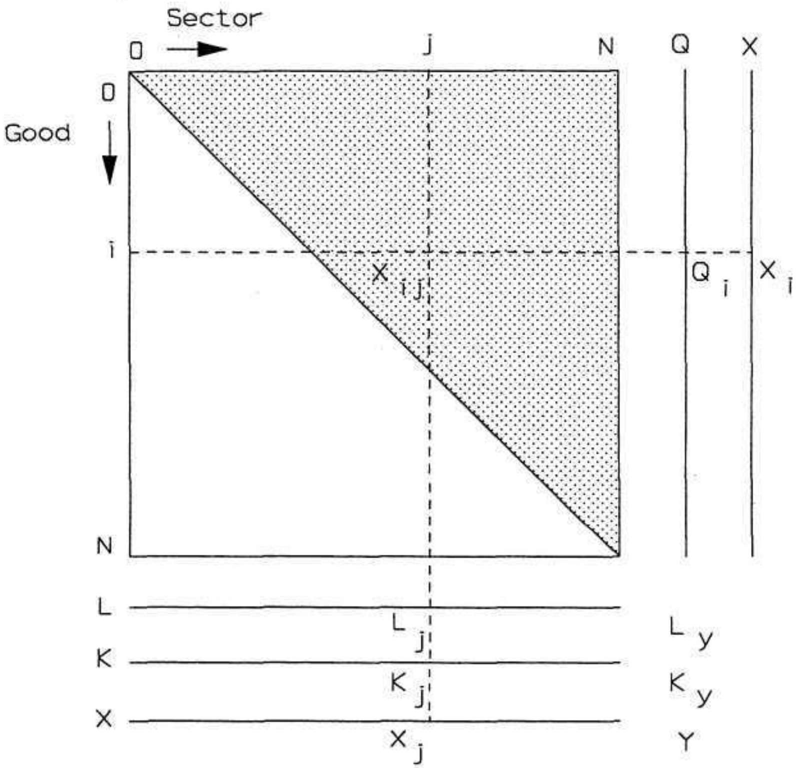
II. The Model

The technological structure of the economy is represented by Figure 1. This is an input-output matrix augmented with the vector of workforce allocation and the vector of capital allocation. The economy consists of N+1 productive sectors:

the final good sector and N intermediate-good sectors. Joint production is excluded and all sectors (and goods) are indexed according to the degree of backward technological integration between 0 and N. This integration increases linearly with the sector's index, so that the input-output matrix is perfectly triangular.

FIGURE 1

STRUCTURE OF TECHNOLOGIES



At any moment in time the final good technology is given by

$$Y = K_y^{\alpha} L_y^{\beta} \int_0^N Q_i^{1-\alpha-\beta}\, di, \tag{1}$$

where K_y is the amount of physical capital allocated to the final good sector, L_y is the labour force allocated to the same sector and Q_i is the intermediate good i used in the same sector. The final good technology is assumed to use the whole range of intermediate goods [0,N].

The final good is demanded for final consumption, C, and also as input for capital good production. It is assumed that forgone consumption of the final good is transformed one to one into capital. Hence, the equilibrium in the final good market is given by

$$Y = C + \dot{K}, \tag{2}$$

where a dot denotes a time derivative, thus a dot over K denotes investment. By simplicity it is assumed that capital is not subject to depreciation.

The technology of the intermediate good j is defined by

$$X_j = K_j^{\alpha} L_j^{\beta} \int_0^j x_{ij}^{1-\alpha-\beta} \, di, \tag{3}$$

where K_j is the amount of physical capital allocated to the production of intermediate good j, L_j is the labour force allocated to the same activity, and X_{ij} is the quantity of intermediate good i used in the production of good j. Notice that all technologies, including the final good technology, are identical except for the fact that some sectors use a longer list of intermediate inputs. In this way, the model captures the observed differences in backward technological integration across sectors.

It will be assumed that intermediate goods are not suitable for final consumption. Hence, the total demand for intermediate good i is given by

$$X_i = \int_i^N X_{ij} \, dj + Q_i. \tag{4}$$

This equation says that intermediate good i is used by the final good sector and those sectors producing intermediate goods with a higher degree of backward technological integration (j > i).

In equilibrium labour must be allocated between the final good sector and the intermediate goods sector:

$$L_y + \int_0^N L_j \, dj = m, \tag{5}$$

where m is the fraction of the workforce allocated to productive activities. The labour force is assumed to be constant and normalized to 1.

