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ARE FINANCIAL VARIABLES INPUTS IN DELIVERED PRODUCTION FUNCTIONS? NO

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Abstract:

Fischer's classic (1974) paper develops conditions under which it is appropriate to use money as an input in a 'delivered' production function. In this paper, we extend Fischer's model I (the Baumol-Tobin inventory approach) by incorporating credit into the analysis. Our investigation of the extended model brings out a very restrictive but necessary implicit assumption employed by Fischer to treat money as an input. Namely, that there exists a binding constraint on the use of money! A similar result holds for our more general model.

I. Introduction

Production functions are often referred to as 'delivered' or 'reduced form' when they capture the effect of financial variables on output by including these variables as inputs. Is it theoretically correct to treat money, or financial variables in general, as an input in a 'delivered' production function? What are the circumstances under which it is appropriate to do so? The object of this paper is to show that, in general, the answer to the first question is no and the answer to the second question includes one condition which is quite limiting and has been completely ignored in the prior literature.

An important reason for considering this issue is the frequent reliance on the assumption of money as an input in a production function for a wide variety of purposes. For example Haliassos and Tobin (1989) note that this procedure is used to argue against the superneutrality of money, which is a subject of interest in recent literature (Fisher and Seater, 1993). Another example is the theoretical work on monetary aggregation by Barnett (1987), who cites Fischer's classic article to support

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the use of this assumption. The choice of appropriate monetary aggregates continues to attract attention (Rotemberg, Driscoll and Poterba, 1995). Finally, there is widespread use of this assumption in empirical work. For instance, almost all empirical studies cited in Sinai and Stokes' (1989) survey simply introduce money as an input. Interestingly, two exceptions to this practice, Nguyen (1986) and Betancourt and Robles (1989), find evidence that financial variables matter in production but cannot be reduced to the role of inputs. Additional evidence is provided by Robles (1995).

In his widely cited article, Fischer (1974) showed that, when money's ability to economize on transaction costs is captured through an index of labor resources released for the production of output, money can be treated as an input in a 'delivered' production function. He also showed that money's ability to economize on transaction costs facilitated production but by itself was not sufficient to treat it as an input[1]. Here we will generalize Fischer's model I, the Baumol-Tobin inventory approach to the demand for money, by incorporating credit into the analysis. This generalization brings together credit needs for working capital purposes and the issue of cash management stressed by Fischer. It is easy to demonstrate in this setting that similar arguments to those which apply to money as an input in a 'delivered' production function also apply to credit[2] We will show in this more general model that a necessary condition for the financial variable to be an input in a 'delivered' production function is that there exists an arbitrary binding constraint on one of the variables in the model. We will also show how this result applies to Fischer's model I and to his model II (the vending machine example).

Our argument proceeds in the following manner. In the next section we present a theoretical model which generates the firm's demand for money and credit when credit is needed for working capital purposes and money economizes on transaction costs by saving on labor resources. In the following section we develop the implications of the model with respect to the use of financial variables as inputs in 'delivered' production functions. Finally, a brief conclusion offers some perspective on the results and on the role of financial variables in production.

II. A Model of the Demand for Credit and Money by Firms

In this section we develop a model of firm behavior that allows us to capture the fundamental role of credit in the productive process as a means of synchronizing the receipt of payments for output and the disbursement of payments for inputs. We shall assume that the firm needs to pay for some or all of its inputs prior to the receipt of payment for its output. Hence, at the beginning of the period, the firm takes out a loan which can be held in terms of money balances or can be placed with other borrowers, for example by purchasing short-term government securities. We will assume that disbursement of the funds for the payment of factors takes place in the middle of the period. At the end of the period the firm repays the principal and interest on the loan using the receipts from the sale of its product. Since the loan is held for the whole period and the "bonds" are held for only half the period, we will also assume that the interest cost of the loan, i, exceeds the return on the securities that the firm purchases, r.

With these assumptions the firm's short-run profits can be written as[3]

$$\pi = pY + rpB - wL - ipLO \tag{1}$$

where Y is real output, p is the price of the product, B is the real value of bonds, L is the amount of labor services, w is the wage rate, and LO is the real value of the loan.

