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OPTIMUM AND REVENUE MAXIMIZING TRADE TAXES IN A MULTICOUNTRY FRAMEWORK

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Abstract:

The traditional literature derives optimum and revenue-maximizing export taxes within two-country models, with one exporter and one importer (Johnson 1950-51, Tower 1977). In reality, most products, including primary products, are exported by several countries. In this paper, we present a theory of trade taxes in a three-country framework. This enables us to deal with strategic interactions among exporting countries. We show that (i) if one of the countries is a Stackelberg leader, both countries improve their welfare relative to Nash equilibrium, and in the symmetric case, the follower's welfare is higher than that of the leader; (ii) the revenue-maximizing Nash tax is larger than the optimum tax for each country; and (iii) welfare may be higher in the revenue-maximizing Nash equilibrium than in the welfare-maximizing Nash equilibrium, a result which cannot arise in two-country models.

I. Introduction

In the traditional literature, the optimum and revenue maximizing export taxes are derived within two-country models (Johnson 1950-51 and Tower 1977)[1]. An important limitation of this approach is that it admits only one exporter (and one importer) of a product[2]. In reality, most products are exported by several countries. Therefore, optimal policy choices of one country are dependent on the choices made by other countries.

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The conventional two-country models are unable to capture this strategic interdependence among countries exporting the same product[3].

This type of policy interdependence is especially important for the developing countries exporting primary products. To take a concrete example, consider the world cocoa market. Cocoa is a relatively homogeneous commodity and is exported by Côte d'Ivoire, Brazil, Ghana, Malaysia, Cameroon, Nigeria, Ecuador, Indonesia and Oceania. Export shares in the world market range from 36% for Côte d'Ivoire, and 20% for Brazil to 2% each for Indonesia and Oceania[4]. Almost all exports originate in developing countries and are sold in developed countries. Clearly, countries such as Côte d'Ivoire and Brazil cannot choose their optimum or revenue maximizing taxes independently of each other. This same issue also arises in other commodity markets such as coffee, tea, cotton, tobacco, groundnuts, copper and oil.

In this paper, we present a simple analysis of optimum and revenue maximizing export taxes in a multicountry model. As many developing countries who export primary products have used export taxes to boost their incomes or tax revenues, it is of interest to see how the welfare and tax revenues under the two objectives compare. Our analysis highlights the importance of policy interdependence among countries competing against each other in the world market; it also enables us to provide a natural interpretation of the fallacy of composition discussed at length in the literature on the terms of trade of primary-product exporters.

For ease of exposition, we limit ourselves to two exporting countries, although the extension to three or more countries is straightforward[5]. In conformity with the contributions by Johnson (1950-51) and Tower (1977), we assume that the importing country does not retaliate. For commodities such as cocoa and coffee which are imported almost exclusively by developed countries, this assumption is entirely reasonable[6].

Previous contributors to the literature have employed general equilibrium models to derive the optimum and revenue maximizing trade taxes. In the present multicountry, game-theoretic context, general equilibrium analysis becomes analytically intractable. Therefore, we adopt a partial equilibrium framework. We also assume linear demand and supply relationships[7]. This assumption is not critical to our results, although it simplifies the analysis considerably.

A key result in the papers by Johnson (1950-51) and Tower (1977) is that the optimum export tax is smaller than the revenue maximizing export tax. Intuitively, in choosing the optimum tax, the government exploits only the monopoly power in the world market. But in choosing the revenue maximizing tax, the government also exercises its monopsony power over the domestic suppliers.

In our multicountry model, we must derive the optimum and revenue maximizing taxes for each country. We demonstrate that if governments are Nash players, the Johnson-Tower result continues to hold in our model in the sense that for each country the Nash optimum tax is lower than the Nash revenue maximizing tax.

A more surprising result obtains in our model with respect to welfare ranking of Nash optimum and Nash revenue maximizing taxes. In the Johnson-Tower model, by definition, welfare of the tax levying country is higher under the optimum tax than under the revenue maximizing tax. By contrast, in our model, the two types of taxes cannot be ranked uniquely with respect to welfare. In particular, *it is possible for welfare to be higher under Nash revenue maximizing export taxes than under Nash optimum taxes.*

Drawing upon the analysis of price setting duopolists in Eaton and Grossman (1986), we can explain this result as follows. Suppose there are two countries, A and B, exporting the same product to the rest of the world. In choosing their optimum taxes under Nash behavior, these countries take a very pessimistic view of the rival's action. As country A raises its tax, country B's optimal response is to raise its own tax as well[8]. But A ignores this fact and assumes that B will not react. In effect, A underestimates the potential demand facing it. The same holds true for B. As a result, Nash taxes turn out to be "too low" in the sense that higher taxes which can improve upon both countries' welfare exist. This undertaxation is similar to the underpricing of goods of price-setting Bertrand duopolists in Eaton and Grossman.

Now suppose that the two countries maximize revenues. We know that Nash revenue maximizing taxes are larger than the corresponding Nash optimum taxes. Therefore, Nash revenue maximizing taxes have the potential for improving upon Nash optimum taxes. Whether or not Nash revenue maximizing taxes actually yield a higher welfare than Nash optimum taxes depends on how much higher the former are relative to the latter. Starting at Nash optimum taxes, increases in tax rates will raise welfare only up to a point. If revenue maximizing taxes lie sufficiently far beyond this point, welfare may decline.