For physical capital we also have a similar equilibrium condition:

$$K_y + \int_0^N K_j \, dj = K, \tag{6}$$

where K is the total stock of physical capital.

Human capital at time t is the accumulated knowledge of technologies defined by the number of existing sectors (goods), N(t). Because our economy's human capital is small compared to more diversified economies, and knowledge is assumed non-excludable, the domestic firms specialize in appropriating foreign technologies. However, this process requires educated agents. Furthermore, the assimilation of new technologies requires new skills. Hence, the process of economic diversification continues as long as the agents allocate some effort to education. In this model we will adopt the education technology of Lucas' (1988) model of human capital accumulation:

$$\dot{N}(t) = \delta\,[1-m(t)]\,N(t). \tag{7}$$

Thus the rate of expansion of varieties (and the process of input-output deepening) is proportional to both the current level of knowledge N(t), and the amount of effort allocated to education as measured by the fraction of labour force which is not working, 1-m(t). The parameter δ is an index of productivity in education.

There is only one source of "felicity" for the representative consumer: the consumption of the final good. We will assume that the consumer is infinitely lived and his/her preferences are additive and separable. A suitable utility function is the following:

$$U = \int_0^\infty e^{-\rho t} \ln\,[C(t)]dt, \tag{8}$$

where ρ is the discount rate.

The previous set of equations define the economic structure of the model.

III. The Instantaneous Equilibrium and the Aggregate Production Function

The mathematical procedure for solving the instantaneous competitive equilibrium is confined to Appendix 1. Here we limit ourselves to examining the solutions for the equilibrium prices, the equilibrium quantities, the factor price frontier and the aggregate production function.

Assuming the final good as numeraire ($p_y = 1$), the reduced-form solution for the relative price of good i is given by

$$p_i = N/i. \tag{9}$$

The solution for the equilibrium quantity of intermediate good i is given by

$$X_i = \frac{1-\alpha-\beta}{\alpha+\beta}\,\frac{Y}{N}\,\frac{i}{N}, \qquad \text{for all } i \in [0, N] \tag{10}$$

These solutions are consistent with the main characteristics of input-output deepening. At any given moment in time -given N, the relative prices of intermediate goods fall with the degree of backward technological integration, i; hence, instantaneously, the demand structure for intermediate goods is biased in favour of sectors that use more intermediate inputs. Along time, the relative price of good i increases with the process of technological diversification, N; hence the input structure shifts in favour of newer intermediate goods, i.e. those whose technologies require more intermediate inputs.

Given prices and quantities of intermediate goods [equations (9) and (10)], and the factor demands [see Appendix 1], we can find the prices of final factors from the equilibrium conditions of the primary factor markets [equations (5) and (6)]. Thus we obtain:

$$r = \frac{\alpha}{\alpha+\beta}\ \frac{Y}{K}, \tag{11}$$

$$w = \frac{\alpha}{\alpha+\beta}\ \frac{Y}{m}. \tag{12}$$

where r is the rental price of capital and w is the wage rate.

The solution for the factor-price frontier is given by the following expression:

$$r^{\alpha}\, w^{\beta} = \alpha^{\alpha}\, \beta^{\beta}\, (\alpha+\beta)^{\alpha+\beta}\, (1-\alpha-\beta)^{1-\alpha-\beta}\, N^{\alpha+\beta} \tag{13}$$

Notice that this frontier shifts to the right with the degree of economic diversification, N.

Substitution of the factor prices into the factor-price frontier yields the aggregate final-good production function of this economy:

$$Y = A\, K^{\frac{\alpha}{\alpha+\beta}}\, m^{\frac{\beta}{\alpha+\beta}}\, N, \tag{14}$$

where $A \equiv (\alpha+\beta)^2\,(1-\alpha-\beta)^{(1-\alpha-\beta)/(\alpha+\beta)}$ is a positive constant. The aggregate technology is homogeneous of degree one in physical capital and labour. However, taking into account human capital (which here corresponds to the number of sectors) the economy experiences increasing returns at the aggregate level. Since instantaneously agents take as given the number of sectors, a competitive equilibrium can be supported.

We have shown that the whole technological structure represented by Figure 1 is reduced under competition to the aggregate production function. This reduction corresponds to Samuelson's transformation of gross output functions into net output functions (Ortiz, 1993, 1995). As proved by Samuelson (1966), the necessary conditions for this transformation are constant returns to scale in the production of intermediate inputs and final goods, no joint production and perfect competition. All these conditions are satisfied by our model.

