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MINIMUM PENSION INSURANCE IN THE CHILEAN PENSION SYSTEM

SALVADOR ZURITA*
Departamento de Administración,
Facultad de Ciencias Económicas y Administrativas,
Universidad de Chile

Abstract:

One of the "social" features of the Chilean individual capitalization pension system is the minimum pension scheme, which guarantees its members a minimum pension irrespective of the funds they accumulate, with the only requirement of twenty years of social security tax payments. The purpose of this paper is to estimate the implicit fiscal subsidy, using an option-based approach. We capture the risk associated to the returns on the pension fund account of a worker by modeling its value as a diffusion process and show the correspondence between the minimum pension insurance and a financial put option. Our results are the present value of the minimum pension benefit, equivalent to 3 percent of Chilean GDP for current active and non-active affiliated workers. These estimates are notoriously higher than previous results based on deterministic models, and strongly suggest the importance of explicitly considering the risk associated to pension assets when estimating the cost to the government of the insurance implied by the minimum pension benefit.

Introduction

The 1981 reform of the pension system in Chile established an individual capitalization pension scheme, in which a worker's pension upon retirement directly depends on the security tax contributions that he has made during his working life.

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However, the system has a social feature, whereby it guarantees its members a minimum pension irrespective of the funds accumulated in their account, with the only requirement of twenty years of social security tax payments. Since health and pension plans are voluntary for self-employed workers, only workers hired with labor contracts are required to become affiliated with the on-going pension system.

Ortúzar (1988) estimated the financial commitment of fiscal funds deriving from the reform of the Chilean pension system, until the year 2030, explicitly including the part attributable to the minimum pension. Also Wagner (1991) simulated the implicit fiscal subsidy in the program for a stationary state which is meant to capture the main characteristics of the Chilean economy around 1990. He found that the implied fiscal subsidy is low in relation to other social security programs in Chile, but quite sensitive to small changes in parameters such as the level of pension funds accumulated over the whole working life and the minimum pension payments. The purpose of this paper is to also estimate the implicit fiscal subsidy. However, instead of estimating this subsidy under different scenarios for the key economic variables, we focus on the risk associated to the rate of return of the pension funds, and we estimate the implicit subsidy modeling this uncertainty using an option-based approach. The organization of the paper is as follows: Section I offers a brief overview of the regulations on minimum pension in Chile. Section II addresses the model and its assumptions and Section III presents a numerical solution based on Monte Carlo techniques. Finally, Section IV examines both our conclusions and suggestions for future research on the subject.

I. Minimum pension in Chile

The first regulation prescribing a minimum pension in Chile was Law N° 10,382 of 1952, which established a minimum pension for all workers affiliated with the Social Security System. In order to qualify for the minimum pension, male workers had to be to be sixty five years old, and female workers fifty five years old. They were also required to have a minimum of social security payments (five hundred weeks of security tax payments for male workers, and 500 weeks for female workers). Later on, in the seventies Law N° 15,386 added minimum pension benefits in case of health impediments (disability insurance), and also in case of death of the worker (survivors insurance), for his or her spouse and children.

In 1975 the military government enacted Law N° 869, extending minimum pension benefits to elderly people (over sixty five) and people with health impediments not covered by the security system. This law confirmed the concept of social benefits for needy people, regardless of their prior contribution.

With the 1981 reform of the pension system, all Chilean workers have an individual capitalization account, accumulating their security tax payments over the years. These funds are managed by a private firm chosen by the worker himself, called "Administradora de Fondos de Pensiones" (Pension Funds Management Companies - AFPs). In the event that a worker's pension payments upon retirement fall bellow the minimum pension, the Government covers the difference. The worker, however, must demonstrate at least twenty years of security tax payments in order to be eligible for this benefit. Both the minimum salary and the minimum pension are determined by law; Table I presents their evolution since 1975.

TABLE I

MINIMUM SALARY AND MINIMUM PENSION IN CHILE: 1975-1993

Year	Minimum pension per month (1)	Minimum salary per month (2)	Min. pen./Min. sal. (1)/(2)
1975	54.2	77.5	0.70
1976	53.0	76.7	0.69
1977	60.7	83.0	0.73
1978	75.2	105.2	0.71
1979	75.2	105.5	0.71
1980	75.0	105.0	0.71
1981	74.0	118.7	0.62
1982	74.5	121.7	0.61
1983	72.2	98.2	0.73
1984	75.2	84.0	0.89
1985	68.5	80.0	0.85
1986	68.0	76.2	0.89
1987	66.0	72.0	0.91
1988	66.7	76.0	0.87
1989	67.2	85.7	0.78
1990	73.0	92.0	0.79
1991	80.7	99.5	0.81
1992	80.2	103.5	0.77
1993	81.7	108.5	0.75

Sources: Superintendency of Social Security (minimum salary and minimum pension).

Note: Both the nominal minimum pension and the minimum salary are expresed in dollars of March 1994 (Banco Central de Chile).

II. The model

In the tradition of the continuous time finance models, we assume that trading takes place continuously, and that there is a unique instantaneous risk-free interest rate at which borrowing and lending take place. We also make the following two simplifying assumptions: (i) individuals are employed during all their working lives, so that there are no lags in the contributions they make to their individual capitalization accounts[1], and (ii) there is no mortality risk. With this setup, the only source of uncertainty left is the risk associated to the return on the pension assets, which we model by the following diffusion process:

$$dA = [\alpha A + \eta(t)]\,dt + A\,\sigma\,dz \qquad (1)$$

where:

A represents a worker's pension funds (his or her individual capitalization account),

α is the instantaneous expected rate of return on the existing assets per unit time,

η is the instantaneous rate of contributions per unit time by employed worker to the pension fund,

σ stands for the instantaneous standard deviation of the return on the assets per unit time, and

d z is an increment to a Gauss-Wiener process.

Equation (1) says that the vâlue of pension funds increases due to a normal rate of return and as the worker contributes new funds. Since there is uncertainty concerning the return on the assets, there is a stochastic component in (1).