A further interesting result is that at least in the symmetric case, Nash optimum taxes cannot yield higher revenues than Nash revenue maximizing taxes. Intuitively, in the symmetric case the Nash optimum tax is the same in the two countries. Starting from this tax rate, as we increase taxes in the two countries equiproportionately, revenues rise monotonically until we reach the joint (cooperative) revenue maximizing tax rate. It can be shown that the Nash revenue maximizing tax lies between this rate and the Nash optimum tax rate. Therefore, Nash revenue maximizing taxes must yield higher revenues than Nash optimum taxes.

Our final result is that under Stackelberg behavior, both the follower and leader benefit. Thus, if welfare maximization is the objective, both countries enjoy a higher welfare when one of them acts as a leader than when they play a simultaneous move Nash game. *Even more surprisingly, the welfare gain of the follower is larger than that of the leader in the Stackelberg equilibrium.*

The remainder of this paper is organized as follows. In Section II, we outline the model and derive Nash optimum taxes. In Section III, we consider Stackelberg and cooperative equilibria and in Section IV we provide a new interpretation of the fallacy of composition. In Section V, we derive Nash revenue maximizing taxes. In Section 6, we rank welfare and revenue maximizing taxes with respect to welfare. For convenience, we focus mainly on the symmetric case in which the two exporting countries are identical. We deal with the nonsymmetric case only briefly. In Section 7, we discuss future applications of our model. An appendix available upon request from the authors contains some technical details.

II. The Model and Nash Optimum Export Taxes

Let there be three countries denoted 1, 2, and 3. Countries 1 and 2 export the good (e.g., cocoa) to country 3. For simplicity, assume that the good is not consumed in exporting countries and not produced in the importing country[9]. The analysis is easily modified to relax this assumption.

Denote the world (i.e., country 3's) demand for the good by

$$Q^D = A - BP \quad (1)$$

where P is the world price and A and B are positive constants. Using lower-case letters to represent supply variables, country i's supply is written

$$q_i^s = a_i + b_i p_i \qquad \text{for } p_i > 0 \tag{2}$$

where b_i is a positive constant and a_i may be positive, zero or negative. Variable p_i denotes the price of the good prevailing in country i. According to the restrictions stated in (2), output cannot be negative. More importantly, a strictly positive price is necessary, though not sufficient, to obtain a strictly positive output. Letting t_i be the ad valorem export tax applicable to the *world* price, we have

$$p_i = p\,(1-t_i) \tag{3}$$

Given this definition of t_i, if we set $t_i = 1$, all sales revenues are taxed away and the price facing producers, p_i, becomes 0.

Net demand facing country 1 may be obtained by subtracting q^S from Q^D. Thus, making use of (3), we have

$$\begin{aligned} Q_1^D &= (A - a_2) - [B + b_2(1 - t_2)]\,P \\ &\equiv A_1 - B_1 P \end{aligned} \tag{4}$$

where $A_1 \equiv A - a_2$ and $B_1(t_2) \equiv B + b_2(1 - t_2)$.

In the present partial equilibrium model with no domestic consumption, welfare maximization is equivalent to profit maximization. Under Nash behavior, each country maximizes its profits taking the other's tax rate as given. Corresponding to (4), we can write the marginal revenue facing country 1 as

$$MR_1 = \frac{1}{B_1(t_2)}\,(A_1 - 2Q_1^D) \tag{5}$$

Social marginal cost in country 1 is given by the height of the supply curve in (2). That is,

$$MC_1 = p_1 = \frac{1}{b_1}\,(q_1^s - a_1) \tag{6}$$

The optimal quantity under Nash behavior is obtained by setting $MR_1 = MC_1$ and $Q_1^D = q_1^S = q_1$. Straight-forward calculations yield optimal q_1 as a function of t_2.

$$q_1 = \frac{A_1 b_1 + a_1 B_1(t_2)}{2b_1 + B_1(t_2)} \tag{7}$$

Recall that a_1 is the intercept of the supply curve on the quantity axis. In general, this intercept may be positive, zero or negative. Therefore, in principle, the numerator of the right-hand side in (7) can be negative. In conformity with (2), we assume that q_1 is nonnegative. That is to say,

$$A_1 b_1 + a_1 B_1(t_2) \geq 0 \tag{8}$$

Next, let us substitute the value of q_1 shown in (7) for Q_1^D and q_1^S in (4) and (6), respectively, and solve for P and p_1. We have

$$P = \frac{1}{B_1(t_2)}\;\frac{A_1 b_1 + B_1(t_2)(A_1 - a_1)}{2b_1 + B_1(t_2)} \tag{9}$$

$$p_1 = \frac{A_1 - 2a_1}{2b_1 + B_1(t_2)} \tag{10}$$

Note that P is the price facing buyers in the world market and p_1 is the price facing suppliers in country 1. In conformity with (2), we assume that p_1 is positive at the equilibrium. Therefore, we write

$$A_1 - 2a_1 > 0 \tag{11}$$

Given (11), $A - a_1 > 0$ necessarily and P is guaranteed to be positive.

