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A SIMPLE RBC MODEL WITH MONEY AND INCOME TAXES

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Abstract:

The empirical Real Business Cycle models started with the pioneer work of Kydland and Prescott (1982). Since then, many work has been done using their approach. This paper is based in this standard real business cycle model to simulate an economy which include stochastic monetary growth (monetary policy) and income taxes (fiscal policy). The results of the simulations are compared with the actual data using the classical metric of standard errors and correlations and, also, with a VAR estimation. The impulse responses of the model with both policies are compared with the corresponding for the data.

Keywords: *Real Business Cycle, VAR, Impulse Response.*

1. Introduction

In current real business cycle research generally the aggregate demand or government policy variables are based on monetary or fiscal policy alone. Examples of those who consider only a fiscal sector are Aiyagari (1991), Aiyagari *et al.* (1990), Braun (1992), Christiano and Eichenbaum (1992a), Greenwood *et al.* (1993), Judd (1989), McGrattan (1989), (1991), (1992), (1993), and McGrattan *et al.* (1993) among others. On the other side we have those who consider only a monetary policy rule. Among them are Bansal and Coleman (1993), Christiano and Eichenbaum (1992b), Coleman (1993), Coleman *et al.* (1993), Cooley and Hansen (1989), Kydland (1989), Lucas (1987), and McGrattan (1991).

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All these models have as mentor the work of Kydland and Prescott (1982). They calibrate a model that captures some of the facts of the U.S. economy, and, as they point out, this "real-side" model is able to reproduce the relative size of the fluctuations in output, consumption and investment. Stochastic shifts in the production function was the motor in their economy. However, nominal variables play no role in the Kydland-Prescott model. This fact creates an incentive to modify this kind of "real" model to increase its performance with respect to the data, especially in order to account for interaction between real and nominal variables.

Consider, first, Christiano and Eichenbaum (1992a). They analyze a RBC model with and without government inside. They considered two specifications to introduce leisure in the utility function. One considers that individual utility is linear in leisure (indivisible-labor model), while the second is that leisure is a nonlinear argument in the utility function (divisible-labor model). Considering an AR(1) time series representation for both the logarithm of the productivity shock and the logarithm of government consumption, they concluded that, "when aggregate demand shocks arising from stochastic movements in government consumption are incorporated,...the model's empirical performance is substantially improved" (page 447).

In the same line, McGrattan ((1989), (1991), (1992)) incorporates taxes on factors of production, and government spending as a proportion of the total output. Using a linear-leisure utility function (indivisible-labor model), and an AR(1) law of motion for the technology shock, McGrattan concludes that there is a great improvement when taxes are included in the model[1].

Finally, and with a very similar structure, Aiyagari *et al.* (1990), addressed a different but related topic. Their paper investigates the effect on some aggregate indicators of changes in government spending, focusing particularly on transitory and permanent effects. They found that both temporary and persistent increases in government consumption increase the interest rate, and have important effects on employment and output[2].

All these articles suggest that incorporating a fiscal sector increases the performance of the standard growth Kydland and Prescott's RBC model, but all of these analyses are made without considering a monetary sector (nominal variables).

Almost simultaneously, and using a dynamic programming approach in the line of Lucas (1990), Coleman *et al.* (1993) builds and simulates a monetary model in which the monetary policy is identified as a stochastic supply schedule for reserves, and where the banks have a demand for reserves as a solution of the optimization process for the dynamic model. Closing the model with this monetary policy equation, they are able to reproduce most of the features of the U.S. data[3].

The point is not to include all the variables of the real world in the analysis, but at least to consider the interaction between fiscal and monetary policy as basic components of the model, as is suggested by Leeper (1993). One step ahead is to allow a maximizing behavior for these "new" agents, in the line of Aiyagari (1991).

In a recent publication Leeper (1993)[4] points out that given that there is a tendency among economic researchers to analyze the behavior of one policy authority at a time, the conclusions of these studies have a hidden assumption about the other policy not considered, an assumption that can yield misleading beliefs about policy effects. This problem arises from the lack of specification of interaction and connection with the other policy authority. In his paper, he considers a model with monetary and fiscal policy jointly, and shows that depending on the monetary and fiscal policy

combinations, the economic implications are different, which implies that a model with only one policy authority will produce different policy recommendations depending on the status of the unspecified policy. For instance, let us assume that the monetary and fiscal policy behavior are represented by $R = R(\pi, y, \theta)$ and $T = \tau(b_{-1}, y, \psi)$, respectively. If the nominal interest rate (R) responds strongly to inflation (π), and direct taxes (T) respond strongly to real debt (b), then real debt does not grow too fast, and the price level is determinated. But if instead we have that direct taxes respond weakly to real debt, then the real debt explodes so that the government becomes insolvent, and no equilibrium exists in which government debt has value.

What it is clear from these lines is that the traditional approach is to consider the monetary and fiscal policy separately. From previous research we know that the performance of ours models will improve if we consider either a fiscal sector or a monetary sector as a component of the models. Then it seems a natural extension to consider monetary and fiscal policy jointly.

Based on the ideas of Chang (1991), Leeper (1991), Leeper (1993), Leeper and Sims (1993), Sims (1994), and Woodford (1994), the basic framework of this study is a standard real business cycle model that includes a public sector which finance his budget with stochastic income and inflationary taxes. Government spending is a stochastic proportion of the output (with mean around 20% based on the actual data). Households and firms behave in a competitive way.

The structure of this paper is as follows. Section 2 describes the model of the economy and solves for an equilibrium. The next section presents the simulation results for three parameterizations of the model. Section 4 compares the welfare costs associated with different levels of inflation taxes for two levels of income taxes. The paper ends with conclusions.

2. Model

The economy to be studied is a version of the indivisible labor model of Hansen (1985) and the cash-in-advance model with cash and credit goods of Stokey and Lucas (1987). The economy has three agents: households that work, consume and invest; firms with constant returns to scale technology; and, the government which finances its spending with inflation and income taxes. As we know, money is valued because it is required to purchase consumption goods.

I assume a continuum of identical infinitely lived households with preferences given by the utility function,

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_{1t}, c_{2t}, l_t), \qquad 0 < \beta < 1$$

where c_{1t} is consumption in the cash good, c_{2t} consumption in the credit good[5], and l_t is leisure in time t. Each household is endowed with one unit of time each period ($0 \leq l \leq 1$), part of which is supplied to a firm that produces the output.

I assume a constant marginal disutility of labor and a generalization of the logarithmic specification used by Cooley and Hansen (1992), that is separability in c_1 and c_2, represented by a utility function of the form:

$$U(c_1, c_2, h) = u(c_1, c_2) - \gamma h$$

$$u(c_1, c_2) = \frac{v}{v-1} \cdot \left(\alpha c_1^{\left(\frac{v-1}{v}\right)} + (1-\alpha) c_2^{\left(\frac{v-1}{v}\right)} \right) \tag{1}$$

where v is either the elasticity of substitution between cash and credit goods at any two points in time[6], or the inverse of the coefficient of relative risk aversion. The variable h is hours of work ($h = 1 - l$). In Cooley and Hansen this parameter was equal to one[7].

At the beginning of the period, households has currency holdings that come from two sources: m_t is currency carried over from the previous period, and a lump-sum transfer given by the government. Thus, purchases of cash goods c_1 must satisfy the cash-in-advance (CIA) inequality constraint,

$$p_t c_{1t} \leq m_t = (M_t - M_{t-1}) \tag{2}$$

where p_t is the price level and M_{t-1} denotes beginning of period (pre-transfer) per-capita money balances.

The resource or flow constraint is given by:

$$c_{1t} + c_{2t} + i_t + \frac{m_{t+1}}{p_t} \leq (1-\tau_h) w_t h_t + (1-\tau_k) r_t k_t + \tau_k \delta k_t + \frac{m_t}{p_t} + \frac{TR_t}{p_t} \tag{3}$$

Hence, the household expenditures include purchases of the cash good (c_1), the credit goods (c_2) and investment (i), and money to be carried into the next period (m_{t+1}). The sources of income include after-tax labor income, after-tax capital rental income, refunds from depreciation, currency carried from the previous period (m_t) and the lump-sum transfer (TR_t).

The law of motion for the household's stock of capital evolves according to:

$$k_{t+1} = (1-\delta) k_t + i_t, \qquad 0 < \delta < 1 \tag{4}$$

The second agent in the economy (firm) produces output y_t according to a constant returns to scale Cobb-Douglas technology[8] where the productivity shock evolves as an AR(1) with root outside the unit circle:

$$\begin{aligned} y_t &= e^{z_t} F(K_t, H_t) = e^{z_t} K^{\theta}_t H_t^{1-\theta}, \qquad 0 < \theta < 1 \\ z_{t+1} &= \rho_1 z_t + \varepsilon_{t+1}, \qquad E[\varepsilon] = 0, \qquad V[\varepsilon] = \sigma^2_{\varepsilon} < \infty \end{aligned} \tag{5}$$

The firm seeks to maximize benefits, so from the first order conditions for the firm's profit maximization problem we obtain that:

$$\begin{aligned} w_t &= e^{z_t} F_2(K_t, H_t) = e^{z_t} (1-\theta) \frac{K^{\theta}_t}{H^{\theta}_t} \\ r_t &= e^{z_t} F_1(K_t, H_t) = e^{z_t} \theta \frac{K_t^{(1-\theta)}}{H_t^{(1-\theta)}} \end{aligned} \tag{6}$$

The role of the government in this economy is to collect taxes and to issue money to finance a sequence of stochastic government expenditures, and to return the unspent income to the households via a lump-sum transfer. Hence the government's budget constraint and law of motion are given by:

$$\begin{aligned} G_t + \frac{TR_t}{p_t} &= \tau_{h,t} w_t H_t + \tau_{k,t} (r_t - \delta) K_t + \frac{M_t - M_{t-1}}{p_t} \\ G_t &= \zeta_t y_t \\ \zeta_{t+1} &= \zeta_0 + \rho_3 \zeta_t + \eta_{\zeta, t+1} \\ \tau_{h, t+1} &= \tau_{h,0} + \rho_4 \tau_{h,t} + \eta_{h, t+1} \\ \tau_{k, t+1} &= \tau_{k,0} + \rho_5 \tau_{h,t} + \eta_{k, t+1} \end{aligned} \tag{7}$$

where the stacked vector $\eta = [\eta_\zeta \eta_h \eta_k]'$ has $E[\eta] = 0, \wedge V[\eta] = \sigma^2_\eta < \infty$.