Under the operation of Chilean law, employed workers are required to contribute 10 percent of their salary to cover pension benefits[2]:

$$\eta(t) = 0.1\, w(t) \qquad (2)$$

where:

w (t) = the worker's taxable monthly salary.

At the end of his working life, individuals use their pension funds to buy an life annuity from an insurance company in order to obtain a pension benefit for life.[3] The insurance company offers a pension rate of payments to retired workers, in exchange for their pension funds. The funds required to guarantee the minimum pensions are:

$$M = m/\delta$$

where:

M = the funds required to buy a minimum pension for life,
m = the minimum pension (defined by law), and
δ = the rate of payments offered by the insurance company.

In the event that the accumulated funds are insufficient to provide the minimum pension, we assume that the government covers the difference. Thus, there are two possibilities at the date of retirement (which we represent by T):

Case 1. If δ A(T) < m, the individual capitalization account is insufficient to provide the minimum pension, and the government must subsidize the worker by M – A (T).

Case 2. If δ A(T) ≥ m, the individual capitalization account makes it possible to buy a life annuity equivalent to or higher than the minimum pension. In this case, there is no cost to the government.

From the previous analysis it is clear that the payoff function of the government at maturity is similar to that of a financial put option, where the pension assets play the role of the underlying asset, and the exercise price is the amount required to buy a minimum life annuity, M. The level of the subsidy implicit in the law, of course, is the present value of the minimum pension liabilities to the government, the value of the put option. Unfortunately, it is not possible to value this put option by using Black and Scholes's formula, since they assume that the underlying asset follows a Brownian motion process, which is different from the diffusion process (1) followed by the pension assets. In the remaining part of this section, we examine two different valuation approaches for the minimum pension insurance.

A. *Arbitrage*

In this section we obtain a partial differential equation using arbitrage arguments, which if solved, would give us the value of the minimum pension insurance. Let P (A, t) represent the government's present value liability involved in the minimum pension, which we model as a put option. Then we can obtain the stochastic process followed by P (.) using Ito's lemma:

$$dP = \left[\frac{1}{2}\frac{\partial^2 P}{\partial A^2} A^2\sigma^2 + \frac{\partial P}{\partial A}(\alpha A + \eta) + \frac{\partial P}{\partial t}\right] dt + \frac{\partial P}{\partial A} A\sigma\, dz \qquad (3)$$

Note that the stochastic components in (1) and (3) are the same Wienner process dz. This allows us to set up a risk-free portfolio, combining the pension assets and the minimum pension put such that a perfect hedge results. Consider an investment of $\$w_1$ in pension assets, which also implies the commitment to invest a rate of η (t) from the present to the date of retirement, T. Such investment has a rate of return of:

$$\frac{dA - \eta\, dt}{A} \qquad (4)$$

The first part in (4), dA/A, represents the capital gains of the investment in pension funds (the appreciation in the value of the pension units), and the second corresponds to the investment at the rate η (t), which plays the role of negative dividends, that is, cash paid, not received, by the investor.

Now consider a portfolio consisting of an instantaneous investment of $\$w_1$ in pension assets, an instantaneous investment of $\$w_2$ in minimum pension insurance, and an instantaneous investment of $w_3 = -(w_1 + w_2)$ in the risk-free asset, which has a rate of return of r, the risk-free rate of interest. Note that this portfolio does not require an initial investment, since:

$$w_1 + w_2 + w_3 = 0 \qquad (5)$$

Also, this portfolio has an instantaneous rate of return of:

$$dx = w_1\left[\frac{dA - \eta\, dt}{A}\right] + w_2\frac{dP}{P} + w_3\, r\, dt \qquad (6)$$

Substituting (1), (3) and (5) into (6), the portfolio's rate of return is:

$$dx = [w_1(\alpha - r) + w_2(\alpha_P - r)]\, dt + [w_1\sigma + w_2\sigma_P]\, dz \qquad (7)$$

where α_P represents the instantaneous expected return per unit time on the minimum pension insurance, defined by:

$$\alpha_P = \frac{1}{P}\left[\frac{1}{2}\frac{\partial^2 P}{\partial A^2} A^2\sigma^2 + \frac{\partial P}{\partial A}(\alpha A + \eta) + \frac{\partial P}{\partial t}\right] \qquad (8)$$

and σ_P represents the instantaneous standard deviation of the return on the minimum pension insurance, defined by:

$$\sigma_P = \frac{1}{P}\left[\frac{\partial P}{\partial A} A\sigma\right] \quad (9)$$

Now choose the investments in the pension funds and the minimum pension insurance such that the coefficient of the Wienner process dz in (7) is zero, and thus the portfolio becomes risk-free:

$$w_1 \sigma + w_2 \sigma_P = 0 \quad (10)$$

Since our portfolio requires no wealth and is risk-free, in equilibrium its expected return must be zero, or else there would be arbitrage opportunities (see (7)):

$$w_1 (\alpha - r) + w_2 (\alpha_P - r) = 0 \quad (11)$$

The no risk condition (10) and the no arbitrage condition (11) imply that:

$$\frac{\alpha - r}{\sigma} = \frac{\alpha_P - r}{\sigma_P} \quad (12)$$

Now substituting (8) and (9) in (12) we obtain the arbitrage pricing equation:

$$\frac{1}{2}\sigma^2 A^2 \frac{\partial^2 P}{\partial A^2} + (r A + \eta)\frac{\partial P}{\partial A} + \frac{\partial P}{\partial t} - r P = 0 \quad (13)$$

Equation (13) can also be derived by an equilibrium argument, which is done in the Appendix. It is almost equal to the Black-Scholes partial differential equation, with the only difference of the additional term η in the coefficient of the first-order partial derivative with respect to the pension assets. This equation has many solutions, depending on the boundary conditions. In our case, we know that at the end of the working life (*i. e.*, $t = T$), the minimum pension insurance will be equal to the maximum between zero and the difference between the funds required to buy the life annuity equivalent to a minimum pension and the accumulated pension funds:

$$P(A, T) = \text{Max } [M - A(T), 0] \quad (14)$$

Since the initial condition (14) is given by the value of the minimum pension insurance at the end of the working life, (13) is a parabolic partial differential equation. It does not have an analytical solution, but numerical solutions are available.