Finally, combining (3), (9), and (10), we can solve for the optimal value of t_1 as a function of t_2. We have

$$t_1 = \frac{A_1 b_1 + a_1 B_1(t_2)}{A_1 b_1 + B_1(t_2)(A_1 - a_1)} \tag{12}$$

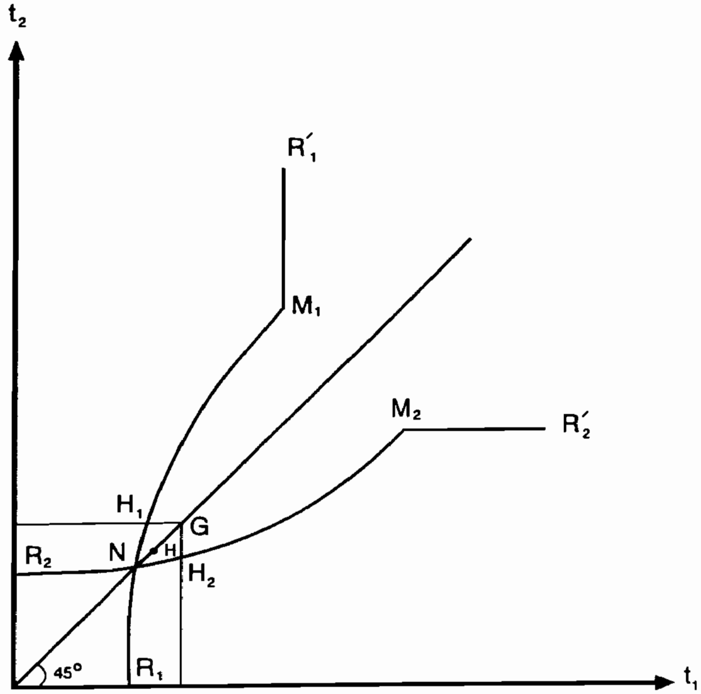
This equation gives us the "reaction function" of country 1. For each value of t_2, (12) gives country 1's optimal export tax. Straightforward manipulations enable us to obtain

$$\frac{dt_1}{dt_2} = \frac{A_1 b_1 b_2 (A_1 - 2a_1)}{[A_1 b_1 + B_1(A_1 - a_1)]^2} > 0, \tag{13}$$

$$\frac{d^2 t_1}{dt_2^2} = \frac{2A_1 b_1 b_2^2 (A_1 - 2a_1)}{[A_1 b_1 + B_1(A_1 - a_1)]^3}\,(A_1 - a_1) > 0, \tag{14}$$

According to (13), as t_2 rises, optimal t_1 also rises. Intuitively, a rise in t_2 implies a larger share of the world market for country 1 and hence more market power. Equation (14) states that as t_2 increases, the optimal t_1 increases at an increasing rate.

FIGURE 1



In figure 1, we represent country 1's reaction curve by R_1R_1'. Note that since we measure t_2 on the vertical axis and t_1 on the horizontal axis, an increasing rate of t_1 with respect to t_2 implies that the reaction curve must become flatter as we increase t_2. For a sufficiently high value of t_2, country 2 ceases to be a potential competitor and country 1 becomes a monopolist. Beyond this value of t_2, shown by point M_1 in Figure 1, country 1's optimal tax rate is constant at the monopoly level. We demonstrate in the appendix that the specific value of t_2 for which country 1 becomes a monopolist is 1 if a_2 is non-negative but less than 1 if a_2 is strictly negative. In the former case, point M_1 is necessarily characterized by $t_1 < t_2$ as shown in Figure 1 while in the latter case the same may or may not hold true.

We can derive country 2's reaction curve in an analogous manner. Assuming that the two countries are identical in all respects, this reaction curve will be the mirror image of country 1's reaction curve along the 45°-line. In Figure 1, we represent

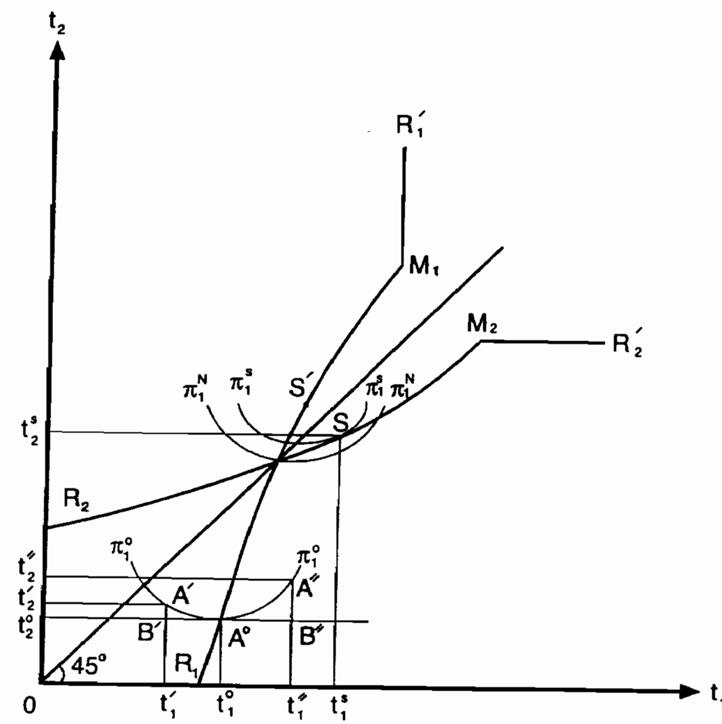
country 2's reaction curve by R_2R_2'. We demonstrate in the appendix that in the symmetric case, point M_1 must lie above the 45°-line and point M_2 below it irrespective of whether the supply curves have a positive, zero or negative intercept on the quantity axis (i.e., $a_1 = a_2 \lesseqgtr 0$). Moreover, the reaction curves must intersect exactly once, yielding a unique Nash equilibrium at point N[10].

