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DOES INCOME INEQUALITY REDUCE GROWTH?

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Abstract:

This paper examines the effects of inequality on the rate of growth of an economy. We assume that it is easier for an individual to achieve a given level of human capital the higher society's average level of human capital. Agents with above average human capital find it relatively more costly to acquire additional human capital, while agents with below average human capital find it relatively cheaper to acquire additional human capital. The existence of such an externality implies that even when there is no income inequality agents will behave inefficiently. In order to achieve the optimal growth rate, a lump sum tax must be combined with a subsidy to investment in education. When incomes are heterogenous, we show that income convergence is attained in the long run. We also show that the effect of inequality on the growth rate of an economy depends on the functional form of the externality. When the externality function is concave, income dispersion reduces the rate of growth. On the other hand, when the externality function is convex, the effect is ambiguous.

I. Introduction

The purpose of this paper is to analyze the effects of income inequality on the growth rate of an economy. Our interest in the subject is motivated by the striking differences in the growth rates of different developing countries.

In the classical development literature, succesful growth was associated with higher income inequality (Lewis (1954)). The classical argument runs as follows.

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Wealthy agents have a lower rate of time preference, consequently they save a larger proportion of their income than poor agents. Thus the savings rate should be higher in countries exhibiting more inequality. The factual evidence does not seem to support this conclusion. In fact, since the 1960's casual observation suggests that the rate of growth in developing countries is negatively related to the degree of income inequality in the country. A good example is provided by comparing Latin American and South East Asia countries. In the first group, slow growth is associated with high income inequality, whereas the second group combines high growth rates and low degrees of inequality (see table I)[1]. These countries differ in other important aspects, so there are alternative explanations for the differences in the growth rates. Nevertheless, it is tempting to explore how growth is affected by income inequality.

Our work is related to the endogenous growth literature pioneered by Romer (1986) and Lucas (1988). In these papers, growth is the consequence of economic forces in a decentralized economy. Their models, by relaxing some of the more restrictive assumptions of the neoclassical growth models, allow for economies that achieve sustained growth. This result depends crucially on the existence of one or more capital goods that are produced without requiring the use, direct or indirect, of non-reproducible factors.

TABLE I

INCOME DISTRIBUTION AND GROWTH

Country	Percentage Share of Income			
	Lowest 20%	Top 20%	Top/Bottom Ratio	Growth GNP/capita, 65-80
Guatemala	9.8	39.4	4.0	0.7
Peru (85)	4.4	51.9	11.8	–0.2
Colombia (88)	4.0	53.0	13.3	2.3
Costa Rica (86)	3.3	54.5	16.5	1.4
Brazil (83)	2.4	62.6	26.0	3.3
Venezuela (87)	4.7	50.6	10.8	–1.0
Average Latin America[2]	3.8	54.5	14.3	1.1
Phillipines (85)	5.5	48.0	8.7	1.3
Malaysia (87)	4.6	51.2	11.1	4.0
Singapore (82)	5.1	48.9	9.5	6.5
Hong Kong (80)	5.4	47.0	8.7	6.2
Taiwan (85)	8.4	37.6	4.5	6.5
Average East Asia	5.8	46.5	8.0	4.9

Sources:

World Development Report 1992. Data for Taiwan comes from Table 8.1 in Bourguignon and Morrisson (1980) and Nafziger (1990). The data are not completelly comparable. For instance, some observations report family income while others correspond to individual income or expenditure.

Ehrlich (1990) has proposed a distinction between those models which consider human capital as the key source of growth and development (as in Lucas (1988)) and those models in which the engine of growth is disembodied technological innovation (as in Romer (1986)). As we are interested in the study of growth in developing countries, it appears that models based on human capital are more relevant as little technological innovation takes place in these countries. Furthermore, there is much evidence that indicates that variables related to human capital are important factors in the growth process. For example Barro (1991) finds that a wide variety of proxies for human capital such as enrollment variables are strongly related to per capita growth rates. These conclusions are shown to be robust by Levine and Renelt (1992) using Extreme Bounds Analysis.

Theoretical explanations for a relation between income distribution and growth have also received attention recently. Some authors (Sachs (1989), Benhabib and Rustichini (1992)) have explored the possibility that more inequality leads to more social instability in a country, and that this instability could in turn reduce the growth rate. A second explanation for the effects of inequality on growth is based on the existence of restrictions in credit markets for poor agents (Banerjee and Newman (1991)). Other authors highlight the effects of taxation: these effects are positive when tax revenues are used for education (Glomm and Ravikumar (1992)) or negative when they are used for redistribution (Persson and Tabellini (1991)).