B. *Risk neutral valuation*

Cox and Ross (1976) suggested a canny procedure to value European options. Noting that the pricing equation (13) is independent of risk preferences, they arrived at the conclusion that risk preferences cannot affect the solution and hence they suggest

making the very simple assumption that all investors are risk neutral. In this case, the expected return on all securities is the risk-free rate of interest, r, since risk neutral investors do not require a premium to induce them to take risks. Similarly, the present value of any cash flow can be obtained discounting it at the risk-free rate of interest.

In our case, the value of the minimum pension insurance at maturity is given by (14), and hence we can value the minimum pension insurance as the expected value of (14) discounted to the present at the risk-free interest rate:

$$P(A, t) = e^{-r(T-t)} \hat{E} [\text{Max } (M - A(T), 0)] \quad (15)$$

where the hat over the expectations operator denotes that the probability distribution used in computing such expectation is not the actual distribution, but rather the distribution that would exist in a world of risk-neutral investors. In such a world, the pension assets would grow at the risk-free interest rate, that is, $\alpha = r$ in (1).

A simple technique to value the minimum pension put by (15) is to use a Monte Carlo simulation, which consists of generating a sample of values for the value of the minimum pension put at maturity, and estimating the expectation (15) simply as their average. One of the first numerical methods proposed in the literature is the Euler method, according to which the stochastic process of the pension assets (1) is discretized as:

$$\Delta A = [r A + \eta(t)] \Delta t + A \sigma \varepsilon \sqrt{\Delta t} \quad (16)$$

where:

ΔA = the change in pension assets, A, in an interval of time Δt; and
ε = a random drawing from a standardized normal distribution.

According to this method, the random path of the pension assets (16) is simulated several times, and the final value of the variable, A (T), is recorded for each of them. This value of the pension assets at retirement is subtracted from the capital needed to guarantee the minimum pension payments for each sample, and the value of minimum pension insurance is obtained as the average of the positive values of this difference, discounted to the present at the risk-free rate of interest according to (15):

$$\hat{P} = e^{-r(T-t)} \frac{1}{n} \sum_{i=1}^{n} \text{Max } [M - A_i(T), 0] \quad (17)$$

where:

$A_i(T)$ = the value of pension assets at retirement age in the sample i (i = 1, 2, ..., n).

An unbiased estimation of the standard deviation of the estimate of the minimum pension insurance is given by:

$$\hat{s}^2 = \frac{1}{n-1} \sum_{i=1}^{n} (e^{-r(T-t)} \text{Max } [M - A_i(T), 0] - P)^2 \quad (18)$$

For large values of n, we can approximate (n −1) by n, and the distribution of:

$$\frac{\hat{P} - P}{\sqrt{\frac{s^2}{n}}} \qquad (19)$$

converges to a standardized normal distribution as n increases. For the values of n in this paper we consider (n = 5,000), the distribution can be regarded as normal and we can obtain confidence limits on the estimate of P based on it. For example, a 95 percent confidence level for the minimum pension insurance P is provided by the following interval:

$$P = 1.96 \frac{s}{\sqrt{n}} < P < P + 1.96 \frac{s}{\sqrt{n}} \qquad (20)$$

Since the length of the interval is a function of n, it can be reduced by increasing n. To reduce the standard deviation by a factor of ten, the number of simulation trials has to be multiplied by one hundred.[4]

III. Numerical solution

In this section we provide a numerical example of the valuation of the minimum pension insurance. Because of its simplicity, and the generally good results obtained in the valuation of European-type options, we use Monte Carlo techniques, specifically the Euler method described in the previous section. This model requires estimates of five parameters: the risk-free rate of interest, the contribution rate, the contribution period, the volatility of the pension assets and the funds needed in order to purchase a life annuity equivalent to a minimum pension from an insurance company. We will make the assumption that the long term rate of interest (real) is of 4 percent yearly. In the following sub-section we suggest reasonable estimates for the remaining parameters of the model.

A. *Estimates of the parameters*

A.1. Rate of contributions (η)

Workers contribute 10 percent of their taxable salary to their individual capitalization accounts. Therefore, a critical variable is the level of salaries over their life cycles. Figures 1 and 2 show the monthly salaries of female and male workers of different ages and with different levels of educational attainment, as reported in the last survey on occupation in the city of Santiago, conducted by the Economics Department of the University of Chile in June 1993. It is obvious that the minimum pension is a subsidy that has zero value for the well educated and well paid Chilean workers, so that we will generate estimations for the relevant sector of low income workers, which have between 0 and 4 years of schooling. An interesting characteristic of such people is that