III. Stackelberg and Cooperative Equilibria

We find it useful at this stage to introduce Stackelberg and cooperative equilibria and compare them to Nash equilibrium. This task is accomplished most conveniently with the help of isoprofit curves introduced in Figure 2.

Consider an arbitrary export tax by country 2, say t_2°. For this tax rate, country 1's best response is t_1° and is given by point A^0 in Figure 2. Denote the country's profit associated with this point by π_1°. Holding country 2's tax at t_2°, any movement in country 1's tax away from t_1° will lower the country's profit. For example, the country's profits at point B' will be less than π_1°. If we now want to hold t_1 at the level indicated by B'

FIGURE 2



but restore country 1's profits back to π_1° we must expand the demand facing it via an increase in country 2's tax to, say, t_2'. Thus, country 1's profits at point A' are the same as at A^0. Analogously, if the country is at a point such as B", profits will be below π_1°. Once again, we can restore profits back to π_1° by raising country 2's tax to a point such as A". By repeating this procedure, we can find all combinations of t_1 and t_2 which yield a profit equal to π_1°. Joining these points, we obtain the isoprofit curve $\pi_1^\circ\pi_1^\circ$.

By construction it is evident, and can be verified algebraically, that the isoprofit curve just derived will have a zero slope at the point where it intersects the reaction curve. For each point on country 1's reaction curve, we can find an isoprofit curve. Thus, we have a family of isoprofit curves. As we move up the reaction curve, the isoprofit curves are associated with higher and higher profits until we reach the monopoly point, M_1. Intuitively, a movement up the reaction curve is accompanied by a higher tax by the rival and hence a larger market for country 1.

We can also derive isoprofit curves for country 2. Without showing them in Figure 2, we note that these isoprofit curves will be strictly convex to the vertical axis. Moreover, each isoprofit curve will have a slope equal to infinity at the point where it interesects R_2R_2'.

Let us now suppose that country 1 is a a Stackelberg leader. This means that the country tries to reach the highest possible isoprofit curve taking country 2's reaction curve as a constraint. It is evident from Figure 2 that the country's optimal choice now is t_1^s. Given this value of t_1, Country 2's best response is t_1^s. Thus, the equilibrium is reached at point S.

Two results may be noted with respect to Stackelberg equilibrium. First, both countries choose higher taxes and enjoy larger profits at this equilibrium than at Nash equilibrium. In terms of the phraseology suggested by Bulow, Geanokoplos, and Klemperer (1985) export taxes are strategic complements in the present model. Intuitively, under Nash behavior, each country acts myopically and ignores the fact that a higher tax by it leads to a higher tax by the rival. As a result each country under-exploit its monopoly power in the market; export taxes are too low. Under Stackelberg behavior, the leader takes into account the follower's behavior and chooses a tax rate higher than that prevailing at Nash equilibrium. Given the positive slope of the reaction curve, the follower also chooses a higher tax. Monopoly power is exploited more fully and profits rise for both countries.

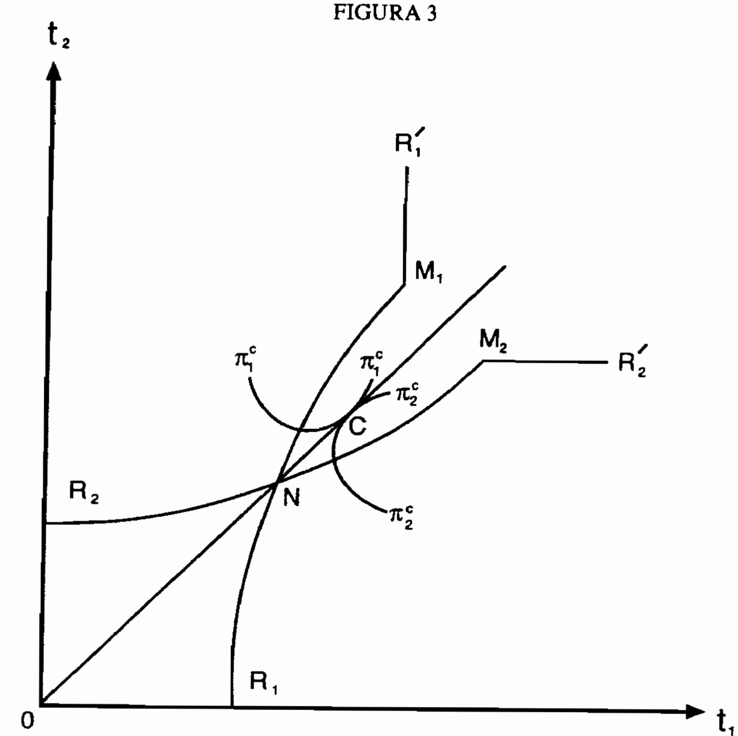
The second result which is more surprising is that at point S, the follower's profits are larger than those of the leader! To demonstrate this point, observe that country 1's profits are given by isoprofit curve $\pi_1^s\pi_1^s$. Correspondingly, country 2's profits are given by its isoprofit curve (not shown) through point S. Given that the reaction curves and associated isoprofit curves of the two countries are symmetric, country 2's isoprofit curve through S will exhibit the same profits as country 1's isoprofit curve passing through point S', where the latter is the mirror image of point S along the 45°-line. Since the isoprofit curve for country 1 passing through S' shows a higher profit than $\pi_1^s\pi_1^s$, country 2's profits exceed country 1's profits at Stackelberg equilibrium, S.