IV. The Dynamic Behaviour

The last section describes the instantaneous supply side of the final good. Let us now turn to the demand side. The consumer maximizes the discounted stream of utility [equation (8)] subject to the aggregate technology [equation (14)], the transition equation of physical capital [equation (2)], and the transition equation of education [equation (7)].

On this basis we have to consider two situations. Firstly, the competitive equilibrium solution and, secondly, the optimal growth path. Given the existence of an externality these two paths are not equal. The first is easy to find: from the aggregate production function one deduces that the human capital externality has no effect on the private marginal productivity of factors -economic diversification increases total factor productivity but this effect is not partially captured by capital or labour. In addition private agents ignore the external benefits from economic diversification and so allocate instantaneously all available time to production. This means that the degree of diversification is fixed and the transition equation of education [equation (7)] disappears. The model collapses into the Ramsey-Cass-Koopmans model of economic growth characterized by decreasing returns to capital and zero growth in the steady state.

In order to find the optimal growth path we assume the existence of some kind of authority -government- that internalizes the externality effect. The Hamiltonian related to this problem is as follows:

$$H(\dots) = \text{Max } \{e^{-\rho t} \ln C(t) + \lambda_1(t) [A\, K(t)^{\frac{\alpha}{\alpha+\beta}} m(t)^{\frac{\beta}{\alpha+\beta}} N(t) - C(t)] + \lambda_2(t)\, N(t)\, [1-m(t)]\, \delta\},$$

where the arguments of the Hamiltonian are $C(t)$, $K(t)$ and the multipliers $\lambda_1(t)$ and $\lambda_2(t)$.

The necessary conditions of maximization are the following:

$$(15) \qquad H_c(\dots) = 0, \quad \text{or } C(t)^{-1} = \theta_1(t)$$

$$(16) \qquad H_m(\dots) = 0, \quad \text{or } \theta_1(t) \frac{\beta A}{\alpha+\beta} \left[\frac{K(t)}{m(t)}\right]^{\frac{\alpha}{\alpha+\beta}} = \theta_2(t)\, \delta$$

where $\theta_1(t)=e^{\rho t}\lambda_1(t)$ and $\theta_2(t)=e^{\rho t}\lambda_2(t)$ are the present value multipliers of the state variables, K and N respectively. Along the optimal path these multipliers obey the following processes:

$$(17) \qquad \frac{\dot{\theta}_1}{\theta_1} = \rho - \frac{\alpha A}{\alpha+\beta} \left[\frac{m(t)}{K(t)}\right]^{\frac{\beta}{\alpha+\beta}} N(t),$$

$$(18) \qquad \frac{\dot{\theta}_2(t)}{\theta_2(t)} = \rho - \delta - \frac{\alpha\, \delta}{\beta}\, m(t).$$

We also need a couple of transversality conditions:

$$(19) \qquad \lim_{t\to\infty} e^{-\rho t}\, \theta_1(t)\, K(t) = 0,$$

$$(20) \qquad \lim_{t\to\infty} e^{-\rho t}\, \theta_2(t)\, N(t) = 0.$$

In general, the dynamics of two-sector models of endogenous growth is complex and not well understood. Although some advances have been made on this subject (Lucas, 1988; Mulligan and Sala-i-Martin, 1992; Barro and Sala-i-Martin, 1993), in many cases analytical solutions are not feasible at all. As one might expect, the dynamics of our model is also complex. Hence, in this section we will examine only the balanced growth path, i.e. the dynamic path characterized by a constant allocation of time between production and education, and constant growth rates of consumption, physical capital and human capital. However, local convergence to this path is examined in the Appendix 2. There we deduce that our economy might converge locally to the balanced path (with unbounded growth) if the education system is sufficiently efficient (high ∂). However, we also show that if education efficiency is too high, the economy does not converge to the balanced growth path -this gives the possibility of increasing growth rates. If education efficiency is low, the economy might experience either negative growth or convergence to a steady state with no growth.