The monetary policy consists in issuing money to finance part of the stochastic government expenditures. This can be represented by the rule:

$$\begin{aligned} M_t &= g_t M_{t-1} \\ \log(g_{t+1}) &= (1-\rho_2) \log(\bar{g}) + \rho_2 \log(g_t) + \bar{\rho}_2 z_t + \eta_{g,t+1} \end{aligned} \tag{8}$$

where $E[\eta] = 0$, $V[\eta] = \sigma^2_{g,\eta} < \infty$, $\bar{g}$ is the mean of the monetary growth rate, $\bar{\rho}_2$ is the sensitivity of the stabilization power of the monetary policy to the activity level (assumed to be negative and obtained by calibration).

The usual change of variables is introduced to induce stationarity for the variables in the model. Let $\hat{m}_t = m_t/M_{t-1}$, $\hat{p}_t = p_t/M_t$, and denote variables in t+1 with a '. In that case, given $k_0 = K_0$, $\hat{m}_0 = 1$ and a sequence of $\{G, \tau_h, \tau_k, g\}_{t=0}^{\infty}$ that satisfy the government budget constraint (7), a competitive equilibrium is a set of sequences for the price level $\{\hat{p}_t\}_{t=0}^{\infty}$, factor prices $\{w_t, r_t\}_{t=0}^{\infty}$, household allocations $\{c_{1t}, c_{2t}, h_t, i_t, \hat{m}_{t+1}, k_{t+1}\}_{t=0}^{\infty}$ and per capita quantities $\{H_t, I_t, K_t\}_{t=0}^{\infty}$ such that the following conditions hold:

(i) Household and firm solve their maximization problem.

(ii) $\hat{m}_t = 1$, $h_t = H_t$, $i_t = I_t$, $k_{t+1} = K_{t+1}$, for all t.

(iii) Market clearing condition: $c_{1t} + c_{2t} + i_t + G_t = y_t$.

Hence, the dynamic programming problem solved by the household can be written as in the system (9), where I, H, P are functions that expresses the relationship between investment, labor, and price level with the state of the economy (z, g, K, λ, τ_h, τ_k).

The method to find these three functions is developed in the paper by Cooley and Hansen (1989) for a monetary economy and in Cooley and Hansen (1992) for an economy with distortions. It assumes that the cash in advance constraint is always binding[9]. Because it is not possible to find equilibrium allocations by solving a planning problem, it is necessary to solve for equilibrium allocations directly by solving a fixed point problem[10]. Using this method we wish to find a recursive competitive equilibrium, which consists of decision rules for the households; a decision rule determining the amount of money the household carries into the next period, $\hat{m}' = m(z, \zeta, \tau_h, \tau_k, g, K, k, \hat{m})$; a set of aggregate decision rules I, H; a function determining the aggregate price level P; and a value function $v(z, \zeta, \tau_h, \tau_k, g, K, k)$, such that: (i) Given the aggregate decision rules and the price level function, the value function satisfies equation (9) and $h(z, \zeta, \tau_h, \tau_k, g, K, k)$, $i(z, \zeta, \tau_h, \tau_k, g, K, k)$ and $\hat{m}'$ are the associated decision rules. (ii) Given the pricing function, individual decisions are consistent with aggregate outcomes: $H = h$, $I = i$, $1 = \hat{m}(z, \zeta, \tau_h, \tau_k, g, K, K, 1)$.

$$v(z, \zeta, \tau_h, \tau_k, g, K, k, \hat{m}) = \max \{u(c_1, c_2, 1-h) + \beta E[v(z', \zeta', \tau_h', \tau_k', g', K', k', \hat{m}') \mid z, \zeta, \tau_h, \tau_k, g, K, k, \hat{m}]\}.$$

s.t. $z' = \rho_1 z + \varepsilon'$

$$\log(g') = (1-\rho_2)\log(\bar{g}) + \rho_2 \log(g) + \bar{\rho}_2 z + \eta'_g$$

$$\zeta' = \zeta_0 + \rho_3 \zeta + \eta'_\zeta$$

$$\tau'_h = \tau_{h,0} + \rho_4 \tau_h + \eta'_h$$

$$\tau'_k = \tau_{k,0} + \rho_5 \tau_k + \eta'_k$$

$$K' = (1-\delta) K + I$$

$$k' = (1-\delta) k + i$$

$$c_1 + c_2 + i + \frac{\hat{m}'}{\hat{p}} = (1-\tau_k) wh + (1-\tau_k) rk + \tau_k \delta k_t + \frac{\hat{m}}{g\hat{p}} + $$

$$+ \tau_k wH + \tau_k (r-\delta) K - \zeta y + \frac{g-1}{g\hat{p}}$$

$$c_1 \leq \frac{\hat{m} + g - 1}{g\hat{p}}$$

$$I = I(z, \zeta, \tau_h, \tau_k, g, K)$$

$$H = H(z, \zeta, \tau_h, \tau_k, g, K)$$

$$\hat{p} = P(z, \zeta, \tau_h, \tau_k, g, K) \qquad (9)$$

The Euler equations to the dynamic programming problem are represented by the following system of seven equations, in addition to the restrictions.

$$c_1 : \quad U'_{c_1} - \lambda\hat{p} - \varphi\hat{p} = 0$$

$$c_2 : \quad U'_{c_2} - \lambda\hat{p} = 0$$

$$h : \quad U'_h + \lambda\hat{p}(1-\tau_h) w = 0$$

$$i : \quad \beta E \left[\frac{\partial v'}{\partial k'} \cdot \frac{\partial k'}{\partial i} \right] - \lambda\hat{p} = 0$$

$$\hat{m}' : \quad \beta E \left[\frac{\partial v'}{\partial \hat{m}'} \right] - \lambda = 0$$

$$\lambda : \quad \hat{m}' = \hat{p}\{(1-\tau_h) wH + (1-\tau_h) rk + \tau_k \delta k + \hat{m}'/g\hat{p} + \tau_h wH + \tau_k (r-\delta) K + (g-1)/g\hat{p} - \zeta y - c_1 - c_2 - i\}$$

$$\varphi : \quad c_1 \leq \frac{\hat{m} + g - 1}{g\hat{p}} = \frac{1}{\hat{p}}, \qquad = \text{if } \varphi > 0$$

In equilibria $\hat{m} = \hat{m}' = 1$, $k = K$, $I = i$, $H = h$. The functions γ, φ are the multipliers for the budget constraint and the cash in advance constraint, respectively. Additionally, the envelope conditions are:

$$\frac{\partial v}{\partial k} = \lambda\hat{p}[(1-\tau_k) r + \tau_k \delta + (1-\delta)]$$

$$\frac{\partial v}{\partial \hat{m}} = \frac{\lambda}{g} + \frac{\varphi}{g}$$

Using the two given marginal conditions for the firm, the law of motion for the stock of capital, the laws of motion for income taxes and proportion of government spending, the production function, and the first order conditions for the household altogether with the envelope conditions, we can solve for all the variables in the economy $\{c_1, c_2, h, i, k, y, G, \hat{p}, r, w, z, \zeta, \tau_h, \tau_k, \lambda, \varphi\}$. For this we employ the usual procedures to reduce the problem to the following system:

$$\lambda = \beta E \left[\frac{\text{Max}\{\lambda', c'_1 \cdot U'_{c'_1}\}}{g'} \right]$$

$$\hat{p} = \text{Min} \left\{ \frac{U'_{c_1}}{\lambda}, \frac{1}{c_1} \right\}$$

$$\varphi = \text{Max}\{0, c_1 \cdot U'_{c_1} - \lambda)$$

Under perfect foresight and steady state, and with the particular preferences and technology given above, it is very easy to solve this system. Once we solved for λ, we can see that the solution for the price level implies that the cash in advance constraint is always binding. To satisfy monotonicity and discounting properties in the fixed point equation, we require the usual restriction over the expected monetary growth, i.e., that $\beta \cdot E[1/g'] \leq 1$. This restriction guarantees a non-negative nominal interest rate[11].

3. Simulations

For the model described above, hours of work, investment, and the price equation is a linear function of the states. Hence, the equilibrium expressions for H, I, and $\hat{p}$ (the inverse of consumption in cash good) are:

$$H = \phi_0 + \phi_1 z + \phi_2 \log(g) + \phi_3 K + \phi_4 \tau_h + \phi_5 \tau_k$$

$$I = \phi_0 + \phi_1 z + \phi_2 \log(g) + \phi_3 K + \phi_4 \tau_h + \phi_5 \tau_k$$

$$\hat{p} = \psi_0 + \psi_1 z + \psi_2 \log(g) + \psi_3 K + \psi_4 \tau_h + \psi_5 \tau_k$$

For the purpose of the simulations the length of the period is one quarter, and in order to make comparisons with previous studies, the parameters values used were:

$$\beta = 0.99,\ \theta = 0.36,\ \delta = 0.025,\ \gamma = 2.86,\ \rho_1 = 0.95.$$

From the monetary policy rule, the values ρ_2, σ_η^2 were estimated for the period 1972:3 to 1993:1 by doing a bootstrapping sample of the parameters obtained in the following AR(1) equation for the money growth:

$$\Delta \log (m_{t+1}) = \xi_0 + \xi_1 \Delta \log (m_t) + \eta_t$$

The bootstrapping results for $\hat{\rho}_2$ and for the standard error of the estimate are given in the following table:

BOOTSTRAPPING (20000 RESAMPLES WITH REPLACEMENT)[12]

Statistic	$\hat{\rho}_2$		$\hat{\sigma}_\eta$	
Sample Mean	0.466531		0.009395	
Standard Error	0.119977		0.000839	
t-Statistic ($\hat{\rho}_2 = 0$)	549.92	(0.00)	1,583.6	(0.00)
Skewness	–0.1144	(0.00)	0.04563	(0.01)
Kurtosis	0.02807	(0.42)	–0.1044	(0.00)

The estimated probability density functions for the parameter ξ_1 and the standard error of the estimate, are plotted in Figure 1a. The mean values marked with a line in Figure 1a, were used as $\hat{\rho}_2$ and $\hat{\sigma}_\eta$ for the simulation. The same procedure was followed for α, ν, β, with plots in Figure 1b.