We can state

Proposition 1. In the present model, Stackelberg equilibrium is characterized by higher profits for each country than Nash equilibrium. More interestingly, in the symmetric case, at Stackelberg equilibrium, the follower's profits are higher than those of the leader.

Next, let us consider briefly the cooperative equilibrium in the symmetric case. We assume that in the cooperative equilibrium, countries maximize their joint profits. Joint profit maximization requires equalization of marginal costs across countries. Therefore, in the cooperative equilibrium, the two countries will choose the same tax rate. In Figure 3, this equilibrium is given by point C. At this point, isoprofit curves of the two countries are tangent to each other along the 45°-degree line. Any movement away from C along the 45°-degree line will yield lower profits for both countries.

FIGURA 3



It may be observed that as we move from Nash equilibrium towards the cooperative equilibrium along the 45°-degree line, each country's profits rise monotonically. Beyond the cooperative equilibrium, profits decline for both countries. In effect, point C gives us the conventional optimum export tax levied jointly by the countries.

IV. Fallacy of Composition: A Disgression

The analysis developed above can be employed to provide a formal interpretation of the fallacy of composition discussed in the context of commodity exports. In Figure

1, suppose that the initial tax rates happen to be below the cooperative-tax rate and are given by point G. Taking country 2's tax rate as given, country 1 can increase its profits by lowering its tax rate to the level indicated by point H_1. Analogously, country 2 can raise its profits by moving its tax rate to the level indicated by point H_2. If both countries act upon this strategy, however, the actual movement will be to neither H_1 nor H_2. Instead, tax rates will move to point H where both countries are unambiguously worse off. Essentially, each country hopes to benefit by undercutting the other but the end result is lower profits for each. Market shares remain unchanged and the price of the product declines transferring gains to importers of the commodity.

V. Revenue-Maximizing Taxes

Let us now turn to a consideration of Nash revenue-maximizing taxes. For many primary-product exporters, government revenue is a major reason for taxing exports. Therefore, it is useful to compare these taxes with profit maximizing taxes derived above.

Country 1's tax revenue is defined as

(15)
$$T = (P - p_1)\, q_1 = \left[\left\{ \frac{A_1}{B_1(t_2)} + \frac{a_1}{b_1} \right\} - \left\{ \frac{1}{B_1(t_2)} + \frac{1}{b_1} \right\} q_1 \right] q_1$$

where the last equality has been obtained by using (2) and (4) after setting $q_1^S = Q_1^D \equiv q_1$.

Maximization of T with respect to q_1 (and hence t_1) at a given t_2 yields the straightforward first-order condition

(16)
$$q_1 = \frac{1}{2} \frac{A_1 b_1 + a_1 B_1(t_2)}{b_1 + B_1(t_2)}$$

Substituting this value of q_1 for Q_1^D and q_1^S in (4) and (6), respectively, we can obtain the world and domestic prices under revenue maximization for a given value of t_2. Thus, we have

(17)
$$P = \frac{1}{2B_1(t_2)} \frac{A_1 b_1 + B_1(t_2)(2A_1 - a_1)}{b_1 + B_1(t_2)}$$

(18)
$$p_1 = \frac{1}{2b_1} \frac{(A_1 - 2a_1)\, b_1 - a_1 B_1(t_2)}{b_1 + B_1(t_2)}$$

In order to ensure $p_1 > 0$, we assume

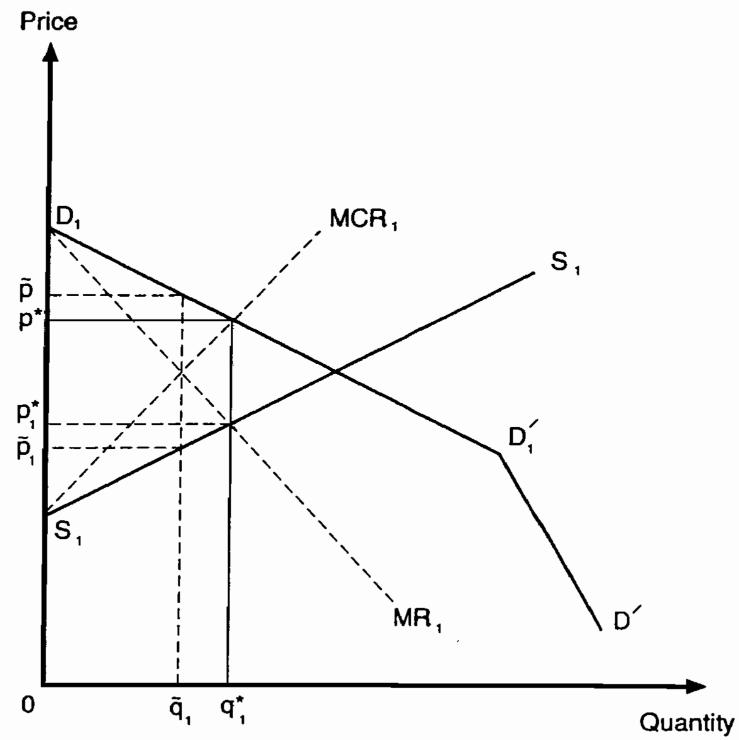
(19)
$$(A_1 - 2a_1)\, b_1 - a_1 B_1(t_2) > 0$$

Equations (17) and (18) can be combined with (3) to solve for the revenue-maximizing tax in country 1 as a function of t_2. We have

(20)
$$t_1 = \frac{(A_1 b_1 + a_1 B_1(t_2))\,[b_1 + B_1(t_2)]}{b_1\,[A_1 b_1 + B_1(t_2)(2A_1 - a_1)]}$$

Comparing equation (16) with (7), we see that *for a given t_2*, the revenue-maximizing output is smaller than the profit maximizing output. Similarly using equations (12) and (20), it can be verified that for a given t_1 the optimal export tax is lower than the revenue-maximizing tax.