The maximization of (1) takes place under a number of constraints. The production constraints are given by

$$Y = F(L_p), \qquad F' > 0, \qquad F'' < 0, \tag{2}$$

$$B = G(L_f), \qquad G' > 0, \qquad G'' < 0, \tag{3}$$

where L_p is the amount of labor services used in the production of real output and L_f is the amount of labor services employed in the purchase and sale of securities. Thus, financial transactions by the firm require the use of real resources and F and G are assumed to be strictly concave functions. The total amount of labor services required by the firm can be written as

$$L = L_p + L_f = F^{-1}(Y) + G^{-1}(B) = H(Y, B) \tag{4}$$

On the financial side, the firm is constrained by the requirement that the value of the bonds that it can purchase must not exceed the amount of the loan, i.e., $pB \leq pLO$. The simplest way to introduce this inequality constraint into the analysis is by introducing a slack variable, M, which represents the nominal money balances held by the firm[4]. Hence,

$$pB + M = pLO \tag{5}$$

Since the loan is costly the firm wants to borrow only that amount which is necessary to cover its needs. This amount will be given by

$$pLO = (wL - rpB) \tag{6}$$

We can now formulate the firm's short-run profit maximization problem as

$$\max L = pY + prB - wH(B,Y) - i\,pLO$$
$$+ \lambda\,[pLO - wH + prB]$$
$$+ \mu\,[pLO - pB - M] \tag{7}$$

The first-order conditions for an interior solution to this problem

($Y > 0, B > 0, M > 0, LO > 0$) are given by

$$L_y = p - wH_Y - lwH_Y = 0 \tag{8}$$

$$L_B = pr - wH_B + lpr - lwH_B - \mu p = 0 \tag{9}$$

$$L_\lambda = pLO - wH + prB = 0 \tag{10}$$

$$L_{LO} = (\lambda - i + \mu)\, p = 0 \tag{11}$$

$$L_M = -\mu = 0 \tag{12}$$

$$L_\mu = pLO - pB - M = 0 \tag{12'}$$

Use of conditions (11) and (12) allow (8) and (9) to be rewritten as

$$wH_Y = p/(1 + i) \tag{8'}$$

$$wH_B = pr \tag{9'}$$

Not surprisingly, these conditions indicate that the firm will supply a level of output and demand an amount of bonds such that the net marginal contribution to revenue of each activity equals the marginal cost of each activity.

An analysis of these conditions reveals that the supply of output function, which is simply the solution to (8)' due to the separability of H, has the following properties,

$$Y = f\,(p, w, i) \tag{13}$$

where $f_p > 0$, $f_w < 0$, $f_i < 0$ and f is homogeneous of degree zero in w and p. Similarly, the demand for bonds function, which is the solution to (9)', will have the following properties,

$$B = g\,(p, w, r) \tag{14}$$

where $g_p > 0$, $g_w < 0$, $g_r > 0$ and g is homogeneous of degree zero in p and w as well as in w and r. While changes in r do not affect the supply of output, changes in i do not affect the demand for bonds.

The real demand for credit or loans can now be obtained by using (13) and (14) in (10), which leads to

$$LO = (w/p)\, H\,[g\,(p, w, r), f\,(p, w, i)] - rg\,(p, w, r) = h\,(p, w, r, i) \tag{15}$$

where

$$h_p = ([p/(1+i)]f_p - (w/p)L)/p \quad < 0 \tag{15a}$$

$$h_w = ([p/(1 + i)]\, f_w + L)/p \quad > 0 \tag{15b}$$

$$h_i = ([p/(1 + i)]\, f_i)/p \quad < 0 \tag{15c}$$

$$h_r = -B \quad < 0 \tag{15d}$$

Since the demand for credit is a particularly important feature of this model, it is useful to interpret (15a - 15d). The first term in (15a) and (15b) represents the change in the real demand for credit as a result of the adjustment in output induced by changes in p and w; the second term in these two conditions represents the change in the real demand for credit as a result of the impact of changes in p and w on the levels of the variables affected by these changes. The signs of the derivatives follow from assuming that the direct effects on the levels of the variables always dominate the indirect effects

due to the adjustments in the level of output. While the effect of a change in i in (15c) is unambiguously negative because there is no direct effect on the level of the variable[5], the effect of a change in r is unambiguously negative because there is no indirect effect through adjustment of output levels.