FIGURE 1a

BOOTSTRAPPING RESULTS FOR THE LAW OF MOTION OF THE MONEY GROWTH

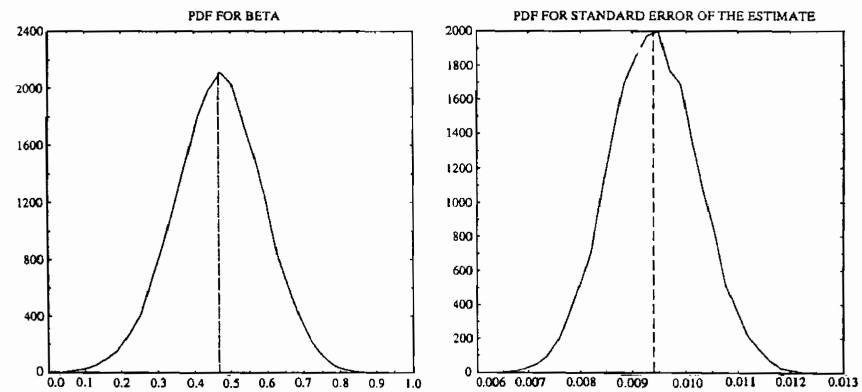
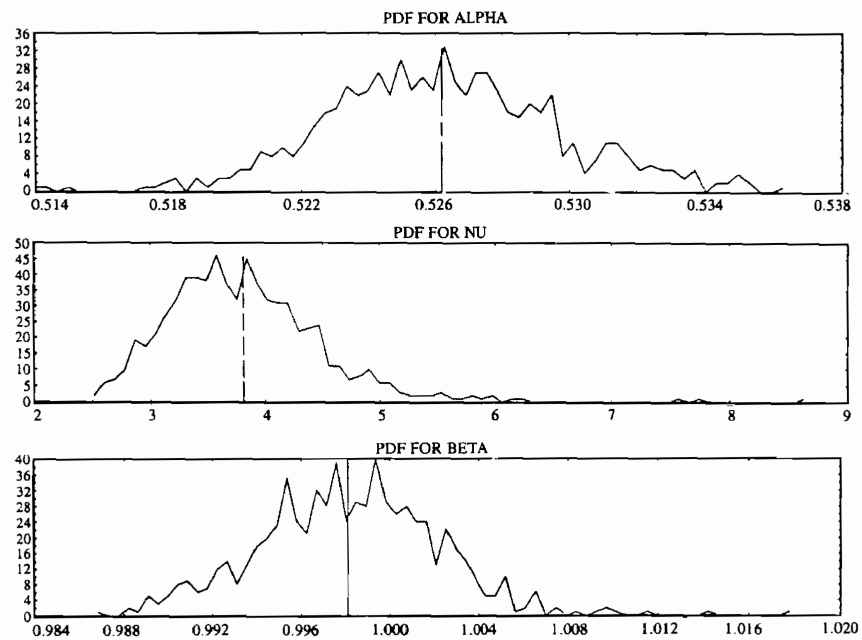


FIGURE 1b

BOOTSTRAPPING FOR α, ν, β



This estimates are compared with the values obtained by Cooley and Hansen (1989) 0.48 and 0.009 respectively, fitting an OLS using 1955:3-1984:1 as sample data for M1. If we use OLS with our sample the estimates are 0.59 and 0.008, respectively. Both parameters are crucial in the volatility of inflation. Given this sensitivity, in order to get accurate estimates of these we used bootstrapping methods[13].

The parameters ß, α, v from the utility function were estimated using the Hansen and Singleton's (1982) GMM technique. The parameter α describes the utility weight of the cash goods in the utility function, while v represents the inverse of the constant relative risk aversion coefficient. ß is the usual discount factor. The intratemporal euler equation is obtained from the first order conditions for c_1 and c_2. The ratio of the marginal conditions produces the following relations:

$$\frac{c_{2t}}{c_{1t}} = [(1-\alpha)/\alpha]^{v} \bullet R^{v}_{t}$$

$$r_{t+1} = \frac{1}{\beta} \bullet \left(\frac{c_{1t+1}}{c_{1t}}\right)^{(1/v)}$$

With a binding cash in advance constraint, c_1 is measured by real balances, c_2 is total consumption minus real money balances, while R_t is the return on three-month Treasury bills and r_t represents the real interest rate. In our model, consumption is consumption of non-durables and services, while real money balances are measured by M1 at constant prices. The instruments used were a constant, one period lagged real consumption growth, current and lagged gross nominal interest rates, lagged real interest rate and consumption velocity of money[14]. The parameters were estimated reasonably well at 0.9981, 0.5262 and 3.8122 with 0.004, 0.003 and 0.68 standard errors, respectively. These values are compared with those obtained for Chari *et al.* (1991). Using quarterly data for the period 1959-1989 and measuring real money balances by monetary base, and consumption by consumption expenditures (including durables), they estimate the previous equation by OLS getting an α equal to 0.43 and v equal to 5.88. The risk aversion parameter in our estimation is around 0.26 while the value obtained by Chari *et al.* was 0.17 (substantially low).

The standard deviations for the error terms are changed for each of the simulated economies in order to match the standard error of the output in the artificial series with the value in the actual data. Each simulation is the same period long as the data sample (85), and it was considered a burning period of 6% of the sample was used, and all the reported statistics are sample means of statistics computed for each of 50 simulations. Each simulated time-series and U.S. data sample was logged and detrended using the Hodrick-Prescott filter[15], following many studies in the real business cycle research area.

Using data from McGrattan (1992), we interpolated the series on capital and labor income taxes to obtain quarterly observations. The data comes from 1947 to 1987. Using this interpolated data, the law of motion for the taxes was estimated as:

$$t'_h = \underset{(0.706374)}{0.001224} + \underset{(81.128820)}{0.996158\tau_h}, \quad see = 0.002383, \quad \bar{\tau}_h = 0.2281$$

$$t'_k = \underset{(0.792283)}{0.004393} + \underset{(147.85904)}{0.990625\tau_k}, \quad see = 0.005982, \quad \bar{\tau}_k = 0.5075$$

where the values in parentheses are the t-statistics under the null that the parameter is zero. The following figure presents the taxes data.

Table 1 shows some of the statistics for the U.S. sample data, and Figure 3 presents some of the series.

Some of the U.S. statistics reported in Table 1 differ from those reported in Kydland and Prescott (1982), Cooley and Hansen (1989) and McGrattan (1993) because of choices of data. As we can see, the correlation between output and price level is negative (–0.55) while the autocorrelations of almost all the series considered are still high until three quarters. Consider the correlation matrix of the innovations in a subset of five variables (output, inflation, interest rate, money, and velocity). The correlation between money and inflation or money and interest rates is close to zero (0.02 and 0.04 respectively), while the correlation between inflation and either velocity or interest rates is somewhat higher (0.34 and 0.32 respectively). The correlation between money and velocity is negative (–0.84) while the correlation between output and interest rates is positive (0.43).

FIGURE 2

MARGINAL INCOME TAXES 1947:1-1987:4

Effective Marginal Capital Tax Rates: 1947:1-1987:4

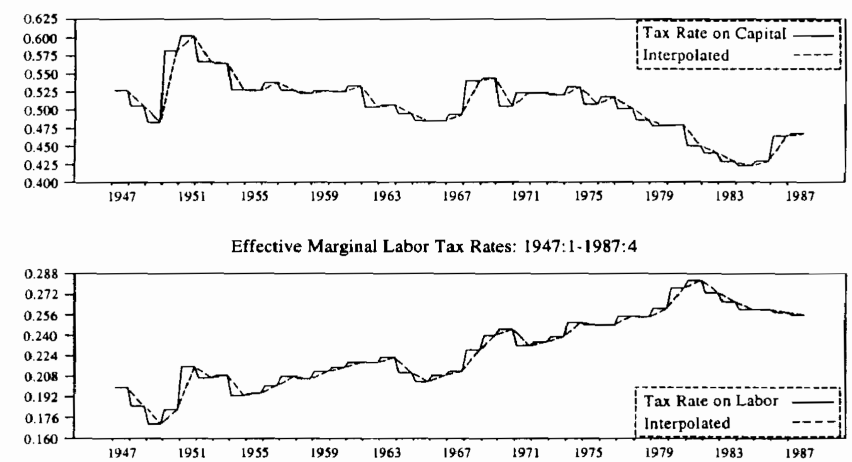


TABLE 1

U.S. SAMPLE STATISTICS; 1972: 1 TO 1993:1

Series	( )	Standard Deviation	1	2	3	4	5	6
GNP	y	1.80	0.86	0.65	0.42	0.21	0.05	–0.10
Inflation	p	0.58	0.53	0.30	0.38	0.15	–0.03	–0.01
T-Bill Rate	r	0.36	0.79	0.53	0.42	0.28	0.12	–0.07
M1	m	2.29	0.89	0.70	0.51	0.30	0.12	–0.05
Velocity	v	2.97	0.91	0.75	0.57	0.35	0.14	–0.05
Consumption	c	0.91	0.87	0.70	053	0.32	0.14	0.01
Investment	i	6.15	0.90	0.72	0.50	0.27	0.07	–0.11
Hours	h	2.39	0.90	0.70	0.46	0.23	0.03	–0.15
Productivity	y/h	0.96	0.81	0.59	0.29	0.05	–0.11	–0.25
CPI	cpi	1.70	0.93	0.81	0.66	0.49	0.30	0.13