FIGURE 1A

MONTHLY WAGE OF MALE WORKERS
Level of Education: 1-4 years

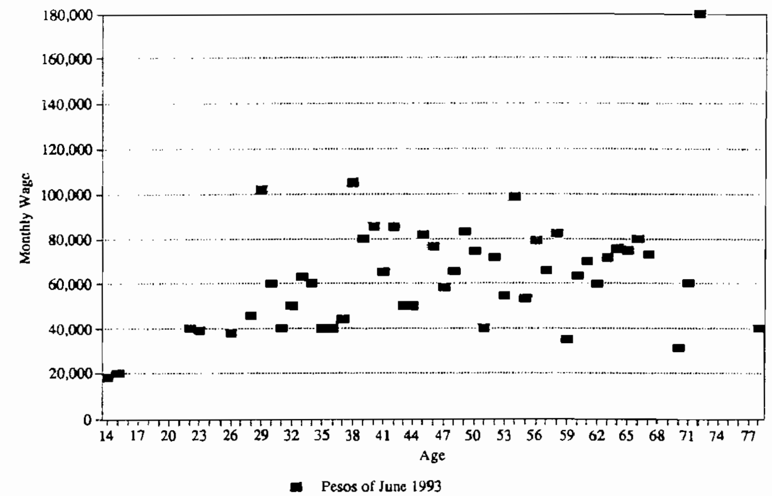

FIGURE 1B

MONTHLY WAGE OF FEMALE WORKERS
Level of Education: 1-4 years

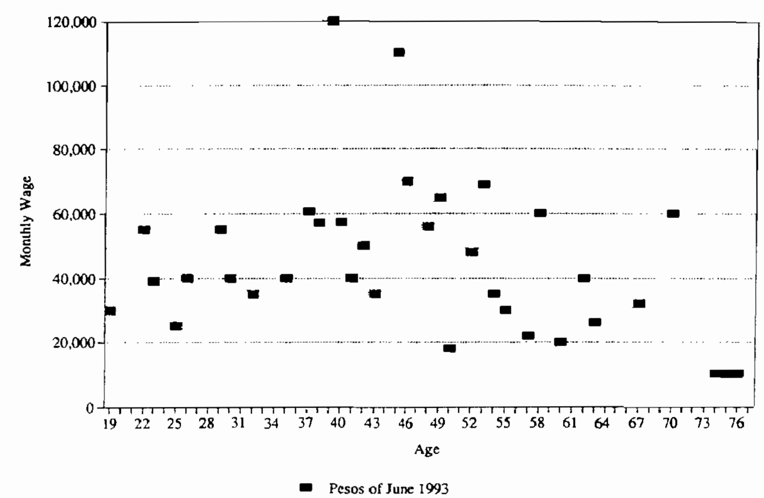

FIGURE 2A

MONTHLY WAGE OF MALE WORKERS
Level of Education: 5-9 years

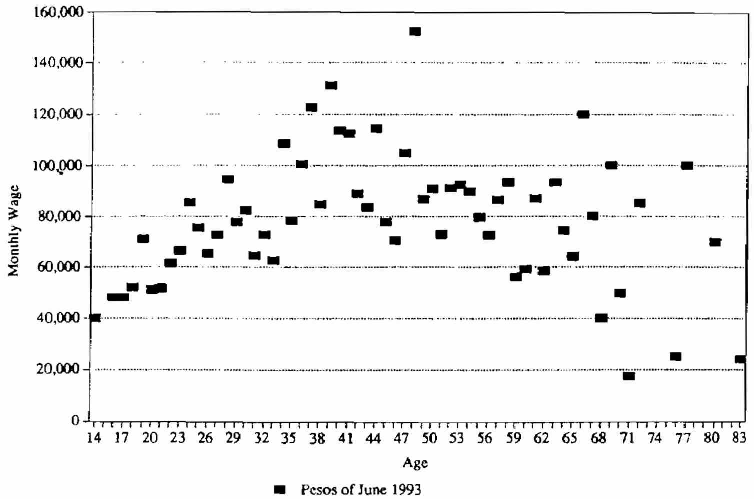

FIGURE 2B

MONTHLY WAGE OF FEMALE WORKERS
Level of Education: 5-9 years

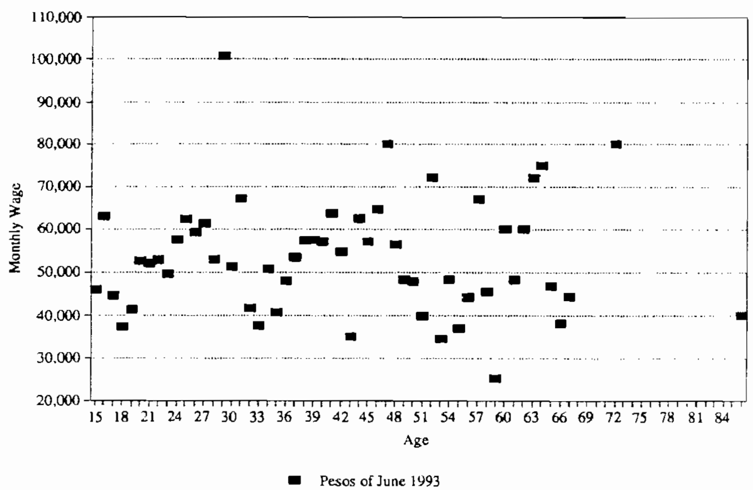

TABLE II

NUMBER OF ACTIVE AFFILIATES BY AGE AND BY INCOME
(September 30, 1993)

a. Female workers

Age\Income	< 24	24-47	47-71	71-141	141-354
< 30	2,683	4,591	6,701	108,690	140,690
30-40	1,149	2,167	3,383	67,634	26,745
40-50	425	893	1,462	37,636	50,335
50-60	126	277	483	16,924	15,483
> 60	20	25	51	1,759	1,340

b. Male workers

Age\Income	< 24	24-47	47-71	71-141	141-354
< 30	6,052	9,748	11,901	157,001	305,024
30-40	2,305	3,876	5,139	104,613	194,228
40-50	857	1,532	2,185	58,199	98,455
50-60	357	697	7,084	29,832	43,244
> 60	98	177	320	9,028	9,712

Source: "Boletín Mensual", Superintendencia de Administradoras de Fondos de Pensiones.

Notes: The table contains the number of active affiliates that contributed in September of 1993, for wages earned in August 1993. Age is measured in years and taxable income is in dollars of March 1994.

the slope of the curve for salaries versus age seems to be zero, in sharp contrast with the more educated workers, where it is clearly positive, and sometimes first positive and then negative, suggesting the familiar u-shape of the life cycle hypothesis. For this reason, we will assume that workers contribute ten percent of their salaries, which are assumed constant during all their working lives. Table II presents the taxable income of the active affiliates in September of 1993. In this table, note that a relatively small number of workers appear with income levels inferior than the minimum salary. They include domestic aide, seasonal and part-time workers, all of which are eligible for the minimum pension benefit and must be considered in our estimates.

A.2. Period of contribution

From Figures 1 and 2 we can also see that male and female workers with less than 5 years of formal schooling seem to start working at about 20 years of age (only comparatively few seem to start working before twenty).