Our approach appeals to the existence of externalities in the process of capital accumulation that gives rise to growth. Our externality, which is similar to the one described in Tamura (1991), is the result of spillovers in the human capital investment technology. Nevertheless we differ from Tamura in that we incorporate production into our model. The introduction of production gives us new results that are absent from previous analysis[3].

In our model, the stock of human capital of an individual depends on the stock of human capital of the parent, on the parent's decision on how much to invest in the education of the descendant and on the average level of human capital in the economy. Thus there is an externality in the production of human capital as the acquisition of human capital is a function of the economy's average stock of human capital. We assume that it is more difficult to increase the human capital stocks of agents with above average wealth, while the converse is true for agents with below average capital stocks[4].

Our first result is that in an homogenous income economy, in the absence of government intervention the steady state rate of growth is lower than optimal. This is expected given that the presence of an externality implies that the private investment decisions of the agents do not take into account the social benefits of their investment, so the society's expenditure in education is lower than optimal (see Stokey, 1991). This result implies that countries that correct for this externality by subsidizing education will achieve higher growth rates.

Next we study the effects of income heterogeinity in this economy. We show that in the long run there is income convergence in this economy. This implies that the growth rate of the economy converges to the steady state rate of growth of an economy with no inequality. Similar results appear in Tamura (1991), in an economy without production. However, in Tamura (1991) there is no convergence with logarithmic utility, because there is no production. Our economy does not follow a Kuznetz (1955) inverted-U path for income distribution; income converges along the whole path of growth. The reason for income convergence in our model is that the externality reduces the efficiency of the investments of the rich. With no externality, inequality would persist.

We also derive conditions on the externality that ensure that inequality reduces the rate of growth. Inequality affects the accumulation of human capital in two ways. First, inequality lowers the savings rate of the wealthy because of decreasing returns to investment for agents with above average human capital. The converse is true for poor agents. We show that the net effect is to decrease the society's savings rate. The second effect depends on the functional form of the externality. When the externality function is concave, income dispersion has a negative impact on the production of new human capital. This effect, coupled to the decrease in aggregate investment, unambiguously reduces the rate of growth of aggregate human capital. On the other hand, if the externality function is convex, income dispersion has a positive effect on the production of human capital. In this case the two effects go in opposite directions and the total effect is ambiguous. Thus it is possible that inequality may make the economy grow faster.

We also show that in countries with non-sophisticated technologies, in the sense that their industries use low human capital-labor ratios, inequality is more likely to have an positive effect on growth. Low income countries may be assumed to have non-sophisticated technologies, whereas high income countries use high human capital-labor ratios in production. In this case inequality is more likely to reduce the growth rate of high income countries than that of low income countries when the externality function is convex.

In our model the savings rate of the wealthy is lower than that of the poor[5]. This result seems to contradict observed facts. This is not the case, however. Investment and savings statistics normally do not include money invested in education. If we were to include physical capital as a factor of production in our model we would find that the rich save proportionally less in human capital but more in physical capital than the poor (see for instance Fischer and Serra (1993))[6].

The rest of the paper is organized as follows. In section 2 we introduce the basic model. In section 3 we analyze the externality. In section 4 we discuss efficiency in the homogenous economy. In section 4 we study the relationship between income inequality and growth.

II. The Basic Model

We study cohorts of agents that live a single period and then die, leaving an inheritance to their descendants. Agents care about the welfare of their descendants so they behave like infinitely lived individuals. Specific production and utility functions are used so that explicit solutions can be obtained.

Production

This is a two factor –unskilled labor and human capital– one final good model. In all periods each individual inelastically supplies one unit of unskilled labor. Human capital is a reproducible factor that requires only human capital as an input. Each agent's production of human capital has the general form

$$h_{t+1} = f(h_t, g_t, \bar{h}_{t+1}), \quad (1)$$

where h_t denotes the agent's stock of human capital, $\bar{h}_t$ the society's average level of human capital and g_t the amount of human capital the agent invests in order to produce new human capital, all variables dated at time t. We use lower case letters for variables referring to individual agents; if necessary we distinguish individual agents with a superindex u.

We define H_t and G_t as the society's stock of human capital and the amount of human capital it spends in education respectively, *i.e.*, $H_t = \sum_u h_t^u$ and $G_t = \sum_u g_t^u$. It follows that the $H_t - G_t$ represents the amount of human capital the economy spends in producing the consumption good.