FIGURE 4



These results can be explained with the help of Figure 4. D_1D_1' represents the excess demand curve (equations (4)) facing country 1 for an exogenously given value of t_2[11]. MR_1 is the marginal revenue curve associated with D_1D_1'. S_1S_1 is country 1's supply curve. The optimum output obtains at the point where MR_1 and S_1S_1 intersect. The optimum per-unit tax is the difference between the demand price p^* and the supply price p_1^* at output q_1^*.

The revenue-maximizing tax is obtained by equating MR_1 to the marginal cost of raising revenue, MCR_1. The latter is, in turn, represented by the curve marginal to the supply curve, S_1S_1[12]. In Figure 4, the MCR_1 curve lies half-way between the vertical axis and S_1S_1. As expected, the intersection of MR_1 and MCR_1 yields a lower output ($= q_1$) and higher tax than those given by the intersection of MR_1 and SS_1[13].

We can now compare the optimum and revenue-maximizing export taxes under Nash behavior. The reaction curve of country 1 under revenue maximization will lie everywhere to the right of R_1R_1' in Figure 1. Analogously, the reaction curve of country 2 will lie everywhere above R_1R_1'. Although we have not shown the reaction curves under revenue maximization to avoid clutter, it is evident that the equilibrium Nash taxes in this case will be higher than under profit maximization.

Figure 1 has been drawn under the assumption that countries 1 and 2 are identical. If the two countries are different, Nash optimum taxes will be different in the two countries. It still remains true, however, that the reaction curves under revenue maximization will move in the direction indicated in the previous paragraph. Therefore, revenue-maximizing taxes will be higher than optimum taxes in the asymmetric case as well. We have

Proposition 2. In each country the revenue maximizing Nash export tax is larger than the welfare maximizing Nash export tax.

VI. Welfare Ranking of Taxes

Nash taxes are derived under the assumption that each country takes the other's tax rate as given. In reality, countries do react to each other's choices. This fact makes Nash taxes inefficient; each country can be made better off in terms of its objective function by appropriate adjustments in Nash taxes.

To illustrate, take the case of optimum taxes under Nash behavior. As country 1 raises the tax rate, it takes country 2's tax rate as given. In reality, country 2 responds by raising its own tax rate. By ignoring this fact, country 1 underestimates the demand it faces and fails to exploit fully its market power. The same argument holds for country 2. Therefore, starting at Nash optimum taxes, welfare of both countries can be raised by raising export taxes appropriately.

This argument raises the following interesting question: since Nash revenue-maximizing taxes are higher than Nash optimum taxes, is it possible for the former to yield a higher welfare than the latter? This is the main issue addressed in the present section.

In the symmetric case, we have shown that as we raise the tax rates along the 45°-line, profits of the two countries rise monotonically until the cooperative equilibrium is reached. Increases in the tax rates beyond this point are accompanied by lower profits. In order to obtain a ranking of the Nash optimum tax and Nash revenue-maximizing tax according to welfare, we must determine the latter's level relative to the cooperative optimum tax.

A comparison of the revenue-maximizing tax and the cooperative optimum tax turns out to be intractable analytically. Therefore, we resort to simulations. In the simulations, we compare all three types of taxes and profits associated with them.

TABLE 1

N°	b_1	t_{opt}	t_r	t_c	Π_{opt}	Π_r	Π_c
1.	1	7.41	46.59	14.75	2872.45	2452.94	2892.86
2.	3	18.48	53.48	35.80	6129.02	6029.34	6371.21
3.	4	22.46	55.35	42.86	6908.76	7096.64	7312.5
4.	8	33.43	60.42	60.30	8179.93	9287.17	9287.203
5.	10	37.21	62.22	65.56	8360.33	9768.01	9801

t_{opt} = Nash optimum tax, t_r = Nash revenue-maximizing tax, t_c (c.i.s.l.c.) = cooperative optimum tax; π_{opt} = welfare (profit) at t_{opt}, Π_r = welfare at t_r, and Π_c = welfare at t_c.

In Table 1, we have shown five cases distinguished by the value of b_1 ($= b_2$). We set $A = 1{,}000$, $B = 10$, $a_1 = a_2 = -10$ and let b_1 ($= b_2$) assume successively values 1, 3, 4, 8 and 10. Observe that as b_1 rises, the supply curve becomes flatter, pivoting around the point $a_1 = -10$ on the quantity axis.

Two interesting points emerge from Table 1. First, the Nash revenue-maximizing tax and the cooperative optimum tax cannot be ranked uniquely. Thus, for low values of b_1 ($= b_2$), the former is larger than the latter. As we increase b_1, all tax rates rise but the cooperative optimum tax rises faster than the revenue-maximizing tax. For $b_1 = 8$, the two tax rates are almost equal and for $b_1 = 10$, the cooperative optimum tax becomes larger.