We will drop the time subscript from variables moving along the balanced path and denote them with a star. First we notice from equation (15) that, along the balanced path, the shadow price of capital, θ_1, grows at a constant rate equal to minus the growth rate of consumption:

$$\frac{\dot{C}^*}{C^*} = -\frac{\dot{\theta}_1^*}{\theta_1^*}$$

Hence, from equation (17) we deduce that, along the balanced path, human capital, N, grows at a fraction $\beta/(\alpha+\beta)$ of the growth rate of physical capital:

$$\frac{\dot{N}^*}{N^*} = \frac{\beta}{\alpha+\beta}\frac{\dot{K}^*}{K^*}$$

Using equation (7) we obtain:

$$\frac{\dot{K}^*}{K^*} = \frac{\alpha+\beta}{\beta}(1-m^*)\delta,$$

where m^* is the fraction of workforce allocated to productive activities along the balanced path. Now, differentiating the equations (15) and (16) with respect to time and using the last equation and the equation (18) we obtain:

$$\frac{\dot{C}^*}{C^*} = \frac{\alpha+\beta}{\beta}\delta - \rho, \tag{21}$$

which is the balanced growth rate of consumption and, as we will see immediately, the balanced growth rate of this economy.

Dividing the transition equation of physical capital [equation (2)] by K(t), and taking into account the aggregate technology [equation (14)] we obtain:

$$\frac{C\,(t)}{K\,(t)} + \frac{\dot{K}\,(t)}{K\,(t)} = A\left[\frac{m\,(t)}{K\,(t)}\right]^{\frac{\beta}{\alpha+\beta}} N\,(t).$$

Notice that along the balanced path the right hand side of this expression is constant, it corresponds to the average product of capital. Since the growth rate of physical capital is also constant along this path, we deduce that physical capital and consumption grow equiproportionally. Thus we can deduce the balanced-path allocation of workforce to production:

$$m^* = \frac{\beta}{\alpha + \beta}\,\frac{\rho}{\delta} > 0 \tag{22}$$

Note that this fraction is always positive. An interior solution ($m^* < 1$) requires the productivity parameter in education to be sufficiently high: $\delta > \rho\beta/(\alpha+\beta)$. Note that this inequality is also the condition for a positive growth rate. Hence, in this model we reach the conclusion that education efficiency and economic growth are closely associated. The links between education efficiency and economic growth are the following. A higher quality of education rises the expectations of future returns because a better education training implies a quicker diversification of intermediate goods which, in turn, increases the productivity of primary factors; thus the effort invested in education also rises leading to a more technologically integrated economic structure and a higher rate of economic growth.

Since the marginal product of capital is constant along the balanced path, the balanced-path rental rate of capital is also constant. By combining equations (11), (14), (15), (17) and (21) we deduce:

$$r^* = \frac{\alpha + \beta}{\beta}\,\delta \tag{23}$$

Thus, this model predicts that the balanced-path rental price of capital is high in countries with high education efficiency (high δ). This result may help to explain why capital does not flow from industrialized countries to non-industrialized countries in order to take advantage of the usually much cheaper available labour -the return to capital is not necessarily higher in countries with cheap labour, but tend to be high in countries with high education standards.

Now, combination of equations (11), (12), (22) and (23) yields

$$w^* = \frac{(\alpha + \beta)^2}{\alpha\,\beta}\,\frac{\delta^2}{\rho}\,K^* \tag{24}$$

Thus, along the balanced growth path, the wage rate grows at the same rate of physical capital and final output. Since along the balanced path physical capital and human capital (the number of sectors) grow together, the last equation implies that countries with a high degree of interindustry technological linkages enjoy high wages. Therefore, our model helps to explain why there exists such a migration pressure from underdeveloped countries to industrialized countries. Dynamic consequences for the real wage can also be deduced from equation (24); since economic growth depends directly on education efficiency [see equation (21)], real wages increase quicker in economies with better educative systems and consequently high rates of technological diversification. This result may help to explain why newly industrialized countries experienced a quick increase in real wages from the beginning of the industrial take-off (Amsden, 1989).

Finally, let us add that the two transversality conditions hold if the discount rate, ρ, is positive. This completes the characterization of the dynamic behaviour of this economy in the steady state.

V. Summary and Concluding Comments

In section II we set up an economy composed of a continuum set of industries characterized by a growing degree of interdependence. They only produce intermediate goods. We also add a single final-good sector that uses the whole range of intermediate inputs. The final good can be used for final consumption or for the production of capital goods. Physical capital and labour are the primary factors used by all sectors.

In section III and Appendix 1 we deduce this economy's aggregate production function, which is understood as the function that maps primary factors into maximum net output. As proved by Samuelson, the necessary conditions for this transformation are constant returns to scale in the production of basic inputs, no joint production and perfect competition (Samuelson, 1966). All these conditions are satisfied by our model.