CORRELATION, STD. DEV. AND CORRELATION MATRIX OF RESIDUALS (ON AND BELOW THE DIAGONAL)

	y	p	r	m	v	c	i	h	y/h	cpi
y	0.0067	0.46	0.36	0.23	–0.23	0.86	0.94	0.93	0.45	–0.55
p	0.25	0.0036	0.62	0.13	0.04	0.25	0.38	0.59	0.61	0.12
r	0.43	0.32	0.0017	–0.20	0.42	0.08	0.24	0.53	0.65	0.43
m	0.19	0.01	0.04	0.0070	–0.92	0.33	0.29	0.13	–0.12	–0.44
v	0.06	0.34	0.20	–0.84	0.0077	–0.35	–0.32	–0.09	0.20	0.70
c						1.00	0.83	0.74	0.22	–0.70
i							1.00	0.90	0.48	–0.62
h								1.00	0.74	–0.39
y/h									1.00	0.06
cpi										1.00

These facts are not consistent through the sample. If we split the sample in 1982:4[16], these facts are no longer valid. Perhaps the most notorius change is in the behavior of velocity and money. Before the 80's, the velocity presented a marked trend, while since early last decade this variable begins to fluctuate with a decreasing trend. During the first sub-sample the correlation between velocity and either inflation or output was positive (0.17 and 0.20 respectively), while in the second sub-sample the same correlations were negatives (–0.12 and –0.76). Another important fact is change in the sign of the correlation between interest rate and money. In the first sub-sample this correlation is negative (–0.56) while in the second sub-sample is positive (0.57). The same happened with the correlation of the innovations. The correlation between inflation and money turns from negative (–0.29) to positive (0.38), while considering all the sample this statistic is around zero (0.01). Something similar happens with velocity and interest rate. Their correlation change from 0.30 to –0.20 (or from 0.62 to –0.12, if we consider the simple correlation sample instead of the innovations).

From studying Figure 4, we can see some of the usual properties in the real aggregates variables[17]. The low volatility of consumption, in comparison with the higher volatility of GNP, and the high negative correlation between velocity and money. In particular, note the high (low) volatility of velocity (GNP) and money since 1982. Before this date there was a high correlation between money and both inflation and GNP. All these procyclical movements disappear after 1982-1983.

The impulse response functions considering all the sample are plotted in Figure 5. The effect of the interest rate on output is negative indicating that if we shock the interest rate upward the activity level is going to decrease (with a lag of two to eight quarters), while the inflation is going to increase in the short run (two quarters). The effect of the monetary expansion on the interest rate doesn't show a liquidity effect[18].

The exercise consists in simulating four economies with different structures for the states variables, each under two specification of the objective function. The first set of four economies assumes a logarithmic utility function without credit goods ($\alpha = v = 1$) as in Cooley and Hansen (1989), while the second set assumes the existence of cash and credit goods and has non-logarithmic utility ($\alpha = 0.53$, $v = 3.21$, from GMM estimation). For these two specifications we simulate four kinds of economies. First we simulate an economy with constant money growth rate (where $\bar{g}$ goes from ß to 1.32), and no taxes, while the second economy assumes an autoregressive process for the money growth rate (AR(1))with mean $\bar{g} = 1.015$, which implies an average inflation of six percent (based on data from 1972: 1 to 1993: 1), and no taxes. The third economy considers an AR(1) process for the monetary growth rate and deterministic taxes, which are obtained taking the average of McGrattan's (1992) data. The last economy is similar to the third economy but now taxes follow an AR(1) process, and government spending is a stochastic proportion of the output, with an average of 20%, which is the actual average figure found in our sample.

Tables 2 and 3 reproduce the results for the experiments in terms of means and standard deviations of the standard errors and correlations of several simulations (for this purpose I considered 50 repetitions). From Table 2 we reproduce the economies of Cooley and Hansen (1989) (simulation 1 and 2). These results confirm that expected inflation has little or no effect on the business cycle. When we introduce stochastic taxes, most of the real variables like consumption, capital stock, hours, and productivity, present statistics similar to the actual data. However, as expected from the structure of the model, the performance of the nominal variables is not so good compared to the real variables. The performance of velocity, in particular, is poor because the model initially has no credit goods. The hours series presents an improvement with respect to the data but still only explains 70% of the standard deviation of the actual data. Productivity explains only 67% of the corresponding actual standard deviation, while for investments this figure exhibits 86%.

The correlation of consumption and output improves with respect to the data if we move from economies with deterministic taxes to the economy with stochastic ones. The same is true for the correlation of output with investment. A common feature of all the models is the poor performance of the correlation of the capital stock and output. All these figures are statistically zero, which goes against the actual observation of 0.65 for our sample. The volatility in taxes makes a good contribution with respect to the variable hours. With stochastic income taxes we capture the volatility in the elasticities of the labor supply, increasing the productivity and hours standard deviations. These results are evident when we try to match the correlations between output and hours and output and productivity. For the first, the actual data (0.93) is explained a hundred percent for the fourth economy, while for the correlation with productivity, the model explains almost 80% of the actual data.

Now consider the model with cash and credit goods, and with a more general functional form for the utility function (see Table 3). In this case the coefficient of relative risk aversion is 0.31 and the weight of the cash good in the utility function is 0.53. These parameter values are compared with the values used by Chari, Christiano and Kehoe (1991) (0.17 and 0.43, respectively). The statistics reported in Table 3 show that the volatility of the money growth has a significant impact on the cyclical properties of the simulated economy. Almost all the effect of this volatility goes to consumption and the price level (as in Cooley and Hansen (1989)), but in addition in our model, the volatility has a big impact on velocity. The correlation of output with consumption, price level, and inflation becomes smaller. However the correlation with velocity is still statistically zero. Once we introduce stochastic taxes, we can see an increase in volatility of consumption (from 0.67 to 0.92), almost no effect on investment and the capital stock, and big increases in the volatility of hours (from 1.79 to 2.38, where the actual figure is 2.39), productivity (from 0.19 to 0.76, where the actual figure is 0.96), the price level (from 2.07 to 2.86, where the actual figure is 1.70), inflation (from 1.47 to 2.04, where the actual figure is 0.58), and velocity (from 1.28 to 1.59, where the actual figure is 2.97).

The correlation structure shows a decrease in the correlation of output with investment (0.96 to 0.92, where the actual figure is 0.94), with hours (0.99 to 0.97, where the actual value is 0.93), with the price level (–0.16 to –0.52, with –0.55 as actual value), and with velocity (0.01 to –0.22, where the actual is –0.23). Hence, the correlations show a marked improvement in terms of consumption, price level, and volatility, while the correlation between output and either investment or the capital stock, doesn't change significantly among the experiments.

In summary, the percentage of explanation in volatility of consumption is almost 100%, investment 105%, capital stock 124%, hours 100%, productivity 80%, price level 168%, and velocity 54%, while in a model without these stochastic components the values were 65% for consumption, 109% for investment, 124% for capital stock, 75% for hours, 20% for productivity, 36% for price level, and 0% for velocity. Hence, the model shows a marked improvement in terms of consumption and hours (because of the stochastic taxes), and in the price level and velocity (because of the stochastic money supply). As we can see the performance in the model improves significantly when we incorporate the stochastic taxes together with volatility in money supply.

Now we study the comparison between a VAR estimation based on actual data and a VAR estimation based on simulated data. Consider the behavior of velocity. In the data the standard deviation and first order autocorrelation of velocity are 2.97 and 0.91 respectively, while the standard deviation of the forecast error from a second-order VAR is 0.0077. For the artificial economies without credit goods (see Tables 2 and 4), the standard deviations are zero. However, when we introduce credit goods into the model, the velocity presents cyclical properties. The standard deviation increases to 1.59 (explaining 54% of the actual volatility), and the standard deviation of the forecast error is around 0.01 (around 130% of the actual figure).

There is a marked persistence in the actual data, that is explained only partially by the model. Simulated output, interest rate and money show persistence, while for velocity and inflation the model does not reproduce persistence. There are a variety of methods to generate persistence in a RBC model. For instance, one can include laws of motion for the state variables with more lags (AR(2) or AR(3)), habit in consumption,

fatigue in leisure, adjustment costs, and more endogenous (with leads and lags) monetary policy functions. These extensions are left for a future research.

The impulse response functions (with standard errors) for the actual and simulated data are reported in Figures 5, 7 and 9 respectively. For the simulated data we consider the versions with and without credit goods, assuming an economy with an AR(1) stochastic monetary growth, taxes, and government spending. From Figures 5 and 9 we observe that the impulse response functions for output are very similar, not only in shape but also in magnitude, between the actual data and the simulated data. While an increase in the interest rate has a negative impact on output (after two quarters) in the actual data, the model presents a null response of output to the same variable. The response of inflation to money presents very similar shapes and magnitudes in both figures.