Finally, the demand for money or real cash balances implicit in this model can now be obtained as a residual using (12)'.

$$m = M/p = LO - B = h\,(p, w, r, i) - g\,(p, w, r) = m\,(p, w, r, i) \tag{16}$$

where

$$m_p = h_p - g_p = \{[p/(1 + i)]\, f_p - pg_p - (w/p)L\}/p \quad < 0 \tag{16a}$$

$$m_w = h_w - g_w = \{[p/(1 + i)]\, f_w - pg_w + L\}/p \quad > 0 \tag{16b}$$

$$m_i = h_i = [p/(1 + i)]f_i/p \quad < 0 \tag{16c}$$

$$m_r = h_r - g_r = -B - g_r \quad < 0 \tag{16d}$$

The effects of changes in p and w on the demand for real cash balances will be more pronounced than on the demand for credit, because the role of gp (gw) is to make the change in real cash balances larger in absolute value than the change in loans. Parenthetically, both the demand for real credit and real money balances are homogeneous of degree zero in p and w. A change in the interest rate on the loan has the same effect on credit as on real money balances and a change in the rate of return on securities has a magnified effect on real money balances relative to credit demand.

III. Implications

We shall use the above model and its modifications to establish five propositions that are relevant for our topic. Perhaps the most important of these propositions is the following one.

Proposition 1: Even though money economizes on transaction costs by saving labor resources and credit is indispensable, it is theoretically incorrect to treat either financial variable as an input in a 'delivered' production function.

In order to establish this proposition consider the following experiment: Suppose one writes the 'delivered' production function as

$$Y = Y\,(L_p, LO) \tag{17}$$

where Y is assumed to satisfy the usual conditions: for instance, $Y_{LO} > 0$, $Y_{LO,\,LO} < 0$. If the rate of return on securities increases, for example, our model says that real output will not change but credit will decrease. Hence, (17) will predict, incorrectly, a decrease in output. Similar considerations arise with respect to changes in any other exogenous variable or if one introduces either bonds or real cash balances as the financial input in (17).

Since financial variables do play a role in production, it is usually possible by imposing some direct or indirect constraint on the model to obtain the result that a financial variable behaves as an input under some circumstances. We capture this possibility for the model of Section II as follows:

Proposition 2: When money economizes on transaction costs by saving labor resources, credit is indispensable and a quantity constraint on the amount of credit is imposed, one can define a 'delivered' production function for real output in which the real amount of credit, but not real money balances or bonds, is treated as an input.

To set the stage for the proof we must first go back to the model of Section II and note that, by assumption, credit (LO) is no longer a decision variable. Instead, it takes on a fixed value (LO^*) which must be less than the optimum if the constraint is binding. Therefore, equation (11) in the first-order conditions is no longer relevant and LO^* replaces LO wherever it appears. The other conditions, ((8)-(12)'), however, remain the same. This implies that the demand for bonds (14) is the same as before. The supply of output, on the other hand, will be determined by equation (10), namely the constraint on loanable funds[6]. Hence, we will have

$$Y = f\ (LO^*, r, w, p) \tag{18}$$

A comparative static analysis of the constraint shows that $f_{LO^*} > 0$, $f_r > 0$, $f_p > 0$, $f_w < 0$. The intuition behind these results is easy to see. For instance if the amount of credit available is increased, other things equal, the supply of output must increase for the constraint to be satisfied. Similarly, an increase in the rate of return on bonds (r) allows the firm to circumvent the quantity constraint, because it needs less credit at any given level of output, and to increase the level of output. An increase in p leads to a rise in output because it increases the nominal amount of funds available to finance wage payments, at given real values of the lending constraint (LO^*) and of the amount of bonds demanded (B). Finally an increase in w forces a decrease in output, because it increases the amount of funds needed to finance labor payments.

In order to establish the main part of Proposition 2 consider the 'delivered' production function defined in (17) and compare it to equation (18) above. The effects of changes in r, p and w on Y are qualitatively the same as the effects of changes in these variables on L_p. Hence, L_p can be used to replace these variables in (18) and no incorrect predictions would be obtained. Of course, LO already appears in both (17) and (18). It can be shown from the constraint in (10) that $\partial^2 Y/\partial L0^2 < 0$; and from the concavity assumptions on F, stated in Section II, it follows that $\partial^2 Y/\partial L_p^2 < 0$.