Given the structure of our economy, the aggregate production function shows that the higher the degree of intersectoral integration (which is here equal to the number of intermediate sectors), the higher the level of productivity. Because private agents take the number of sectors (and goods) as given, they perceive constant returns to scale in their own activities, making a competitive equilibrium possible. The allocation of effort to education increases the number of sectors and then the society's aggregate technology is characterized by increasing returns to scale.

In section IV we show that allocation of effort to education is not achieved automatically in a competitive economy because in our model education has no private effects on the productivity of labour or capital. In addition, private agents ignore the external effect of education on economic diversification. Thus, a government subsidy that fills the gap between the private and the social valuation of education might decentralize the optimal allocation of resources. In this situation, the aggregate technology is characterized by increasing returns to physical capital, labour and human capital so that unbounded growth is made possible. This feature depends critically on the achievement of some minimum level of education efficiency. Thus government subsidy to education is a necessary but not sufficient condition for long-run economic growth. In this case, the model yields a direct relationship between real wages and the degree of intersectoral integration in the economy. We also find that the steady-state rental rate of physical capital increases with the coefficient of education efficiency.

These results are interesting because they may explain, at least partially, why capital does not flow from industrialized countries to non-industrialized countries and why there exist migration pressures from underdeveloped countries to industrialized countries. They also may help to explain why newly industrialized countries, which invested heavily in education, experienced through learning intense processes of technological diversification and quick increases in real wages.

The analysis in section IV is focused on the characteristics of the balanced growth equilibrium, which implies constant growth rates. However, in the Appendix 2 we hypothesize that for very high coefficients of education efficiency, the economy might enjoy increasing rates of growth. We also postulate that for very low coefficients of education efficiency the economy might converge to zero growth. The Appendix 2 also shows that there exists an intermediate range of coefficients of education efficiency which is consistent with balanced path equilibrium. An analysis of global stability could provide valuable insights into the process of economic development.

Notes

1. Intermediate goods may be not transable because of prohibitive tariffs and/or large transport costs. Even if intermediate goods are transable, domestic production may still be preferable because of the strategic importance of quick and opportune provision (Porter, 1990) or the importance of technological domestic linkages for the transmission of economic externalities (Barstelman, Caballero and Lyons, 1991).
2. It may be convenient to state that Hirschman's vision of development does not imply a rigid line of industrialization; in fact, he does consider the possibility of different ways of development according to initial conditions. Furthermore, he even considers the possibility of jumping steps and following quite particular industrialization paths (Hirschman, 1958).

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APPENDIX 1
The Instantaneous Solution for the Competitive Equilibrium

We assume that firms are price takers and maximize profits. The solution of this problem for the sector producing the intermediate good j yields the following factor demands:

$$K_j = \alpha\, p_j\, X_j / r, \tag{A1.1}$$

$$L_j = \beta\, p_j\, X_j / w, \tag{A1.2}$$

$$X_{ij} = [(1-\alpha-\beta)\, p_j / p_i]^{\frac{1}{\alpha+\beta}}\, K_j^{\frac{\alpha}{\alpha+\beta}}\, L_j^{\frac{\beta}{\alpha+\beta}}, \quad i \in [0, j] \tag{A1.3}$$

where r is the rental price of capital, w is the wage rate, and p_j is the price of good j.

Substitution of these factor demands into the production function of good j [equation (3)] yields a function for the price of this good in terms of the rental price of capital, the wage rate and all prices of intermediate goods used in the production of good j:

$$p_j^{-1/(\alpha+\beta)} = a\, r^{-\alpha/(\alpha+\beta)}\, w^{-\beta/(\alpha+\beta)} \int_0^j p_i^{1-1/(\alpha+\beta)}\, di,$$

where

$$a \equiv [\alpha^\alpha \beta^\beta (1-\alpha-\beta)^{1-\alpha-\beta}]^{1/(\alpha+\beta)}$$

Differentiating this equation with respect to j and integrating afterwards between zero and i yields the solution for the price of the intermediate good i:

$$pi = \frac{r^{\frac{\alpha}{\alpha+\beta}}\, w^{\frac{\beta}{\alpha+\beta}}}{(\alpha+\beta)\;\; a\, i}. \tag{A1.4}$$

We are able to obtain this solution because the output of good 0 is zero: non integrated sectors do not produce output [see equation (3)], hence the only meaningful price of good 0 is infinity.