TABLE 2

STANDARD DEVIATIONS[19] AND CORRELATIONS FOR ARTIFICIAL ECONOMIES:
$v = \alpha = 1, \hat{\rho}_2 = 0$

Series	Data 71-93	Economy with Constant Money Growth Rate and no Taxes ($\bar{g}$ = ß –1.32)		Economy with Autoregressive Money Growth Rate and no Taxes ($\bar{g}$ = 1.015)		Economy with Autoregressive Money Growth Rate and Constant Taxes ($\bar{g}$ = 1.015)		Economy with Autoregressive Money Growth Rate[20] and Stochastic Taxes ($\bar{g}$ = 1.015)	
	Mean	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
y	1.80	1.80	0.28	1.80	0.28	1.80	0.28	1.80	0.32
c	0.91	0.51	0.10	0.64	0.09	0.71	0.10	0.95	0.11
in	6.15	5.93	1.09	6.05	1.07	5.87	1.08	5.34	1.19
k	0.45	0.50	0.12	0.50	0.12	0.49	0.12	0.47	0.14
h	2.39	1.39	0.22	1.39	0.22	1.26	0.20	1.68	0.31
y/h	0.96	0.50	0.09	0.51	0.09	0.59	0.10	0.64	0.09
cpi	1.70	0.50	0.09	1.95	0.36	1.98	0.37	2.79	0.54
p	0.58	0.31	0.03	1.30	0.10	1.33	0.10	1.85	0.16
v	2.97	0.00	0.00	0.01	0.00	0.01	0.00	0.01	0.00
r	0.36	0.00	0.00	0.96	0.08	0.96	0.08	1.19	0.11
Correlations									
y-c	0.86	0.86	0.05	0.67	0.10	0.77	0.08	0.82	0.05
y-i	0.94	0.99	0.00	0.97	0.01	0.97	0.01	0.94	0.03
y-k	0.65	0.06	0.06	0.05	0.07	0.03	0.07	–0.02	0.08
y-h	0.93	0.98	0.00	0.98	0.00	0.99	0.00	0.93	0.03
y-y/h	0.45	0.87	0.02	0.86	0.03	0.94	0.01	0.35	0.19
y-cpi	–0.55	–0.87	0.02	–0.22	0.20	–0.27	0.19	–0.67	0.13
y-p	0.46	–0.59	0.02	–0.14	0.12	–0.15	0.12	–0.35	0.08
y-v	–0.23	0.00	0.11	0.00	0.16	0.00	0.16	–0.35	0.12
v-p	0.04	0.00	0.05	0.92	0.01	0.91	0.01	0.92	0.01
r-p	0.42	0.00	0.05	0.92	0.01	0.91	0.01	0.92	0.01

TABLE 3

STANDARD DEVIATIONS[21] AND CORRELATIONS FOR ARTIFICIAL ECONOMIES: $v = 3.21, \alpha = 0.53, \hat{\rho}_2 = 0$

Series	Data 71-93	Economy with Constant Money Growth Rate and no Taxes ($\bar{g}$ = ß −1.32)		Economy with Autoregressive Money Growth Rate and no Taxes ($\bar{g}$ = 1.015)		Economy with Autoregressive Money Growth Rate and Constant Taxes ($\bar{g}$ = 1.015)		Economy with Autoregressive Money Growth Rate[22] and Stochastic Taxes ($\bar{g}$ = 1.015)	
	Mean	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
y	1.80	1.80	0.29	1.80	0.29	1.79	0.29	1.80	0.37
c	0.91	0.59	0.14	0.67	0.13	0.69	0.11	0.92	0.15
in	6.15	6.68	1.25	6.76	1.24	6.12	1.15	6.49	1.79
k	0.45	0.56	0.16	0.56	0.14	0.52	0.13	0.56	0.18
h	2.39	1.79	0.29	1.79	0.29	1.68	0.28	2.38	0.50
y/h	0.96	0.19	0.05	0.19	0.05	0.17	0.04	0.76	0.14
cpi	1.70	0.61	0.14	2.07	0.36	2.09	0.37	2.86	0.51
p	0.58	0.22	0.03	1.47	0.11	1.50	0.11	2.04	0.17
v	2.97	0.00	0.00	1.28	0.11	1.28	0.11	1.59	0.15
r	0.36	0.00	0.00	0.96	0.08	0.96	0.08	1.19	0.11
Correlations									
y-c	0.86	0.40	0.09	0.34	0.11	0.68	0.08	0.56	0.10
y-i	0.94	0.97	0.01	0.96	0.01	0.96	0.01	0.92	0.04
y-k	0.65	0.17	0.07	0.17	0.08	0.12	0.08	0.13	0.11
y-h	0.93	0.99	0.00	0.99	0.00	0.99	0.00	0.97	0.01
y-y/h	0.45	0.10	0.12	0.10	0.13	0.65	0.11	−0.66	0.13
y-cpi	−0.55	−0.60	0.03	−0.16	0.20	−0.25	0.20	−0.52	0.19
y-p	0.46	−0.85	0.01	−0.12	0.11	−0.14	0.11	−0.24	0.09
y-v	−0.23	0.00	0.11	0.01	0.16	0.01	0.16	−0.22	0.14
v-p	0.04	0.00	0.09	0.90	0.01	0.89	0.01	0.89	0.02
r-p	0.42	0.00	0.09	0.90	0.01	0.89	0.01	0.89	0.02

TABLE 4

AUTOCORRELATIONS AND CORRELATION MATRIX OF THE INNOVATIONS IN A 5 VARIABLES VAR

Autocorrelations for the Fourth Artificial Economy: $v = \alpha = 1, \hat{\rho}_2 = 0$

	lag 1	lag 2	lag 3	lag 4	lag 5	lag 6
y	0.74	0.56	0.33	0.23	0.17	0.11
p	0.06	0.06	−0.19	−0.34	−0.07	−0.07
r	0.38	0.03	−0.31	−0.43	−0.17	−0.06
m	0.79	0.45	0.11	−0.10	−0.14	−0.08
v	0.38	0.03	−0.31	−0.43	−0.17	−0.06

Correlations Matrix of the Innovations and Correlation Matrix (over the diagonal)

Economy 1	Std. Dev	y	p	r	m	v
y	0.0096	1.00	−0.59	0.14	−0.01	−0.14
p	0.0023	−1.00	1.00	−0.06	−0.02	−0.06
r	0.0000	−0.24	0.24	1.00	0.94	1.00
m	0.0000	−0.10	0.10	0.76	1.00	0.94
v	0.0000	−0.24	0.24	1.00	0.77	1.00
Economy 2	**Std. Dev**	**y**	**p**	**r**	**m**	**v**
y	0.0092	1.00	−0.03	0.14	0.04	0.14
p	0.0101	−0.13	1.00	0.93	0.17	0.93
r	0.0072	0.08	0.98	1.00	0.36	1.00
m	0.0071	0.08	0.98	1.00	1.00	0.36
v	0.0001	0.08	0.98	1.00	1.00	1.00
Economy 3	**Std. Dev**	**y**	**p**	**r**	**m**	**v**
y	0.0091	1.00	−0.04	0.14	0.05	0.14
p	0.0103	−0.19	1.00	0.91	0.17	0.91
r	0.0072	0.09	0.96	1.00	0.36	1.00
m	0.0071	0.09	0.96	1.00	1.00	0.36
v	0.0001	0.09	0.96	1.00	1.00	1.00
Economy 4	**Std. Dev**	**y**	**p**	**r**	**m**	**v**
y	0.0093	1.00	−0.29	−0.20	−0.55	−0.20
p	0.0126	−0.62	1.00	0.90	0.14	0.90
r	0.0078	−0.42	0.95	1.00	0.31	1.00
m	0.0077	−0.42	0.95	1.00	1.00	0.31
v	0.0001	−0.42	0.95	1.00	1.00	1.00

TABLE 5

AUTOCORRELATIONS AND CORRELATION MATRIX OF THE INNOVATIONS IN A 5 VARIABLES VAR

Autocorrelations for the Fourth Artificial Economy: $v = 3.21$, $\alpha = 0.52$, $\hat{\rho}_2 = 0.07$

	lag 1	lag 2	lag 3	lag 4	lag 5	lag 6
y	0.78	0.62	0.43	0.30	0.20	0.08
p	–0.05	0.00	–0.16	–0.33	0.02	–0.06
r	0.38	0.03	–0.31	–0.43	–0.17	–0.06
m	0.79	0.45	0.11	–0.10	–0.14	–0.08
v	0.38	0.03	–0.31	–0.43	–0.17	–0.06

Correlations Matrix of the Innovations and Correlation Matrix (over the diagonal)

Economy 1	Std. Dev	y	p	r	m	v
y	0.0093	1.00	–0.85	0.14	–0.02	0.14
p	0.0015	–1.00	1.00	–0.11	–0.01	–0.11
r	0.0000	–0.21	0.21	1.00	0.94	1.00
m	0.0000	–0.06	0.06	0.77	1.00	0.94
v	0.0000	–0.20	0.21	1.00	0.77	1.00
Economy 2	**Std. Dev**	**y**	**p**	**r**	**m**	**v**
y	0.0091	1.00	–0.03	0.15	0.07	0.15
p	0.0116	–0.03	1.00	0.90	0.10	0.90
r	0.0072	0.09	0.99	1.00	0.36	1.00
m	0.0071	0.09	0.99	1.00	1.00	0.36
v	0.0097	0.09	0.99	1.00	1.00	1.00
Economy 3	**Std. Dev**	**y**	**p**	**r**	**m**	**v**
y	0.0090	1.00	–0.05	0.15	0.08	0.15
p	0.0118	–0.13	1.00	0.88	0.10	0.88
r	0.0073	0.10	0.97	1.00	0.36	1.00
m	0.0071	0.10	0.97	1.00	1.00	0.36
v	0.0097	0.10	0.97	1.00	1.00	1.00
Economy 4	**Std. Dev**	**y**	**p**	**r**	**m**	**v**
y	0.0091	1.00	–0.23	–0.12	–0.50	–0.12
p	0.0148	–0.45	1.00	0.84	0.10	0.84
r	0.0078	–0.27	0.93	1.00	0.31	1.00
m	0.0076	–0.27	0.93	1.00	1.00	0.31
v	0.0103	–0.27	0.93	1.00	1.00	1.00

4. Welfare costs

In this section we report the welfare costs of the inflation tax in economies that differ in the level of incomes taxes, and compare these estimates with others studies. This exercise is similar to the one reported in Cooley and Hansen (1992), with the difference in the magnitude of the coefficient α. In their (1989) model the total present consumption is not divided between cash and credit goods. In our model if $\alpha = 1$ we obtain their utility function. Moreover, the logarithmic utility function is reached in our model if we assume that $v \to 1$. We report the welfare costs for economies with $\alpha = 0.52$ and $v = 1$. I use the same methodology as Cooley and Hansen (1989), comparing the steady states under the different economies with the Pareto optimal allocation. All the experiments assume for simplicity that $\zeta = 0$. The following tables exhibit the steady state values for different levels of inflation in economies with only inflation taxes, or in economies with both income and inflation taxes. Table 6.1 is comparable to Table 2 in Cooley and Hansen (1989), and Table 6.2 to Table 1 in Cooley and Hansen (1991).