In order to establish the second part of this proposition, first notice that the demand for real cash balances under the credit constraint will be given by

$$m = M/p = LO^* - g\ (r, p, w) = m\ (LO^*, r, p, w) \tag{19}$$

where $m_{LO^*} > 0$, $m = -g_r < 0$, $m_p = -g_p < 0$, $m_w = -g_w > 0$. If one were to use real money balances in (17), this equation would become

$$Y = Y\ (m, L_p) \tag{17'}$$

As long as m changes because the credit constraint is altered, i.e., LO* changes, (17)' would yield the same predictions as (18). Nevertheless, real money balances can also

be affected by r, p, and w, as can be seen from (19). In this circumstance, if r increases, for example, L_p increases because the effects of changes in r, p and w on Y are qualitatively the same as those on L_p and Y increases; by contrast (17)' can predict either a decrease or an increase in Y, because an increase in r also decreases m, by (19), which in turn decreases Y. Thus, whether output increases or decreases depends on whether the effect of the rate of return on securities on the demand for labor is stronger or weaker than its effect on the demand for real cash balances as well as on the marginal products of these variables in the 'delivered' production function. Similar arguments can be made with respect to the inappropriateness of using bonds in a 'delivered' production function.

The model of Section II generates Fischer's model I as a special case if one assumes that credit is not indispensable, for example because the firm is paid in advance for its output. In this case the following proposition holds.

Proposition 3: When credit is not indispensable and money economizes on transaction costs by saving labor resources, it is theoretically incorrect to define a 'delivered' production function with real money balances as an input.

To establish this proposition first note that the model of Section II specializes to this case in the following manner. Since credit is not needed, it plays no explicit role in the firm's choice and the firm's problem becomes to choose the level of output and to allocate its cash between real money balances and bonds. That is, equations (10) and (11) are no longer relevant, because loans are zero, and the amount of cash available from advanced payments (pY) replaces the nominal value of loans in (12)'. Thus, the demand and supply functions become

$$Y = f\ (p, w);\ B = g\ (p, w, r);\ \text{and}\ M/p = Y - B = m\ (p, w, r)$$

Now writing a 'delivered' production function such as (17)' leads to theoretically incorrect predictions. For instance, if the rate of return on bonds changes we see that the level of output does not change but m does change. Therefore, (17)' would predict a change in output as a result of the change in m when none would take place.

Proposition 3 is inconsistent with the assertion that when money economizes in transaction costs by saving on labor resources one can write a 'delivered' production function using money as an input. The reason for this inconsistency is that one additional assumption is required to obtain this result. This assumption is implicit in Fischer's argument and we make it explicit below.

Proposition 4: When credit is not indispensable, money economizes on transaction costs by saving on labor resources and there is a constraint on total labor availability, it is appropriate to postulate a 'delivered' production function with money as an input.

To establish this proposition note that if the total amount of labor, $L = L_p + L_f$ is fixed, then an increase in the rate of return on bonds, for example, leads to an increase in L_f, which in turn implies a decrease in L_p as well as a decrease in m; and, therefore, both terms in (17)' lead to the now accurate prediction that Y decreases.

Incidentally, the same arguments would hold if the production function were written in terms of the total amount of labor (L) available rather than in terms of productive labor (Lp) as in (17)', i.e., as

$$Y = f\ (L, m) \tag{20}$$

In the case of Proposition 3, an increase in r would not change Y but it would change L and m. Thus, for (20) to be correct L and m would have to change in such a way as to leave Y unchanged. There is no mechanism that will ensure this outcome and, therefore, (20) will yield incorrect predictions. In the case of Proposition 4, it becomes appropriate to write (20) because L is fixed by assumption and m and Y will always move in the same direction in this circumstance.

How sensitive are these results to changes in assumptions? Not very much in our opinion. Thus, we conclude this section with a conjecture that directs attention to this robustness.

Proposition 5: Whenever it is theoretically appropriate to include a financial variable as an input into a production function, we conjecture that a necessary but not sufficient condition will be imposing some arbitrary binding constraint in the model which ensures this result.