Let us turn now to the final good sector. Here as well firms are price takers and maximize profits. We choose the final good as numeraire ($p_y = 1$). The factor demands in this sector are given by the following equations:

$$K_y = \alpha Y / r, \tag{A1.5}$$

$$L_y = \beta Y / w, \tag{A1.6}$$

$$Q_i = [(1-\alpha-\beta) / p_i]^{\frac{1}{\alpha+\beta}}\, K_Y^{\frac{\alpha}{\alpha+\beta}}\, L_Y^{\frac{\beta}{\alpha+\beta}}, \quad i \in [0, N]. \tag{A1.7}$$

Now we substitute these factor demands into the production function of the final good [equation (1)], and obtain a function relating all the prices in this economy. Using the above solution for the price of intermediate good i we solve for the factor-price frontier [equation (13)].

By combining the factor-price frontier [equation (13)] and the equation for the relative price of the intermediate good i [equation (A1.4)], we deduce the reduced-form solution for the relative price of good i [equation (9)].

Let us now solve for the quantities. We begin with the determination of intermediate good demands. Combining the equation for the equilibrium of the market of good i [equation (4)], the factor demands of sector i [equations (A1.1), (A1.2) and (A1.3)], the price of good i [equation (A1.4)], the demands for intermediate goods of the final good sector [equation (A1.7)], and the factor-price frontier [equation (13)] we deduce:

$$Xi = \frac{1-\alpha-\beta}{\alpha+\beta}\, i^{\frac{1}{\alpha+\beta}} \int_i^N \frac{X_j}{j^{1+1/(\alpha+\beta)}}\, dj + \frac{1-\alpha-\beta}{\alpha+\beta}\, \frac{Y}{n} \left(\frac{i}{N}\right)^{\frac{1}{\alpha+\beta}}$$

By differentiating this expression with respect to i we find that the gross output of good i, X_i, is unit elastic with respect to its degree of backward technological integration, i. Hence we can solve for the equilibrium quantities of intermediate goods [equation (10)].

APPENDIX 2
Local Stability in the Dynamic Model with Optimal Allocation

The system of differential equations driving the dynamics of this economy is as follows:

$$\dot{C}/C = \hat{\alpha}\ K^{\hat{\alpha}-1}\ m^{1-\hat{\alpha}}\ N - \rho,$$
$$\dot{m}/m = \delta/\hat{\alpha} + \delta m/\ (1-\hat{\alpha}) - C/K,$$
$$\dot{K} = K^{\hat{\alpha}}\ m^{1-\hat{\alpha}}\ N - C,$$
$$\dot{N} = N\ (1-m)\ \delta.$$

In order to obtain this system we have made the following transformations. First, the coefficient A in the production function [equation (14)] has been normalized to 1. We also denote $\hat{\alpha} \equiv \alpha\ (\alpha+\beta)$. The first two differential equations are obtained by differentiating equations (15) and (16) with respect to time, afterwards we substitute for the growth rates of the shadow prices of physical capital and human capital [equations (17) and (18), respectively]. The last two differential equations in the system above are the transition equation of physical capital [equation (2)] and the transition equation of education [equation (7)].

If the allocation of workforce to production (m) is constant, the second differential equation implies that physical capital and consumption grow at the same rate. From the first and the third equation we deduce that this growth rate is constant. In section IV we showed that the balanced growth rate of physical capital and consumption is given by $g \equiv (1-\hat{\alpha})^{-1}\delta-\rho$, the balanced growth rate of human capital is $g_N=(1-\hat{\alpha})g$, and the balanced path allocation of workforce to production is $m^*=(1-\hat{\alpha})\rho/\delta$. We will adopt the usual convention that starred variables are on the equilibrium growth path. We will also find it convenient to define the following balanced path constants: $\Delta=(K^*/m^*)^{\hat{\alpha}-1}N^*=\delta/(\hat{\alpha}(1-\hat{\alpha}))$ and $(C^*/K^*)=\rho+\delta/\hat{\alpha}$.