The welfare cost of 10% inflation in the economy without taxes and with $v = 1$ is 0.21 percent of GNP. This value must be compared with the 0.39 found by Cooley and Hansen (1989), and also with the estimates of 0.3 percent provided by Stanley Fisher

TABLE 6.1

STEADY STATES AND WELFARE COSTS ASSOCIATED WITH ALTERNATIVES GROWTH OF MONEY ($\alpha = 0.52$, $v = 1$)

	Economy with Only Inflation Tax							
Inflation	–4%	0%	5%	10%	20%	50%	100%	200%
g	β	1	1.012	1.024	1.047	1.107	1.189	1.316
y	1.115	1.109	1.102	1.095	1.083	1.053	1.016	0.969
c	0.829	0.825	0.819	0.814	0.805	0.783	0.756	0.720
c_1	0.439	0.434	0.429	0.424	0.415	0.392	0.365	0.330
c_2	0.390	0.390	0.390	0.390	0.390	0.390	0.390	0.390
i	0.286	0.284	0.282	0.281	0.278	0.270	0.260	0.248
k	11.433	11.373	11.300	11.232	11.106	10.796	10.420	9.935
h	0.301	0.299	0.297	0.296	0.292	0.284	0.274	0.261
	Welfare and Revenue Consequences							
Seigniorage	–0.0044	0	0.0052	0.0100	0.0185	0.0378	0.0581	0.0792
Seign/output	–0.0040	0	0.0047	0.0091	0.0171	0.0359	0.0572	0.0818
Welfare Cost of Inflation (%C)	0	0.0764	0.1753	0.276	0.4802	1.0993	2.0928	3.8516
Welfare Cost of Inflation (% GNP)	0	0.0764	0.1303	0.205	0.3571	0.8174	1.5562	2.8641
Seig/Tot Revenues	1	1	1	1	1	1	1	1
Welfare Cost of Policy (%C)	0	0.0764	0.1753	0.276	0.4802	1.0993	2.0928	3.8516
Welfare Cost of Policy (% GNP)	0	0.0764	0.1303	0.205	0.3571	0.8174	1.5562	2.8641

(1981) or 0.45 percent obtained by Robert Lucas (1981) (measure of the area under a money demand function). Now, if we consider the model with taxes, this value increases to 0.57 (when v = 1) percent of the GNP. In this case the values found by Fisher and Lucas underestimate the actual welfare cost of inflation relative to the model of this paper. Consider, first, the welfare cost of policy when the inflation tax is zero. In our model this cost arises to 13.93% of GNP while Cooley and Hansen estimated it at 16.84% of GNP. Now, the welfare cost of inflation almost doubles when we consider capital and labor income taxes. With 10% inflation, the inflation tax in the model without income taxes is 0.28% of consumption while in the model with taxes this value almost triples (0.71%). See Figure 3 for a comparison with others studies.

If we consider the welfare cost of the policy (inflation tax and income taxes), the estimates reported by Cooley and Hansen (1989) are uninformative because they did not include taxes in their model. It is important to note the magnitude of financing that the government has in terms of seigniorage at different levels of inflation. For instance, if the annual inflation rate is 5 percent, then the proportion of the revenues that comes from seigniorage is between 1.1 and 1.2 percent of the total revenues (around 0.3 percent of GNP). Meanwhile, if the inflation level increases up to 50 percent, the seigniorage revenues are between 7.8 and 8.1 percent of the total revenues (around 2

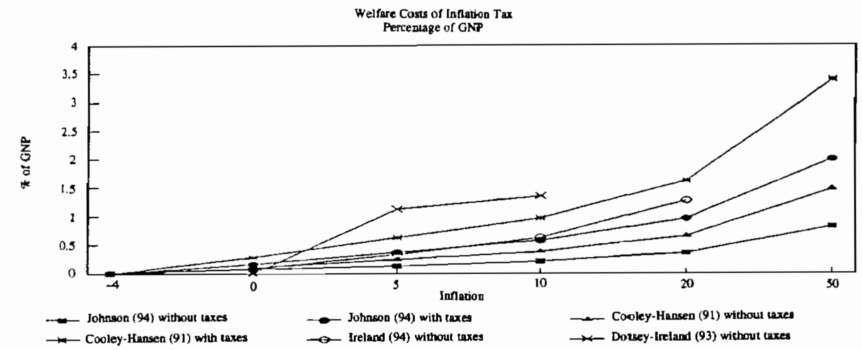
TABLE 6.2

STEADY STATES AND WELFARE COSTS ASSOCIATED WITH ALTERNATIVES GROWTH OF MONEY ($\alpha = 0.52$, v = 1)

	Economy with Capital and Labor Income Taxation							
Inflation	–4%	0%	5%	10%	20%	50%	100%	200%
g	B	1	1.012	1.024	1.047	1.107	1.189	1.316
y	0.691	0.688	0.683	0.679	0.671	0.653	0.630	0.601
c	0.554	0.551	0.547	0.544	0.538	0.523	0.505	0.481
c_1	0.293	0.290	0.286	0.283	0.277	0.262	0.244	0.220
c_2	0.261	0.261	0.261	0.261	0.261	0.261	0.261	0.261
i	0.138	0.137	0.136	0.135	0.134	0.130	0.126	0.120
k	5.506	5.477	5.442	5.401	5.348	5.199	5.018	4.784
h	0.215	0.214	0.213	0.211	0.209	0.203	0.196	0.187
	Welfare and Revenue Consequences							
Seigniorage	–0.0030	0	0.0035	0.0067	0.0123	0.0253	0.0388	0.0529
Seign/output	–0.0043	0	0.0051	0.0098	0.0184	0.0387	0.0616	0.0881
Welfare Cost of Inflation (%C)	0	0.2063	0.4617	0.7102	1.1883	2.4920	4.3364	7.2450
Welfare Cost of Inflation (% GNP)	0	0.1652	0.3698	0.5688	0.9517	1.9959	3.4730	5.8024
Seig/Tot Revenues	–0.0192	0	0.0218	0.0413	0.0747	0.1453	0.2129	0.2790
Welfare Cost of Pocili (%C)	17.146	17.387	17.686	17.977	18.537	20.065	22.225	25.632
Welfare Cost of Policy (% GNP)	13.732	13.925	14.165	14.400	14.846	16.070	17.800	20.529

FIGURE 3

WELFARE COSTS: A COMPARISON AMONG STUDIES



percent of GNP). Based on a general equilibrium approach with transaction cost technology, Dotsey and Ireland (1993) estimate that the welfare cost of inflation is about 1.34% of GNP, while in a recent publication which studies the links between financial structure and economic growth, Ireland (1994) estimates the welfare cost as 0.62% of GNP. The values reported by Ireland (1994) are very similar to the ones found in our model with taxes, with inflationary levels between five and ten percent (see Figure 3).

5. Conclusions

The purpose of this study is to extend the standard RBC models to include stochastic monetary and fiscal instruments. Specifically, we structured an RBC economy which includes inflation and income taxes, to mimic some of the standard features of the U.S. data. One conclusion that arises from the simulation and from the welfare cost estimates is that implications of the model are very sensitive to the parameters like the coeficcient of relative risk aversion (for cash and credit goods), and the specification of the fiscal and monetary policy.

The inclusion of stochastic money growth and taxes improves the ability of the model to produce behavior similar to that of the actual data. The effect on the unexpected inflation is not neutral when we link the monetary and fiscal policy, through the budget constraint. Moreover, the sensitivity of the monetary policy to the productivity shock, makes the economic activity more inelastic to the productivity shock. The model is able to explain most of the volatility in selected macroeconomic variables like consumption (100%), investment (105%), employment (100%), productivity (80%), the capital stock (124%), the price level (168%), and velocity (54%).