Under current law, a Chilean worker needs to make security tax payments only 20 years in order to qualify for the minimum pension benefit. There are a variety of reasons why a worker may not contribute until retirement age, including: workers that decide not to work at the current salary, unemployment, and evasion of security tax payments (licit in the case of self-employed workers and illegal in other cases). This accounts for the difference between the number of workers affiliated with the new pension system, which is over four million six hundred thousand people, that have made contributions to their individual capitalization accounts at one time or another, and the number of active affiliates, of some two million three hundred thousand workers, that are currently making security tax payments. Table III reports the historical frequency of contributions of female and male affiliated workers during the thirteen years that the private pension system has been in operation. It also reports the estimated total number of years of security contributions on the assumption that the historical frequency for the first thirteen years is representative of the whole life of the workers. On this assumption, the last two rows of Table IIIa (female) and the last row of Table IIIb (male) correspond to workers with less than twenty years of contributions and who consequently are not eligible for the minimum pension benefit, and so we concentrate on estimating the subsidy for the rest of the population. Also from Table III we can compute the average contribution time, weighting the estimated years of contribution by the respective number of workers. In this way, the expected contribution time is of 38.49 years for female workers, and of 42.52 years for male workers.

For the workers that will be entitled to benefit from the minimum pension insurance, and in order to simplify the estimations, we will assume that all of them start working when twenty years old and work until their expected contribution time.[5]

A.3. Volatility of pension assets (σ)

Our data base for estimating the volatility consists of daily data from July to October, 1993, for the 21 pension funds in the industry. We estimate the annual volatility of the rate of return of the different pension funds in the usual way: Define

$n + 1$ = number of daily observations

S_i = value of the pension fund share at the end of the i^{th} day ($i = 0, 1, 2, ..., n$)

u_i = $\ln (S_i/S_{i-1})$

Note that $S_i = S_{i-1} e^{u_i}$, then ui is the continuously compounded return (not annualized) in the i^{th} interval for $i = 1, 2, ..., n$. An unbiased estimate of the standard deviation of the continuously compounded rates of return is:

$$s = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (u_i - \bar{u})^2} \qquad (21)$$

where:

$$\bar{u} = \frac{1}{n} \sum_{i=1}^{n} u_i \qquad (22)$$

TABLE III

FREQUENCY OF CONTRIBUTIONS OF AFFILIATED WORKERS
(Both active and non-active)

a. Female workers

Frequency of contributions	Years of contributions (estimated)	Number of female workers (estimated)
100%	40.0	949,851
92%	36.8	347,308
85%	34.0	115,124
77%	30.8	81,775
69%	27.6	61,205
61%	24.4	48,679
54%	21.6	39,134
46%	18.4	29,218
39%	15.6	82,528
Total (a)		1,755,003

b. Male workers

Frequency of contributions	Years of contributions (estimated)	Number of female workers (estimated)
100%	45.0	1,549,756
92%	41.4	566,660
85%	38.3	187,834
77%	34.7	133,423
69%	31.1	99,862
61%	27.5	79,423
54%	24.3	64,144
46%	20.7	47,671
39%	17.6	134,650
Total (b)		2,863,425
Total (a) + (b)		4,448,245

Source: "Boletín Mensual", Superintendencia de Administradoras de Fondos de Pensiones.

Note: The number of female and male workers in each category was calculated on the assumption that the percentage of female and male workers is the same for each frequency, and equal to the population percentages of 38% for female workers and 62% for male workers.

In order to obtain annual volatilities the standard deviation (21) was multiplied by the square root of 365:[6]

$$s^* = s \sqrt{365} \qquad (23)$$

The results are reported in Table IV. Excluding the pension fund Plan Vital, which was bought out by Invierta, and had to sell off an important part of its portfolio since

TABLE IV

ANNUAL VOLATILITIES

AFP	Annual volatility
CONCORDIA	1.9%
CUPRUM	1.9%
EL LIBERTADOR	1.9%
FUTURO	2.8%
HABITAT	2.4%
INVIERTA (*)	9.0%
MAGISTER	1.8%
PLAN VITAL (*)	24.0%
PROTECCION	8.4 %
PROVIDA	2.0%
SANTA MARIA	5.1%
SUMMA	9.5%
UNION	1.9%
BANSANTANDER	2.3%
PREVIPAN	2.0%
LABORAL	2.3%
BANGUARDIA	2.0%
QUALITAS	1.9%
FOMENTA	2.2%
NORPREVISION	7.1%
GENERA	6.8%

(*) During this period there was a merger between Invierta and Plan Vital, and Plan Vital sold out an important part of its portfolio as many workers left the AFP. This explains their high volatilities.

Note: The volatilities were calculated using the daily quotations from July to October, 1993, excluding week-ends, and in annual terms.

many workers left it, the average volatility was 3.8 percent, which is perhaps rather low. For this numerical example we will use the estimated volatility of four and a higher volatility of seven percent in order to perform sensitivity analyses.[7]

A.3. The required capital (M)

In order to estimate the funds required to purchase a minimum pension, we obtained quotations from some insurance companies. Obviously, the required capital M depends on the composition of the family: age of the spouse, and number and age of the children. For illustrative purposes, we only consider the basic case of a couple without dependent children.

A typical quotation is as follows (this particular one comes from an insurance company in Santiago, but the others are relatively similar):

Gender	Minimum Amount, M
Male (65 years old and a wife of 60)	US$ 13,625
Female (alone)	US$ 14,500

Since at the present the minimum pension is US$ 82, the implicit δ is equal to 82/13,625 = 0.6 percent monthly (or 7.2 percent yearly) in the case of male workers and 82/14,500 = 0.56 percent monthly (or 6.8 percent yearly) in the case of female workers, the difference being attributable to the fact that female workers retire at sixty years whereas male workers retire at sixty five years old, and to the different life expectancy of male and female workers. In the case of a female worker, the quotation does not consider a husband, since it is assumed that her husband, if alive, has his own pension.