Considering every alternative model is obviously infeasible. Therefore, we justify our conjecture by noting several cases, besides the ones already discussed, where the conjecture holds true. First, consider Fischer's model II, the vending machine example. Fischer concludes on the basis of this model that it is appropriate to include money as an input in a production function. In order to do so, however, he excludes in a footnote the solution that leads to stocking more change than necessary in the machine. That is, he imposes the binding constraint that change must be needed into the solution and then money behaves as an input. Similarly, if we consider the result in his model I (Proposition 4 above) it is easy to show that it is not very robust to changes in assumptions. For example, Proposition 4 no longer holds if one were to relax the assumption that the advance payments entail no costs to the firm. One way of doing so is by incorporating credit into the analysis as in the model of Section II. Imposing a constraint on total labor availability in that model does not result in 'delivered' production functions that lead to correct predictions, as is shown in Appendix 1.

A similar situation arises if one were to assume that the firm borrows from the public through an advance on sales. As long as the firm pays the public the same interest rate that it would have paid the bank for the fixed term loan, the results are exactly the same as those in Section II and 'delivered' production functions with financial variables included are theoretically incorrect. It is possible, however, to obtain a result similar to Proposition 2 in this model by assuming that the fraction of sales the firm can borrow from the public as an advance is limited to be below what it would like to borrow. In that setting one can write a 'delivered' production function with credit (but not money or bonds) as an input. The only difference with the case underlying Proposition 2 is that the production function in such a setting would be of the Leontief type. This is shown in Appendix 2.

IV. Concluding Perspectives

Our discussion leaves little doubt that in general 'delivered' production functions with financial variables as inputs lead to theoretically incorrect predictions. Moreover, the additional assumptions required to generate theoretically correct predictions are somewhat arbitrary, highly restrictive and sensitive to changes in other assumptions.

For empirical purposes the consequences of these findings are not that serious, considering the available technology. They simply suggest the introduction of financial variables in production functions or cost functions as state of nature variables, rather than forcing them to operate as inputs. This is quite feasible and frequently done for other variables. On the theoretical side the implications are more damaging, because they suggest a need to establish very carefully the purpose and role of the financial variable in the model rather than treating it as an input. Such a process tends to go against the tendency toward simplicity and generality in theoretical work.

We can put our results in perspective with a brief discussion of the two basic functions of financial variables in the productive process that are captured by the model of Section II. The role of credit in the production process captured by this model is a well recognized one, namely as a means of synchronizing receipts and payments over the production period. Indeed, this function of financial variables is the focus of the working capital literature, e.g., Calvo and Thoumi (1984). If bonds cannot be held real money balances become a perfect proxy for credit in the performance of this role. On the other hand, if firms can obtain credit from a credit line at the same rate as from a fixed term loan both money and bonds become irrelevant to the performance of this function in the productive process.

Interestingly, the latter case highlights that the demand for money and bonds in this model of the firm arises to solve a financial synchronization problem, i.e., the need to reconcile the timing of the receipt of the loan with the disbursement of its funds. It is in this context that money's ability to economize on transaction costs makes it an attractive alternative to bonds. This financial synchronization function of money and bonds is not widely recognized although it is certainly encompassed in earlier work, e.g., Clower and Howitt (1978) show how "bunching" costs lead to a positive demand for money and bonds.

We conclude by noting an implication of the credit function. Firms in transition economies have two institutional mechanisms to solve the synchronization problem between payments and receipts: money or credit from the Central Bank and inter-enterprise credit. In the absence of bonds a cut in money by the Central Bank operates as the imposition of a credit constraint on firms. When interenterprise credit is taken into account, the constraint can become more or less binding depending on whether the Central Bank's activity directly affects all or a subset of firms. The former situation provides a rationale for Calvo's (1995, Part VI) conjecture that a cut in money may lead to a larger cut in credit and serious production difficulties.

