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THE EFFECT OF TAXES ON GROWTH INVESTMENTS: A NOTE

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Abstract

The standard text-book presentation of growth investments does not include the treatment of taxation, an issue that was first addressed by Brenner and Venezia (1983). While their analysis can be considered complete for the no-reinvestment case, their conclusion for the reinvestment case is somewhat misleading as the effect of taxes was not separated from that of inflation. The present note explicitly shows that the introduction of taxation has no effect on the optimal rotation.

Going beyond the traditional text-book presentation, as illustrated in Allen (1938), Baumol (1972), Bierman and Smidt (1993), Henderson and Quandt (1971), Hirshleifer (1970), Simon and Blume (1994) and Varian (1990); Brenner and Venezia (1983) analyzed the effects of both taxes and inflation on growth investments. With regard to the isolated effect of taxation, it was shown that, for the case of no-reinvestment, if the function which describes the net real cashflow along time is concave, the optimal duration of the growth investment will be shortened.

For the case of reinvestment, however, those two authors did not investigate the effect of taxation by itself. Rather, their presentation analyzed the effects of the simultaneous presence of taxes and inflation, and it was concluded that an increase in the rate of inflation always increases the duration of the investment cycle.

Focusing attention on the isolated effect of taxation, the purpose of this note is two-fold. First, for the case of no-reinvestment, it will be shown that the introduction of taxation may reduce the optimal duration of the growth investment

even if the cashflow function is not concave. Second, for the reinvestment case, it will be shown that the introduction of taxation has no effect on the optimal duration of the investment cycle.

I. The No-Reinvestment Case

Adopting the notation in Brenner and Venezia [4], let

$F(T)$ = net present value of the investment
$f(T)$ = net real cashflow if investment is terminated at time T
i = real rate of interest (as measured in instantaneous terms and assumed to be positive)
C = cost of investment
T = time to termination of investment

Assuming the presence of a corporate tax rate φ and that the asset can be depreciated, for tax purposes, in a lump sum at the time of termination, it follows that:

$$F(T) = -C + \{(1-\varphi) f(T) + \varphi C\} e^{-iT} \tag{1}$$

where $f(T) - C$ is the taxable income.

The first-order condition for the maximization of $F(T)$ implies the following extension of the so-called Jevons' formula (as pointed out by Baumol [2, p. 451], it appears that the English mathematician W. S. Jevons was the first author to study growth investments):

$$i = \frac{(1-\varphi) f'(T)}{(1-\varphi) f(T) + \varphi C} \tag{2}$$

On the other hand, observing that $0 \leq \varphi < 1$ and that taxation can only occurs if the taxable income is positive, if we denote the optimal duration by T^*, the second-order condition requires that:

$$(1-\varphi)\{f(T^*) f''(T^*) - [f'(T^*)]^2\} + \varphi C f''(T^*) < 0 \tag{3}$$

a condition that may hold even if $f''(T^*) > 0$.

Differentiating (2), holding constant the interest rate, it follows from (3) that, if $f'(T^*) > 0$, which must hold as i was assumed to be positive, we have:

$$\frac{dT^*}{d\varphi} = \frac{C f'(T^*)}{(1-\varphi)\{(1-\varphi)[f(T^*) f''(T^*) - [f'(T^*)]^2] + \varphi C f''(T^*)\}} < 0 \tag{4}$$

That is, the introduction of taxation shortens the duration. A result which may occur even if the cashflow function is not concave at the optimal duration.

II. The Reinvestment Case

Supposing the possibility of reinvestment in the same asset, at the same conditions, it follows that the present value of an infinite chain of reinvestment cycles may be written as (the case of reinvestment is also known as the rotation problem and, according to Hirshleifer [6, p. 89], its solution was originally published by Faustmann in 1849):

$$R(T) = \sum_{k=1}^{\infty} F_k(T)\, e^{-i\,(k-1)T} = F_1\,(T) \,/\, (1-e^{-iT}) \tag{5}$$

$$\text{as } F_k(T) = F_1(T) = -C + \left\{(1-\varphi) f(T) + \varphi\, C\right\} e^{-iT},\ \text{k} = 1,2$$

Proceeding to the maximization of $R(T)$, it is easily seen that the optimal duration of each cycle has to satisfy the following first-order condition:

$$i = \frac{(1 - e^{-iT})\, f'(T)}{f(T) - C} \tag{6}$$

The point that should be stressed is that the above expression does not depend on the tax rate φ. Therefore, contrary to what happens in the no-reinvestment case, the introduction of taxation has no effect on the optimal duration T^{**} of each cycle in the reinvestment case.

To conclude the analysis, it is interesting to compare the solutions T^* and T^{**}. To do this it is convenient to rewrite the first-order condition for the reinvestment case as:

$$(1 - \varphi) f'(T^{**}) = i\left\{(1 - \varphi) f(T^{**}) + \varphi\, C + R(T^{**})\right\} \tag{7}$$

Taking into account that the reinvestment case makes sense only if $R(T^{**}) > 0$, if we assume that $f(T)$ is strictly increasing and concave, a direct comparison of (2) and (7) leads to the conclusion that $T^{**} < T^*$ (it suffices to verify that the assumption of $T^{**} > T^*$ leads to a contradiction).That is, the possibility of an indefinite number of reinvestments shortens the optimal duration of the cycle.

III. Conclusion

If wine ageing or forest growing, the two classical examples of growth investments, are seen as one-shot opportunities, the introduction of taxation will have the effect of reducing the optimal time of holding the asset. On the other hand, if any one of those two activities is considered as an ever-reproducing process, the introduction of taxation has no effect on the optimal rotation.

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