B. *Estimation of the minimum pension benefit*

As stated in the foregoing section, we will make the simplifying assumption that workers start working at twenty years of age and work all their lives until retirement, which is at 38.49 years for female workers and of 42.52 years for male workers.[8]

On the assumptions made in the previous section, we will simulate the following two discrete stochastic processes for pension assets (expressed in dollars of March 1993):

$$\Delta A = [0.04\, A + \eta]\, \Delta t + 0.04\, A\, \varepsilon \sqrt{\Delta t} \qquad (24)$$

corresponding to the estimated volatility of 4 percent yearly, and:

$$\Delta A = [0.04\, A + \eta]\, \Delta t + 0.07\, A\, \varepsilon \sqrt{\Delta t} \qquad (25)$$

corresponding to a volatility of 7 percent yearly.

In both cases, we will consider time intervals of $\Delta t = 0.01$ years (that is, we assume that prices can change every 3.65 days, or approximately, twice a week). Our goal is to estimate the cost to the government of the minimum pension benefit.

B.1. Estimation procedure

The estimation procedure consists of the following steps:

1. Our first goal is to obtain an estimate of the minimum pension benefit for each group of workers in Table II (defined by gender, age, and taxable income). In order to do this, we will need to simulate the future path of the pension assets according to (24) and (25), starting from the current level of the pension assets. For workers with twenty years of age we can assume $A(0) = 0$ (at the beginning of the working life in $t = 0$, there has been no tax payments at all), but we must deal with the starting pension assets for the older workers in Table II.

Since there are no statistics available at the required level of disaggregation, we make the assumption that the pension funds increase at an expected rate of $\alpha = 5\%$ yearly (continuously compounded). We also assume that the workers in each category of taxable income earn the average income of the category, that is to say, the first group in Table II earns US$ 12, the second earns US$ 35, the third earns US$ 59, the fourth earns US$ 106, and the last one earns US$ 247.

The assumption of a rate of return equal to an expected rate of growth of five percent is equivalent to assuming that there is no volatility in (1), in which case the past evolution of the pension assets can be described by the differential equation:

$$A'(t) = \alpha A(t) + \eta$$

with the initial value A(0) = 0. The solution to this ordinary differential equation is:

$$A(t) = \frac{\eta}{\alpha}\left[-1 + e^{\alpha t}\right]$$

Table V reports the accumulated funds according to this assumption.

2. With the previous input, we obtain an estimate of the minimum pension insurance performing 5,000 simulations of a Fortran program that computes the average minimum pension insurance put (17) and the standard deviation of the minimum pension insurance (18) simulating the path of the pension assets according to (24) and (25). In this way, we have the cost for each type of active affiliates, by gender, age, and taxable income (and for volatilities of the returns of the pension assets of 4 percent and 7 percent).

The results are reported in Table VI, where the first number in each cell represents the average, and the number in brackets represents the standard deviation of the five thousand simulastions computed of the minimum pension insurance.[9] As should be expected, the cost to the government of the minimum pension benefit is higher for female workers than for male workers of similar age and taxable income, due to the fact that males work during longer periods on average and have a lower life expectation upon retirement. Also very reasonably, the minimum pension insurance is consistently more valuable if pension funds have a volatility of 7 percent instead of 4 percent. The effect of age may need an explanation: For low income workers, the higher the age group, the more valuable the minimum pension insurance; but for relatively higher income groups, the cost of the minimum pension insurance first increases and then decreases with the age group. In general, the effect of time to maturity of a financial put option is not clear: whereas for American type options, the longer the time to maturity, the more valuable the option, in the case of European type options of the kind considered here, the effect may be valid either way, since the holder of a European option can only exercise it at maturity, and an early exercise of the option might be optimal. In the case of a put on a stock that does not pay dividends, early exercise of

TABLE V

EXPECTED PENSION FUNDS

Years (t)	η = 14.4	η = 42.0	η = 70.8	η = 127.2	η = 296.4
5	82	239	402	723	1,684
15	322	938	1,582	2,842	6,622
25	717	2,092	3,526	6,335	14,763
35	1,369	3,994	6,733	12,096	28,185

Note: The assumed contribution rate is in dollars of March 1994 per year, and the expected pension funds are in dollars of March 1994. These figures are computed on the assumption that pension funds grow at a continuosly compounded rate of 5% per year.

TABLE VI

ESTIMATES OF MINIMUM PENSION INSURANCE BY GENDER, AGE AND INCOME

a. Female workers and volatility of 4%

Age\Income	< 24	24-47	47-71	71-141	141-354
< 30	3,451 (58)	2,785 (169)	2,092 (284)	758 (452)	0 (3)
30-40	5,126 (86)	4,090 (251)	3,008 (424)	950 (644)	0 (0)
40-50	7,586 (116)	5,924 (339)	4,191 (571)	938 (793)	0 (0)
> 50	11,195 (104)	8,481 (303)	5,649 (511)	423 (545)	0 (0)

b. Male workers and volatility of 4%

Age\Income	< 24	24-47	47-71	71-141	141-354
< 30	2,677 (63)	1,988 (184)	1,265 (308)	146 (234)	0 (0)
30-40	3,970 (95)	2,895 (278)	1,773 (466)	146 (287)	0 (0)
40-50	5,863 (134)	4,140 (391)	2,345 (657)	77 (233)	0 (0)
> 50	8,623 (154)	5,818 (444)	2,892 (755)	3 (43)	0 (0)

c. Female workers and volatility of 7%

Age\Income	< 24	24-47	47-71	71-141	141-354
< 30	3,452 (103)	2,787 (301)	2,096 (500)	870 (653)	6 (61)
30-40	5,126 (154)	4,092 (447)	3,015 (738)	1,134 (930)	6 (79)
40-50	7,587 (205)	5,926 (599)	4,195 (1,003)	1,215 (1,169)	2 (44)
> 50	11,196 (182)	8,482 (532)	5,651 (896)	699 (900)	0 (0)