Appendix 1

THE EFFECT OF A CONSTRAINT ON LABOR AVAILABILITY

Imposing a constraint on the total amount of labor available to the firm modifies the optimization problem as follows

$$\max L = pY + rpB - wL^{*} - ipLO + \lambda\,[pLO - wL^{*} + rpB] + \mu\,[pLO - pB - M] + \gamma\,[L^{*} - H(B, Y)] \qquad (A1)$$

The first-order conditions for an interior optimum will be given by

$$L_Y = p - \lambda H_Y = 0 \tag{A2}$$

$$L_B = rp + \lambda rp - \mu p - \gamma H_B = 0 \tag{A3}$$

$$L_\lambda = pLO - wL^* + rpB = 0 \tag{A4}$$

$$L_{LO} = (\lambda - i + \mu)\, p = 0 \tag{A5}$$

$$L_M = -\mu = 0 \tag{A6}$$

$$L_\mu = pLO - pB - M = 0 \tag{A6'}$$

$$L_\gamma = L^* - H(B, Y) = 0 \tag{A7}$$

Use of (A6), (A5) and (A2) in (A3) leads to

$$r(1 + i) = H_B/H_Y \tag{A8}$$

This equation and the labor constraint (A7) must be solved simultaneously to obtain the demand for bonds function and the supply of output function. The solution of these two equations yields

$$B = g(r, i, L^*) \tag{A9}$$

$$Y = f(r, i, L^*) \tag{A10}$$

where $g_r > 0$, $g_I > 0$, $g_{L^*} > 0$ and $f_r < 0$, $f_I < 0$, f_{L^*}.

The signs follow from a comparative statics analysis of (A7) and (A8).

In this model, the demand for credit will be given by

$$LO = (w/p)L^* - rB = h(L^*, w/p, r, i)$$

where $h_{w/p} > 0$, $h_r < 0$, $h_I < 0$ and h_{L^*}?

The last sign is ambiguous. Using the previous comparative statics analysis and some manipulation yields

$$\partial LO/\partial L^* = h_{L^*} = wH_{BB}(H_Y)^2 + \{(H_B)^2 H_{YY}H_Y / (1 + i)\}\, [w(1 + i) - p/H_Y] \tag{A11}$$

The reason for the ambiguous sign is the expression in square brackets in the second term of (A11). In the absence of a constraint this second term is always zero and an increase in labor usage increases credit. In the presence of a binding constraint, however, one would expect that the contribution of labor to marginal costs $[w(1 + i)]$ will be smaller than its potential contribution to revenues through the sale of the product, i.e., the value of its marginal product (p/H_Y). Nevertheless, $H_{L^*} > 0$ if the first term dominates the second in (A11).

The demand for money in this model will be given by

$$m = LO - B = h(L^*, w/p, r, i) - g(L^*, r, i) = m(L^*, w/p, r, i)$$

where $m_r < 0$, $m_i < 0$, $m_{w/p} > 0$, m_{L^*}?

Note that $m_{L^*} = h_{L^*} - g_{L^*}$; therefore a relaxation of the constraint can result in an increase or a decrease in the demand for real money balances. Thus, an increase in the wage rate increases the demand for money and it does not affect the level of output, as can be seen from (A10). Therefore if one writes a delivered production as $Y = Y(L, m)$, it will yield predictions inconsistent with the model in this Appendix because there will be circumstances in which real money balances increase and output does not change.

Appendix 2

BORROWING THROUGH AN ADVANCE ON SALES

A. *Unconstrained Case*

When the firm borrows from its customers, it chooses the fraction of sales needed to cover the amount of the loan, thus $\alpha pY = pLO$. Hence profits can now be defined as

$$\pi = pY + rpB - wH - i\alpha pY \tag{B1}$$

The firm's problem is to choose Y, B, m and α subject to the financial constraint, (5) in the text, and to the restriction that what it borrows just covers its needs, i.e., $\alpha pY = wH - rpB$. Consequently, the objective function can be written as

$$\max L = pY + rpB - wH - i\alpha pY + \lambda(\alpha pY - wH + rpB) + \mu(\alpha pY - pB - M) \tag{B2}$$