FIGURE 4

CYCLICAL BEHAVIOR OF U.S. ECONOMIC TIME SERIES,
1972: 1 - 1933: 1. DEVIATION FROM TREND

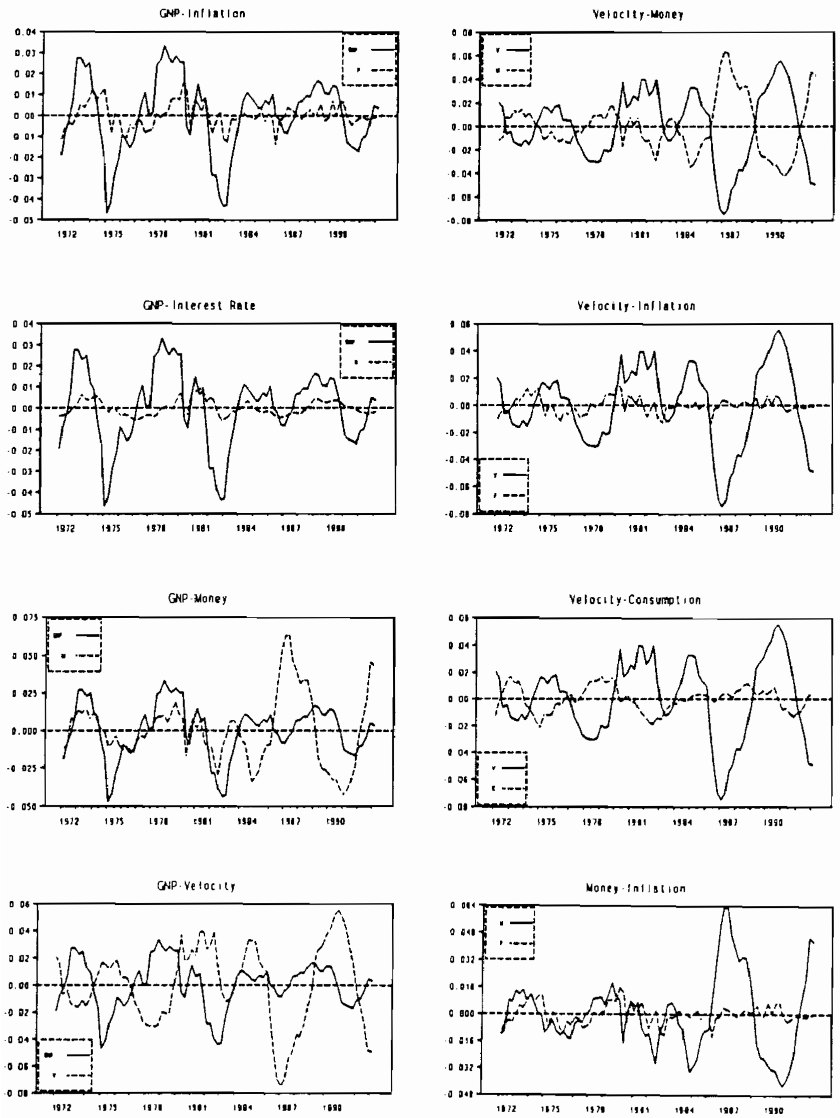


FIGURE 5

IMPULSE RESPONSE FOR SELECTED MACROECONOMIC VARIABLES
(GNP - P - R - M - V)

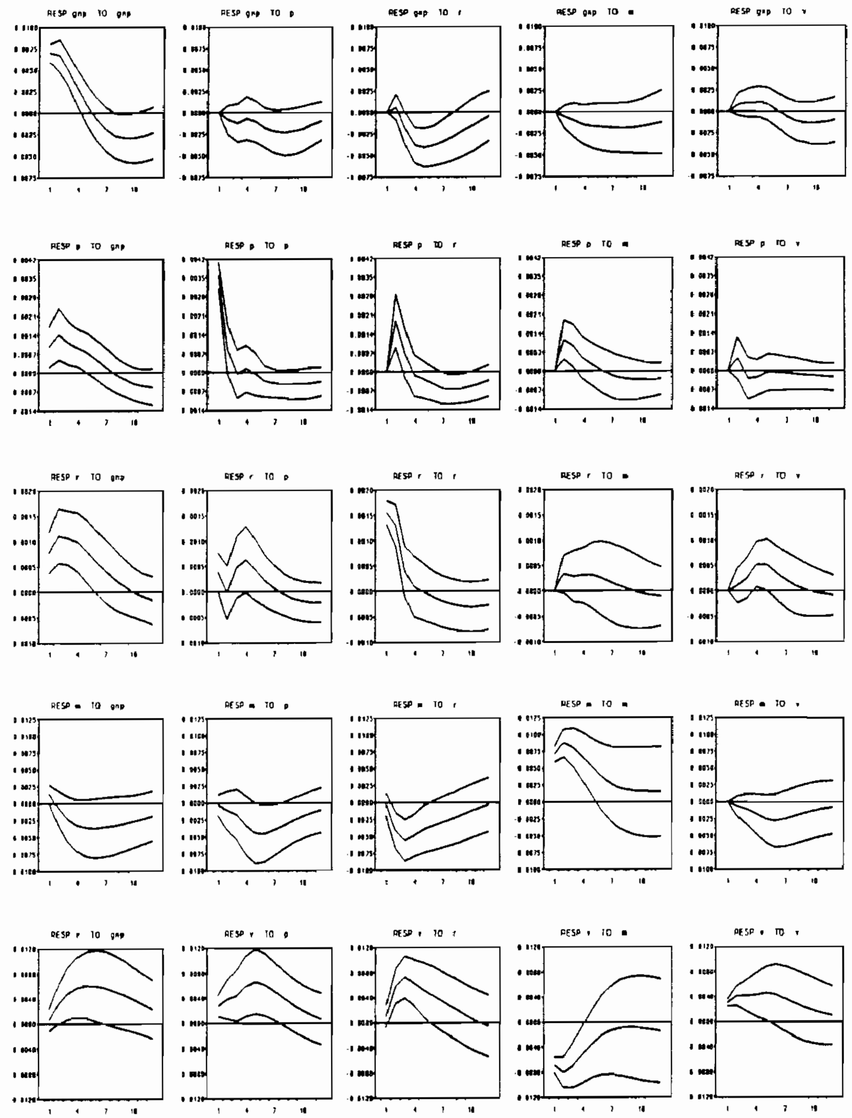


FIGURE 6

CYCLICAL BEHAVIOR OF SIMULATED DATA ($g = 1.015$, $\alpha = \nu = 1$).
DEVIATION FROM TREND

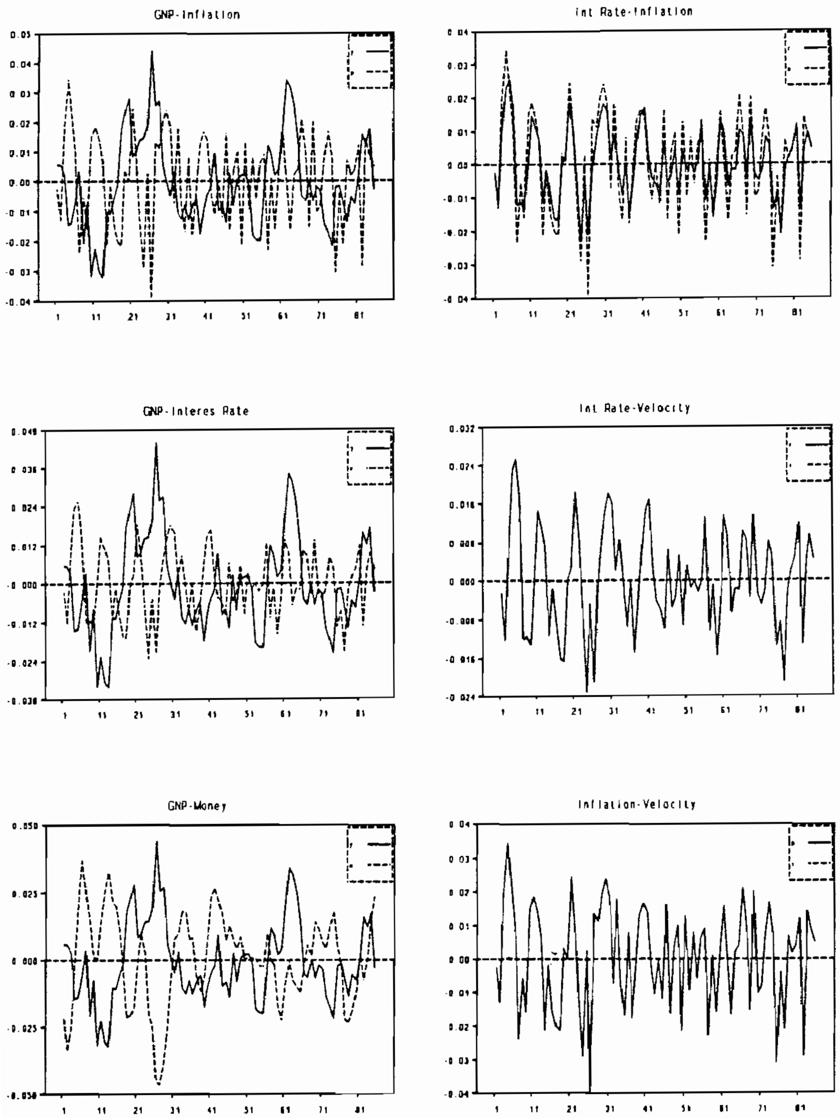


FIGURE 7

IMPULSE RESPONSE FOR SELECTED SIMULATED VARIABLES
($g = 1.015$, $\alpha = \nu = 1$)