d. Male workers and volatility of 7%

Age\Income	< 24	24-47	47-71	71-141	141-354
< 30	2,678 (113)	1,992 (326)	1,283 (511)	306 (411)	1 (22)
30-40	3,972 (170)	2,900 (493)	1,800 (768)	369 (554)	1 (23)
40-50	5,863 (238)	4,143 (692)	2,378 (1,085)	311 (605)	2 (1)
> 50	8,623 (271)	5,818 (789)	2,912 (1,273)	86 (328)	0 (0)

the option might be optimal, depending on the value that may be attained by the stock price. But in our particular case, it would even be more likely that the holder of the put might find it optimal to exercise it earlier if he could, since the underlying asset is one that pays negative dividends (that is to say, the periodic contributions to the individual capitalization account are similar to dividends that are paid, not received by the holder of the minimum pension put). In this case, the older the group age, the less time to maturity of the minimum pension put, which makes the put more valuable since early exercise is very likely to be optimal. Of course, this effect is not as important for the workers with higher taxable income, since for them the probability that the government will eventually have to cover any difference is significantly lower, and therefore it is most likely that the put will never be exercised.

3. With these estimations, we compute the cost to the government of the minimum pension benefit for the female and male active affiliates, simply by multiplying the number of workers in each category (from Table II), times the estimated minimum pension insurance for each type (from Table VI), and then by adding up across the different types of age and taxable income. In this way we can compute the cost to the government of the subsidy for female and male workers. The value of the minimum pension insurance for female active affiliates was estimated with 95 percent probability in US$ 270 ± 1.9 millions if the volatility of pension assets is 4 percent, and US$ 314 ± 3.1 millions if the volatility is 7 percent. In like manner, the value of the minimum pension insurance for male active affiliates was estimated with 95 percent probability in US$ 169.8 ± 1.4 millions if the volatility of the pension assets is 4 percent, and US$ 236.5 ± 2.9 millions if the volatility is 7 percent.

4. As argued in the preceding section, an important part of the now non-active affiliated workers will also be likely to be eligible for the minimum pension benefit, and we can estimate this proportion from the historical frequency of contributions reported in Table III. Accordingly, we need to extrapolate our estimations for the population of affiliated workers, which we do by multiplying the previous estimates of minimum pension insurance for female and male workers, times a factor, defined as the quotient of affiliated workers eligible for the minimum pension benefit[10] to the total number of active affiliates. In the case of female workers, this factor is 2.38, and in the case of male workers it is 1.81, reflecting the fact that married female workers generally stay longer periods out of the working force to devote more time to their children. With this adjustment, we obtain a minimum pension insurance for affiliated female workers of US$ 642.6 ± 4.7 millions and US$ 747.5 ± 7.4 millions (with volatilities of pension assets of 4 percent and 7 percent, respectively), and a minimum pension insurance for affiliated male workers of US$ 307.4 ± 2.5 millions and US$ 428.0 ± 5.3 millions (also with volatilities of 4 percent and 7 percent, respectively).

5. We obtain the total cost to the government by adding the cost of the minimum pension benefit to female and male affiliated workers, which is US$ 950.0 ± 7.1 millions and US$ 1,175.5 ± 12.8 millions, with volatilities of pension funds of 4 percent and 7 percent respectively (all with 95 percent probability). In 1992 the Chilean GDP was of US$ 31.195 billion and hence the present value of the minimum pension benefit for Chilean workers that are currently affiliated represents 3.05 percent and 3.77 percent of GDP (with volatilities of 4 percent and 7 percent, respectively).

Wagner (1991) is the only previous study that estimated the cost to the government of the minimum pension insurance. He presents several scenarios, though his basic one corresponds more closely with our assumptions (individuals start working at twenty years of age, pension funds grow at an expected rate of 5 percent, and the discount rate is 4 percent). In contrast with our approach, he is concerned with a long term steady state of the economy, and he assumes a density of security tax payments of 61% for males that retire at 65 years of age and 56% for females that retire at 60 years of age, a minimum pension of US$ 66, and life expectations of 14 years for male workers and 20 years for female workers after retirement. With these assumptions, he obtains that the yearly cost to the government of the minimum pension benefit is of US$ 7.33 millions yearly, which corresponds to a present value of minimum pension liabilities to the government of US$ 183 million in all (discounted at his assumed discount rate of 4 percent). This figure is notoriously lower than our estimate of a present value of the pension liabilities of current affiliates of US$ 950 million, so even if the two studies are not directly comparable (his corresponding to a steady state of the Chilean economy, and ours to the current affiliates), we suggest three reasons that explain the difference:

i) Wagner uses a minimum pension of US$ 66, which corresponds to the average of the minimum pensions for the period 1987-1990, but the average over the next three years increased to US$ 81. In our study we obtained quotations from insurance companies that were based on the current minimum pension, which is of US$ 82 (24 percent higher), and for this reason the minimum amount required to finance a minimum pension should be higher, and hence the present value liability to the government is obviously higher.

ii) Wagner considers the life expectation upon retirement, and uses this average in order to compute the capital needed to finance the minimum pension. In this way, he excludes the longevity risk, that is, the risk that a retired worker may live more than the average. In contrast, in our study we use the quotations for a life annuity from an insurance company, which duly considers this risk. The cost of the insurance necessarily increase the funds needed to finance the minimum pension, and hence the present value liability to the government should be higher if we include the longevity risk.

iii) And last, but not least, Wagner models the minimum pension insurance in a deterministic setup, and hence does not consider the risk associated to pension funds. His model is equivalent to a financial put option on an underlying asset with zero volatility, but the volatility of pension assets is not zero, and hence he must underestimate the true cost to the government.[11]

IV. Conclusions

In this paper we have proposed a methodology for estimating the cost to the government of the minimum pension benefit, which is based on a contingent claims analysis. We have shown that the minimum pension insurance is analogous to a financial put option, and we have used a risk neutral argument to price it by resorting to Monte Carlo techniques.