Second, the Nash optimum and revenue-maximizing taxes cannot be ranked uniquely in terms of welfare (profits). For $b_1 = 1$ and $b_1 = 3$, the Nash optimum tax is associated with higher welfare but for $b_1 = 4$ and larger, the ranking is reversed. Interestingly, when $b_1 = 4$ or 8, the revenue-maximizing tax is *larger* than the cooperative tax but yields higher profits than the Nash optimum tax.

The reason for increases in the tax rates in response to an increase in b_1 can be explained with the help of Figures 1 and 4. In Figure 4, let us think of S_1S_1 as the combined supply curve of the two countries and of D_1D_1 as the world demand curve. An increase in b_1 pivots S_1S_1 in the clockwise direction around its intercept on the quantity axis. This change, in turn, increases the optimal quantity of output. Since the vertical distance between the demand and marginal revenue curves increases and P declines as quantity rises, the implicit tax rate increases unambiguously. Thus, the cooperative optimum tax rises with a rise in b_1 ($= b_2$).

An analogous reasoning applies to Nash optimum and revenue maximizing taxes. Essentially, for a given t_2 (t_1) the optimum t_1 (t_2) rises with a rise in b_2 (b_1). In Figure 1, the reaction curve R_1R_2 shifts to the right and R_2R_2 shifts up. Nash optimum taxes rise.

We now state

Proposition 3. Revenue maximizing and welfare maximizing Nash taxes cannot be ranked uniquely with respect to welfare. In particular, it is possible for welfare to be higher at the revenue maximizing Nash equilibrium than at the welfare maximizing Nash equilibrium.

A natural question which arises at this point concerns the comparison of various taxes with respect to tax revenue. Interestingly, here it can be shown that at least in the symmetric case Nash revenue-maximizing taxes must always yield a higher revenue than Nash optimum taxes. We know that the Nash revenue-maximizing taxes are lower than the cooperative revenue-maximizing tax. We can also show that revenues rise monotonically as we raise the tax rates along the 45°-line up to the cooperative revenue-maximizing tax rate. But since Nash optimum taxes are below Nash revenue-maximizing taxes, revenues associated with the former will be smaller than those associated with the latter.

Finally, what happens when the two countries are not identical? Once again, we find it useful to report some simulations to address this question. In Table 2, we report the Nash optimum, Nash revenue-maximizing and cooperative optimum taxes and associated profits when countries differ with respect to intercept or slope parameters. In the first four cases, the two supply curves are parallel but country 2's curve lies to the right of country 1's. In the last two cases, quantity intercepts are the same but country 2's curve is flatter than that of country 1. In all cases, at a given price, country 2 produces more than country 1.

TABLE 2

$A = 1{,}000$, $B = 10$, $a_2 = -10$, $b_1 = 1$.

		t_{opt}	t_r	t_c	Π_{opt}	Π_r	Π_c
$a_1 = -50, b_1 = 1$	Country 1	33.9	57.9	63.4	7268	8688	8265
	Country 2	36.8	61.9	63.4	8779	10086	10593
$a_1 = -100, b_2 = 1$	Country 1	29.9	52.7	60.8	6011	7429	6480
	Country 2	36.2	61.6	60.8	9318	10489	11583
$a_1 = -200, b_1 = 1$	Country 1	22.7	42.7	55.8	3858	5209	3360
	Country 2	35.3	61.0	55.8	10445	11314	13563
$a_1 = -300, b_1 = 1$	Country 1	16.2	33.4	51.1	2182	3387	840
	Country 2	34.5	60.6	51.1	11639	12164	15543
$a_1 = -10, b_2 = 10$	Country 1	5.2	44.7	50.9	1064	1089	450
	Country 2	46.0	65.1	50.9	18077	18333	19402
$a_1 = -10, b_2 = 20$	Country 1	4.5	43.6	66.7	1387	1437	1013
	Country 2	64.0	73.7	66.7	14728	14469	15593

t_{opt} = Nash optimum tax, t_r = Nash revenue-maximizing tax, t_C = cooperative optimum tax; t_{opt} = welfare (profit) at t_{opt}, Π_r = welfare at t_r, and Π_c = welfare at t_c.

Several points emerge from Table 2. First, country 1's Nash taxes are consistently lower than those of country 2. Thus, the country with larger supply, country 2, imposes higher taxes. Second, profits for country 1 are consistently lower under the cooperative tax than under at least one of the remaining taxes. Thus, country 1 is unlikely to agree to joint profit maximization unless it is offered compensation. Of course, as expected, joint profits *are* maximized under cooperative taxes[14]. Third, for the larger country, country 2, profits are higher under cooperative taxes in all the cases considered. Thus, there is an asymmetry between the larger and the smaller country. The larger country gains from joint profit maximization in all the cases considered but the smaller one does not. Finally, for the smaller country, country 1, Nash revenue-maximizing taxes yield a higher welfare than Nash optimum taxes in all the cases. By contrast, the larger country's welfare is higher under Nash revenue-maximizing taxes than under Nash optimum taxes in the first five cases but not in the last one[15].

VII. Concluding Remarks

In this paper, we have made four contributions. First, we have derived optimum and revenue-maximizing export taxes in a three-country framework. Although we have considered only two exporting countries, the analysis extends to three or more exporters in a straightforward manner. Indeed, in Panagariya and Schiff (1991), we derive Nash optimum taxes for nine cocoa exporters. Second, we have demonstrated that if one of the countries is a Stackelberg leader, both countries improve their welfare relative to Nash equilibrium. More interestingly, in the symmetric case, the follower's welfare is higher than that of the leader. Third, we have shown that as in the traditional literature, the revenue-maximizing Nash tax is larger than the optimum Nash tax. Finally, we have shown that the actual welfare may be higher in the revenue-maximizing tax Nash equilibrium than in the optimum tax Nash equilibrium. This result cannot arise in the traditional two-country analyses of optimum and revenue-maximizing export taxes.