First-order conditions for an interior solution are given by

$$L_Y = p - wH_Y - i\alpha p + \lambda\alpha p - \lambda wH_Y + \mu\alpha p = 0 \tag{B3}$$

$$L_B = rp - wH_B - \lambda wH_B + \lambda rp - \mu p = 0 \tag{B4}$$

$$L_M = -\mu = 0 \tag{B5}$$

$$L_\mu = \alpha pY - pB - M = 0 \tag{B5'}$$

$$L_\lambda = \alpha pY - wH + rpB = 0 \tag{B6}$$

$$L_\alpha = -ipY + \lambda pY + \mu pY = 0 \tag{B7}$$

Use of (B5) and (B7) in (B3) and (B4) leads to

$$p/(1+i) = wH_Y \quad \text{(B8)}$$

$$rp = wH_B \quad \text{(B9)}$$

Equations (B8) and (B9) are identical to (8)' and (9)'. Hence, all of the results derived from these two equations are the same as in Section II. The demand for credit can be found from (B6) by using $\alpha pY = pLO$. Therefore, it will be identical to what we found in Section II and, consequently, the demand for real money balances from (B5)' will also be the same. Finally, (B6) can be used to find the optimal value of the fraction to be borrowed from customers, i.e.,

$$\alpha = (wH - rpB)/pY \quad \text{(B10)}$$

B. Variable Quantity Constraint Case

First we must define what we mean by the imposition of a credit constraint under this alternative credit mechanism. The most plausible assumption is that the fraction of sales which the firm can borrow is not subject to the control of the firm. For our purposes, all that matters is the firm's inability to control the fraction of sales which it borrows; whether this fraction is determined by custom or the market and who happens to be the actual lender does not affect our immediate argument. In this situation, the firm's optimization problem can be written as

$$\max L = pY + rpB - wH - i\alpha \bullet pY + \lambda\,[\alpha \bullet pY - wH + rpB] + \mu\,[\alpha \bullet pY - pB - M], \quad \text{(B11)}$$

where α^* is the fixed fraction of its sales that the firm can borrow. In arriving at (B11), we have used the relation between sales and the loan, $\alpha \bullet pY = pLO$, to eliminate the loan from appearing explicitly in the analysis.

Incidentally, we will assume that $\alpha^* < \alpha^o$ where α^o is the optimal fraction as determined in Part A of this Appendix.

First-order conditions for an interior solution are given by

$$L_Y = p - wH_Y - i\alpha \bullet p + \lambda\alpha \bullet p - \lambda wH_Y - \mu\alpha \bullet p = 0 \quad \text{(B12)}$$

$$L_B = rp - wH_B - lwH_B + \lambda rp - \mu p = 0 \quad \text{(B13)}$$

$$L_B = -\mu = 0 \quad \text{(B14)}$$

$$L_\lambda = \alpha \bullet pY - wH + rpB = 0 \quad \text{(B15)}$$

Use of (B14) in(B13) yields $rp = wH_B$. Therefore, the demand for bonds will be the same as in part A of this Appendix or in Section II of the text (14). Use of (14) in (B15) yields the supply of output function

$$Y = f\,(a\bullet, r, w, p) \quad \text{(B16)}$$

A comparative static analysis of the constraint in (B15) reveals that $f_{\alpha\bullet}$, f_r, f_r are positive and f_w is negative as

$$(wH_Y - \alpha \bullet p) \quad \text{(B17)}$$

is positive and viceversa. This condition represents the net marginal contribution of a unit of output to the firm's credit needs. If its marginal contribution to credit needs through an increase in labor demand (wH_Y) exceeds its marginal contribution to credit available ($\alpha \bullet p$), then the signs of the output supply function will be the same as before. In this case, the imposition of the credit constraint, which can be viewed as a lowering of $a\bullet' = a^o$ to $\alpha\bullet'' < a^o$, will lead to a reduction in output. On the other hand, if the net marginal contribution of a unit of output to the firm's credit needs is negative, $wH_Y < \alpha \bullet p$, the imposition of this type of credit constraint leads the firm to increase its level of output beyond what is optimal in the unconstrained situation and reverses all of the signs of the output supply function.

Finally, condition (B12) can be rewritten as

$$p\,(1 - \alpha \bullet i) = wH_Y + \lambda\,(wH_Y - \alpha \bullet p), \quad \text{(B18)}$$

and compared to the corresponding unconstrained condition, which can also be rewritten as

$$p\,(1 - \alpha \bullet i) = wH_Y + i\,(wH_Y - \alpha \bullet p) \quad \text{(B19)}$$

If condition (B17) is positive, we know that the value of Y in (B19) is greater than in (B18); hence condition (B18) can be used to solve for l and, together with (B19), also used to show that $\lambda > i$. If condition (B17) is negative, we know that the value of Y in (Bl9) is less than in (B18); and the relation between (B18) and (B19) can be used to show that $\lambda < i$. The reason is that in this current case relaxing the credit constraint contributes less to profits than the cost of credit because it entails a reduction in output.