Following this methodology, we estimated the cost of the minimum pension benefit in 3.01 percent of GDP, although if the volatility is higher than estimated, the cost may increase to 3.77 percent of GDP. These estimates are much higher than previous results based on a deterministic setup, and strongly suggest the importance of explicitly modeling the risk associated to pension assets when estimating the cost to the government of the insurance implied by the minimum pension benefit. However, the specific estimates that we have obtained should be regarded more as an illustration of the method rather than precise estimations of the cost of the minimum pension insurance, for at least four reasons: (i) the new pension system has been in operation for only thirteen years, which is a far too brief experience to be representative of the stationary state of the Chilean pension system, (ii) for the time being, the composition of the workers affiliated to the new system is asymmetric, because the fraction of male workers between sixty and sixty four years old and female workers between fifty five and sixty years old is significantly lower, (iii) we have assumed that, depending on their gender, all individuals work the same constant expected working life, regardless of their income level. However, since the minimum pension will benefit low income workers, and considering that with high probability this group is affected by more frequent and longer unemployment periods and higher rotation in their occupations, it is very likely that their frequency of contributions is lower than the average and hence the value of the minimum pension put is underestimated and (iv) finally, in order to simplify the problem, we have reduced the working periods of individuals by their expected lags in contributions by 1.5 and 2.5 years for women and men, respectively. In actual fact, they are lags in contributions, though not in terms of growth of the accumulated funds, thereby overestimating the value of the pension put, since the pension assets had less time to grow in our model than in reality. Obviously, a more bona fide description of reality would include the possibility of lags in the contributions of the workers due to unemployment, job rotation, changes in salaries, etc, during the worker's life cycle. From a modeling point of view, such features could be added by regarding the contribution rate as a stochastic process in itself, which should also be estimated from appropriate data.

Appendix

THE VALUATION EQUATION

The valuation equation (9) can also be derived in an equilibrium context. From (3) and (4) the systematic risk of the minimum pension insurance and the pension funds is related by:

$$\beta_P = \frac{A}{P} \frac{\partial P}{\partial A} \beta_A \tag{A1}$$

According to the intertemporal capital asset pricing model of Merton (1977), the expected return of any asset must lie on the Security Market line:

$$\mu_i - r = \beta_i \; (\mu_M - r) \tag{A2}$$

where:

μ_i = Expected rate of return of asset i,
β_i = Systematic risk of asset i,
μ_M = Expected return of the market, and
r = Instantaneous risk-free interest rate.

Substituting (A1) in (A2):

$$\mu_P - r = \frac{A}{P} \frac{\partial P}{\partial A} \; [\mu_A - r] \tag{A3}$$

Finally, substitute the expected return on the minimum pension insurance $\mu_P = \alpha_P$, defined in (8), and the expected return on the pension assets $\mu_A = \alpha + (\eta/A)$ in (A3) to obtain the arbitrage pricing equation (13).

Notes

1 For simplicity, this assumption is presented in a stronger form than what is really needed. We will incorporate the expected periods of unemployment in our estimations, but we will neglect the uncertainty associated with it. We argue that such uncertainty is quantitatively smaller than the uncertainty related with the return on the assets of the pension funds, and we focus in estimating the impact of the later variable.

2 The worker can contribute more, if he the so wishes. Here we assume that voluntary contributions are negligible.

3 There is a second alternative which involves the worker's scheduled withdrawal of his pension funds. In order to simplify the estimations, this alternative is not considered.

4 The length of the interval can also be reduced by using variance reduction techiques, as the control variate technique suggested by Boyle (1977), which focuses in reducing the size of the standard deviation, increasing the precision of the stimates. Moreover, the Euler method can be classified as a strong approximation scheme, since it attempts to simulate both the distribution of pension assets at the end of the period and the sample path of the variable. In contrast, a weak approximation scheme aims only at obtaining the distribution at the end of the period, but does not attempet to replicate the path of the random variable. There are potentially important economies of estimation time in using a weak approximation scheme, as in the method suggested by Milstein, based on the uniform distribution for each step (see Platen, 1992). Since we are dealing with European type options we are only interested in the distribution of pension assets at retirement age, and Milstein's method seems attractive. We leave these improvements for further research.

5 This assumption implies that female workers retire 1.5 years and male workers 2.5 years before the age prescribed by law, which, of course, is not possible in the cases we are considering (in order to retire earlier workers must have accumulated pension funds sufficient to finance 1.1 times the minimum pension).

6 There is an issue as to whether time should be measured in calendar days (the choice that we made here) or in trading days. In any event, the volatilities estimated here are quite low, and we suspect that by considering a longer period we may well obtain higher numbers.

7 There are at least three reasons why the volatility of the pension funds should be significantly lower than that of the stocks: (i) by law, only a maximum of 30% of the portfolio can be invested in stock and hence the pension funds consist mainly of fixed-income securities, (ii) the value of the pension funds is computed using the average of the last ten days of the prices of the stock in the portfolio which reduces the volatility by a factor of 1 over square root of 10, and (iii) pension funds are allowed only to invest in the less risky stock in the economy, even though this last regulation will tend to be softened with the last bill of law on the reform of the capital market approved by Congress. However, a volatility of 3.8% is still low, and perhaps is not representative of the volatility prevailing in the long term.

[8] A more realistic (but also more complex) way to model the workers' security tax payments is assume that they follow a diffusion process. This allows for unemployment and changes in salaries between the working life of 40 years for female workers and 45 years for male workers.

[9] Note that the standard deviation of the average is the number reported in brackets divided by the square root of five thousand.

[10] That is to say, those whose estimated years of contributions in Table III exceed twenty years.

[11] The value of any option unambiguously increases with the volatility of the underlying asset, since the density of the option is the truncated density of the underlying asset (that is, truncated at the level of the exercise price of the option). For example, in the case of a put option like the one we are considering, a higher volatility of pension assets implies that with greater probability we will observe much higher and also much lower accumulated pension funds than the funds required to finance the minimum pension, M. However, while much higher accumulated pension funds will have no effect in the value of the minimum pension but (since the government would not have to finance any difference anyway), if the accumulated pension funds are much lower than the exercise price M the value of the pension put will grow, reflecting the greater amounts that the government will have to cover in this case.

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