An important question concerns the validity of our results when the demand and supply curves are nonlinear. We note that in this case the reaction curves are not necessarily positively sloped[16]. However, if the sufficiency conditions for a positive slope are satisfied, the revenue-maximizing Nash tax will be larger than the Nash optimum tax. Hence, the welfare ranking of the two types of taxes remains ambiguous.

In our future work, we propose to extend this analysis in a number of directions. First, we plan to consider games when countries take their rival's quantity rather than tax rate as given. Second, we will study the implications of smuggling for Nash equilibria. In this case, each government must take into account the possibility of tax evasion via illegal exports if its tax rate happens to be dramatically different from that of its rival. Finally, we plan to consider the implications of timing of production versus tax decisions along the lines suggested by Eaton and Grossman (1986).

Notes

1 Also see Bhagwati and Srinivasan 1973, and Panagariya 1982. The standart practice in the literature is to formulate the problem in terms of tariffs rather than export taxes. But in view of the Lerner symmetry theorem, the two types of taxes are equivalent. For reasons that will become apparent later, we prefer to formulate the problem in terms of export taxes.

2 The literature being referred to here assumes that commodities are homogeneous and that there is no intra-industry trade.

3 A different kind of interdependence arises in two-country models when the two countries tax the export of (or imports from) each other. This type of interdependence has, of course, been analyzed amply in the literature. Thus, see Johnson (1954), Gorman (1958), Horwell (1966), Kuga (1973), Tower (1975), Weymark (1980) and Thursby and Jensen (1983).

4 These figures relate to the year 1986.

5 In Panagariya and Schiff (1991), we derive Nash optimum taxes in a ten-country model of the world cocoa market. Contrary to what we do in this paper, the 1991 paper does not deal with alternative taxes such as Nash revenue-maximizing taxes, cooperative optimum taxes or Stackelberg optimum taxes.

6 Developed countries have generally not resorted to retaliatory taxes on exports of these commodities. In the case of coffee, they have in fact indirectly encouraged export taxes via the International Coffee Agreement.

7 The reader may note some resemblance between the basic structure of our model and that in the recent international-duopoly literature (e.g., Brander and Spencer 1985). However, the nature of our problem is fundamentally different form that of the duopoly problem. Inthe latter case, *firms* exert market power and, under Cournot assumption, export too little relative to the country's optimum. Hence optimal policy is an export subsidy. By contrast, our model has the conventional feature that firms are perfectly competitive and export too much relative to the optimum. Therefore, optimal policy is an export tax.

8 A higher tax by A implies a larger market and hence a greater market power for B.

9 As noted earlier this assumption is indeed valid for cocoa.

10 The relationship between welfare under welfare maximizing taxes and welfare under revenue maximizing taxes is examined in the general (non-linear) case in Panagariya and Schiff (1994). The 1994 paper does not deal with any of the other issues examined in this paper.

11 At point D_1', country 2's supply declines to zero. The kink at D_1' arises due to the fact that below this point the demand curve facing country 1 coincides with the world demand curve.

12 Given that the supply curve is upward sloping, the government's cost MCR_1 of obtaining an additional unit of output is higher than the price. For instance, if producers produce 10 units at price 10 and 11 units at price 11, then the marginal cost of the eleventh unit is 11 but the costo of acquiring an eleventh unit is $MCR = 11^2 = 10^2 = 21$.

13 The essential point is that when revenue rather than welfare is the objective, the government exercises both monopoly power in the world market and monopsony power inthe domestic market.

14 Note that alternative taxes which yield higher profits for both countries relative to Nash taxes do exist. For example, a small equi-proportionate increase in Nash optimum taxes will lead to higher profits for both countries.

15 We remind the reader that these results are based on simulations and need not hold in general.

16 However, despite intense efforts, we were unable to obtain negatively-sloped reaction curves in the general case (see Panagariya and Schiff, 1994).

17 Initial results (Panagariya and Schiff, 1992) indicate that the Nash solution is more restrictive in the case os export quotas than with export taxes, and that welfare is higer inthe case of quotas (the latter based on simulations of the cocoa industry). This result differs from that of the traditional analysis where the same effect is obtained in the case of an export tax and of anexport quota. In the case of strategic interaction, the equivalence between taxes (or tariffs) and quotas disappears. Tower (1983) obtains the same results as in our 1992 paper in the case of a Stackelberg game, at least for the Stackelberg leader. He does not examine the effect of the tax versus quota for the follower.
The logic of this result is as follows: the more inelastic the excess-demand curve of the follower, the better for the leader because then the follower will levy a higher tax. This will result in a higher world price for the leader. Also, the higher tax by the follower gives more market power to the leader. How can the leader make the excess-demand of the follower more inelastic? By setting an optimal quota rather than an optimal tax, by using an optimal ad-valorem tax rather than a specific tax, and by setting the optimal fixed price rather than the optimal tax. On the other hand, if the leader is a monopolist and the follower is the bilateral trading partner, then the leader wants the follower to have an elastic excess-demand curve, and all the results are reversed.

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