Linearizing the above system around the balanced path equilibrium, we obtain the following system of transition equations:

$$\begin{bmatrix} \dot{C} \\ \dot{m} \\ \dot{K} \\ \dot{N} \end{bmatrix} = \begin{bmatrix} g & \dfrac{\hat{\alpha}(1-\hat{\alpha})\ C^*\Delta}{m^*} & -\dfrac{\hat{\alpha}(1-\hat{\alpha})\ C^*\Delta}{K^*} & \dfrac{\hat{\alpha}C^*\Delta}{N^*} \\ -\dfrac{m^*}{K^*} & \dfrac{\delta m^*}{1-\hat{\alpha}} & \dfrac{m^*C^*}{(K^*)^2} & 0 \\ -1 & \dfrac{(1-\hat{\alpha})\ \Delta K^*}{m^*} & \hat{\alpha}\Delta & \dfrac{\Delta K^*}{N^*} \\ 0 & -\delta N^* & 0 & (1-\hat{\alpha})g \end{bmatrix} \begin{bmatrix} C-C^* \\ m-m^* \\ K-K^* \\ N-N^* \end{bmatrix} \quad \begin{bmatrix} gC^* \\ 0 \\ gK^* \\ g_N N^* \end{bmatrix}$$

With two predetermined variables, the stocks of physical and human capital (K and N), and two forward-looking variables, the choices of workforce allocation and consumption (m and C), saddle-path stability requires that the transition matrix should be characterized by two negative eigenvalues and two positive eigenvalues when evaluated along the balanced path. Thus the determinant of the transition matrix along the balanced path should be positive for saddle-path stability. Analytical solutions for the eigenvalues are extremely difficult to obtain -if not impossible. Hence, we will only examine the sign of the matrix determinant. After some algebra we obtain:

$$\det = \frac{\delta}{\hat{\alpha}}\, g \left\{ -\frac{(1-\hat{\alpha})}{\hat{\alpha}}\,\delta^2 + \rho\left[(1-\hat{\alpha})\left(\frac{2}{\hat{\alpha}}+1\right)+\frac{\hat{\alpha}}{1-\hat{\alpha}}\right]\delta \right. \\ \left. -\rho^2\left[2\,(1-\hat{\alpha}^2)+\hat{\alpha}\right]\right\}$$

Because the growth rate, g, depends on δ, the determinant is a polynomial expression of the fourth order in the coefficient of education efficiency, δ. The discriminant of the quadratic expression between curly brackets depends exclusively on $\hat{\alpha}$ and is always positive, i.e. the roots of this expression are real. All roots are positive except $\delta = 0$. These features determine the shape of the determinant function which is shown by Figure 2.

Assuming that the coefficient of efficiency in education is positive ($\delta>0$), one of the positive roots, **a**, **b** or **c**, corresponds to zero growth ($g = 0$). The highest root of the quadratic expression in curly brackets is higher than $(1-\hat{\alpha})\rho$, which is the root value associated with zero growth. Thus, the delta value associated with zero growth is either the value at point **a** or the value at point **b**. Three situations may arise:

(1) Efficiency coefficients below the root **a** would imply negauve growth. Agents might allocate instead all their time to production and converge to zero growth. Notice that if all the workforce is allocated to production, m=1, human capital, N, would be fixed. Thus the dynamic system is reduced to two variables, physical capital, K, and consumption, C, and the transition matrix would be characterized by two eigenvalues of opposite signs, which gives a negative determinant. The dynamics then might resemble the dynamics of the standard Ramsey-Cass-Koopmans model of growth with decreasing returns.

(2) If the zero-growth root is at point **a**, only coefficients of education efficiency in the range **bc** might be consistent with balanced growth; in this case the economy as a whole might experience constant returns to scale as the coefficient Δ is constant, implying constant marginal productivity of physical capital. For coefficients in the range **ab** the efficiency in education is so low that the system might be trapped again in a corner solution with no effort in education; in this case the economy might converge to a steady state with no growth. Now, for efficiency coefficients above the point **c**, the economy cannot converge locally to the balanced growth path. In this case the economy might experience increasing growth rates because the growth rate of human

capital is so high that the economy might experience increasing returns to scale.

(3) If the zero-growth root is at point **b** there are only two possibilities. For delta values in the range **bc** the economy might be consistent with balanced growth, as analyzed in the preceding situation. For delta values above the point **c**, the economy is not consistent with balanced growth; again, this situation might induce increasing rates of growth.

The above analysis is focused on local stability. The issue of global stability is not addressed at all. Thus, we do not know the behaviour of this economy for dynamic paths which are outside of the vicinity of the balanced growth path.

FIGURE 2

THE DETERMINANT OF THE TRANSITION MATRIX