With the present type of credit mechanism constraint, there will be a demand for credit by the firm which is given by

$$LO = \alpha \bullet Y = \alpha \bullet f\,(\alpha\bullet, r, p, w) = h\,(\alpha\bullet, r, p, w), \quad \text{(B20)}$$

where $h_{\alpha\bullet}$, hr, hp are positive and hw is negative if (B17) is positive and viceversa. To illustrate the rationale for these signs, let us consider explicitly the effect of a change in the fraction of sales that the firm can borrow on the demand for credit, i.e.,

$$h_{\alpha\bullet} = Y + f_{\alpha\bullet} = Y + pY/(wH_Y - \alpha \bullet p) = (pY\,(1 - a^*) + wH_Y Y)/(wH_Y - \alpha \bullet p)$$

The reason for the ambiguity in the sign is exactly the same as in the case of the supply of output function. If output increases as a result of relaxing the credit constraint, $(wH_Y - \alpha \bullet p) > 0$, then the demand for credit will increase; if output decreases as a result of relaxing the constraint, $(wH_Y - \alpha \bullet p) < 0$, the demand for credit decreases.

The demand for real cash balances can now be derived from the financial constraint

$$m = LO - B = h(\alpha^*, r, p, w) - g(r, p, w) = m(\alpha^*, r, p, w), \tag{B21}$$

where $m_{\alpha^*} < 0$, $m_r < 0$, $m_p < 0$, and $m_w > 0$ if $(wH_Y - \alpha^* p) < 0$; and

$m_{\alpha^*} > 0$, m_r ?, m_p ?, and m_w ? if $(wH_Y - \alpha^* p) > 0$

When output increases as a result of imposing the constraint, the effects of the price variables on the demand for real cash balances are the same as in the fixed quantity constraint model of Section III in the text; when output decreases, however, the effects of these variables are ambiguous.

Finally, we present the implications of this model for the use of financial variables in a 'delivered' production function in terms of an additional proposition.

Proposition A1: In the presence of a binding restriction on the fraction of sales that the firm can borrow, it is appropriate to use credit as an input in a delivered production function if and only if this production function has a Leontief form; in this situation, neither real money balances nor real bond holdings are adequate proxies for credit.

Since by the definition of the credit constraint $\alpha \cdot pY = pLO$, it follows trivially that the firm operates as if subject to the following production function, $Y = (1/\alpha^*) LO$. Furthermore, since the demand for credit and for real cash balances need not always move in the same direction, the second part of the proposition also follows rather directly once it is noted that changes in the demand for bonds will also differ from changes in the demand for credit.

Notes

1 For other studies of the firm's demand for money stressing its role as a means of economizing on transaction costs, see those surveyed in Bernstein (1978).

2 Given the recent interest on credit theory versus money theory, e.g., Blinder and Stiglitz (1983) and Brunner and Meltzer (1988), it is useful to be aware of both the similarities and the differences in the productive roles of the two variables.

3 Parenthetically the main results would not be affected if we included variable inputs other than labor or fixed inputs into the analysis, but the exposition is considerably simplified by leaving them out.

4 The use of this procedure for dealing with inequality constraints is suggested in the context of consumer theory by Green (1976, Ch. 7). An alternative but equivalent way of imposing the constraint is to assume directly that the proceeds from the loan are held as money balances or bonds, which can be expressed as $M = pLO - pB$; and to impose the inequality constraint that money balances must be nonnegative, i.e., $M \geq 0$, which ensures that $pLO - pB$.

5 If one were to assume that interest on the loan had to be paid in the middle of the period, when labor payments are made, one would also obtain a direct effect through the amount of the loan which would be opposite in sign to the indirect effect in the text. Therefore, in this case the nature of the credit finance mechanism can result in an increase in the cost of credit (the loan) leading to an increase in the demand for credit by the firm. In the text an increase in the cost of credit always leads to a decrease in the demand for credit because we are assuming interest is paid at the end of the period and, thus, there is no direct effect of a change in the interest rate.

6 Incidentally, condition (8) now determines the value of λ, that is the marginal contribution to profits of relaxing the quantity constraint on credit.

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