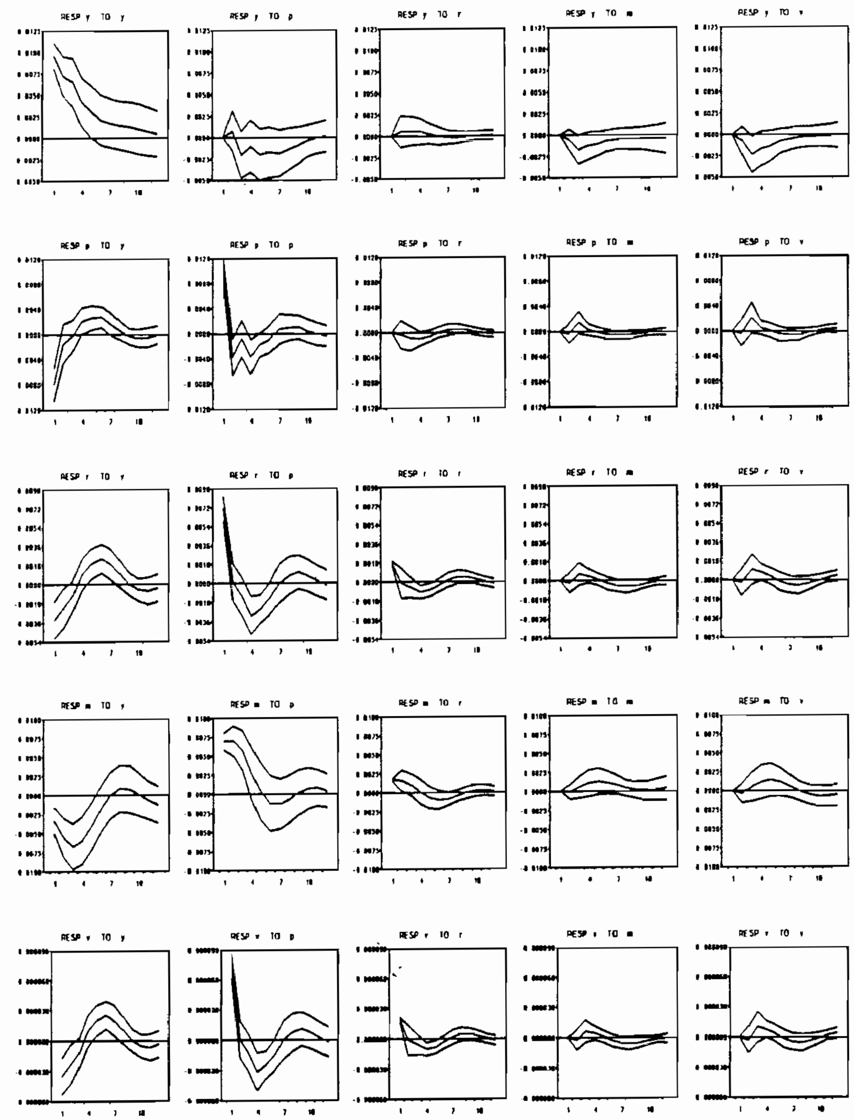


FIGURE 8

CYCLICAL BEHAVIOR OF SIMULATED DATA ($g = 1.015$, $\alpha = 0.53$, $\nu = 3.21$). DEVIATION FROM TREND

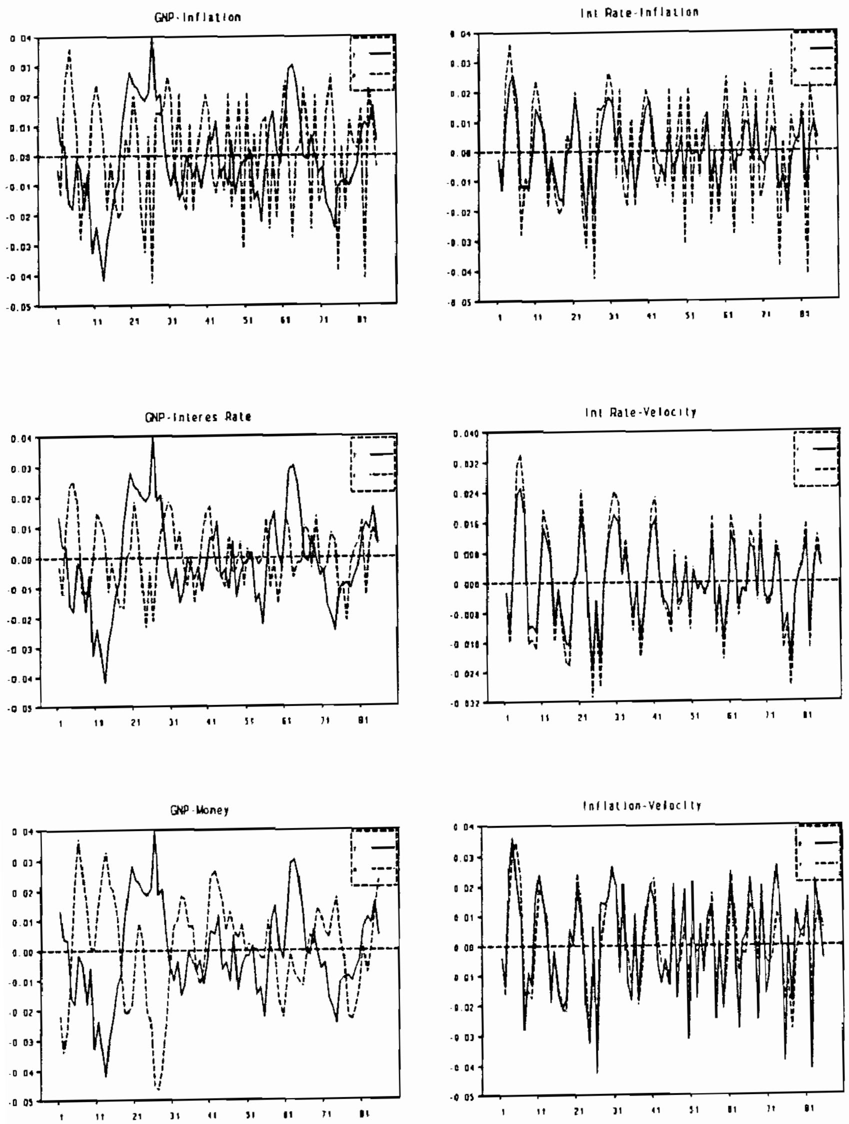
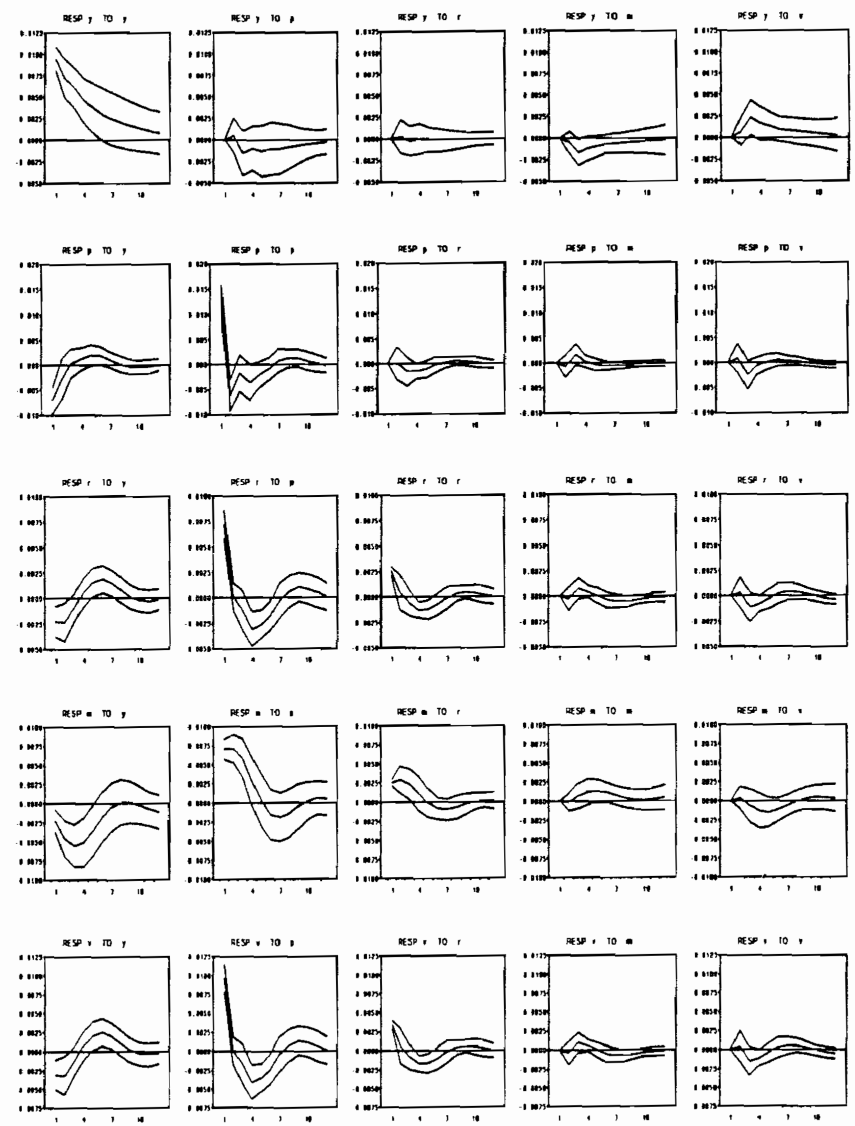


FIGURE 9

IMPULSE RESPONSE FOR SELECTED SIMULATED VARIABLES
($g = 1.015$, $\alpha = 0.53$, $\nu = 3.21$)



Notes

1 McGrattan (1991) modified a basic model in two directions. In the first model, taxes are included. In the second, money via cash-in-advance is incorporated. The final conclusion is that the improvements of the monetary model are small relative to that found with the tax model.
2 Here, as in Christiano and Eichenbaum (1992), the government purchases are financed solely via lump-sum taxes. The extension is to consider distorting income taxes instead of lump-sum taxes.
3 The comparison is based on a VAR estimation for the actual data and a VAR using the simulated data from the model, altogether with the first two central moments of each variable.
4 See also Leeper (1991) and Leeper and Sims (1993).
5 Investment and leisure are also credit goods.
6 Separability implies that consumption of the credit good remains unchanged by inflation.
7 Estimates of the coefficient of relative risk aversion vary substantially but usually lie around or below unity.
8 Some of the capital letters are used for per capita variables that are determinated in the equilibrium, and small letters for control variables.
9 To ensure that the nominal interest rate is not negative we require that the monetary growth rate, g, must be greater than or equal to the discount factor ß. See Lucas and Stokey (1987).
10 See Stokey and Lucas with Prescott (1989) chapter 18.
11 See Lucas and Stokey (1987).
12 Significance levels are in parenthesis.
13 See Mooney and Duval (1993).
14 Similar estimation was done by Bohn (1991).
15 See Prescott (1986) for the method of detrending, and King and Rebelo (1993) for some critiques.
16 Since then, the Fed changed his policy from targeting nonborrowed reserves to indirectly targeting the funds rate (Leeper and Gordon (1992)). For a discussion about this point see Balke and Emery (1994).
17 For a detailed illustration of post-World War II U.S. non-monetary economic time series see Huffman (1994).
18 This effect is not shown in a VAR even when the federal funds rate is considered instead of the three month treasury bill rate. See Leeper and Gordon (1992) and (1993), Christiano (1991), Christiano and Eichenbaum (1992b), and Coleman *et al.* (1994).
19 In percent.
20 For this economy we used $rho_2 = 0.007$.
21 In percent.
22 In this economy we used $rho_2 = 0.007$.

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