The production function for the final good is:

$$X_t = L^\alpha (H_t - G_t)^{1-\alpha},\ 0 \le \alpha \le 1 \quad (2)$$

where X_t, is the amount of final good and L is the number of individuals in the economy, all variables dated at time t. Choosing the price of the final good as numeraire, in equilibrium factor prices are given by:

$$v_t = (1-\alpha)\,[L/(H_t - G_t)]^\alpha \quad (3)$$

$$w_t = \alpha\,[L/(H_t - G_t)]^{\alpha-1} \quad (4)$$

where v_t is the return on human capital and w_t is the wage rate.

Consumption. The utility of each agent is given by

$$V(h_t) = \mathrm{Max}_{\{c_t\}}\ \{u(c_t) + \gamma V(h_{t+1})\} \quad 0 \le \gamma \le 1 \quad (5)$$

where ct is the agent's consumption in period t and γ the agent's rate of time discount. The consumption expenditure of an agent during period t satisfies

$$c_t = w_t + v_t (h_t - g_t) \quad (6)$$

Using (3) and (4), individual consumptions may be expressed as:

$$c_t = L^\alpha (H_t - G_t)^{1-\alpha} \left[\frac{\alpha}{L} + \frac{(1-\alpha)(h_t - g_t)}{H_t - G_t}\right] \quad (7)$$

The expression between brackets in (7), which we denote ε_t, is the fraction of aggregate consumption consumed by an agent, therefore $\sum_u \varepsilon_t^u = 1$. For reasons of tractability we assume logarithmic preferences, *i.e.*

$$u(c_t) = a \log c_t \quad (8)$$

III. Externalities in Human Capital Production

We now specify the technology for the production of human capital. We want a technology that corresponds to the notion that there is an externality in the acquisition of human capital. In particular, we would like the cost of acquiring human capital to be decreasing on the average level of human capital. The explanation for this externality is that in the educational process there are spillovers, so that individuals benefit from the level of education in society. We also believe that agents who have above average human capital find it more difficult to acquire additional human capital, because for agents close to the frontier of knowledge, additional human capital is difficult to achieve (see Tamura (1991)). On the contrary, those agents that have a stock of human capital below average find easier to increase their stock of human capital. A simple specification is

$$h_{t+1} = (\bar{h}_{t+1})^{1-\delta} (\rho h_t + g_t)^{\delta}, \quad 0 \le \delta \le 1 \tag{9}$$

In this equation, the level of human capital of the parent has a positive influence on the stock of the descendant, reflecting the effects of rearing on education.

We can rewrite (9) in a more illustrative way,

$$h_{t+1} = (\rho h_t + g_t) \left(\frac{\bar{h}_{t+1}}{h_{t+1}}\right)^{1/\delta - 1} \tag{10}$$

The second expression in the right hand side (RHS) of equation (10) represents our externality function. Note that if δ is less or equal than one half, the externality function is convex, otherwise it is concave. This distinction will become important later.

Rearranging terms we have that

$$h_t - g_t = (1+\rho) h_t - (h_{t+1})^{1/\delta} (\bar{h}_{t+1})^{1-1/\delta} \tag{11}$$

Agents are assumed to have perfect foresight on the path of the average human capital stock. Consequently, assuming an interior solution each individual's Euler equation is given by:

$$\frac{\partial u(c_t)}{\partial \delta h_{t+1}} + \gamma \frac{\partial u(c_{t+1})}{\partial h_{t+1}} = 0 \tag{12}$$

In what follows we assume interior solutions for all individuals. Now, from (7), (8) and (11)

$$u(c_t) = a \log \left(L^{\alpha}(H_t - G_t)^{1-\alpha} \left[\frac{\alpha}{L} + \frac{(1-\alpha)\left((1+\rho)h_t - (h_{t+1})^{1/\delta} (\bar{h}_{t+1})^{1-1/\delta}\right)}{H_t - G_t} \right] \right) \tag{13}$$

Recalling the definition of ε, equation (12) becomes

$$\frac{1}{\varepsilon_t (H_t - G_t)} = \frac{\gamma \delta (1+\rho)}{\varepsilon_{t+1} (H_{t+1})} \left(\frac{\bar{h}_{t+1}}{h_{t+1}}\right)^{1/\delta - 1} \tag{14}$$

and the transversality condition is

$$\lim \gamma^t \frac{a(1-\alpha)}{\varepsilon_t} \frac{1+\rho}{H_t - G_t} h_t = 0 \tag{15}$$

Given the properties exhibited by function u[7], the solution of each agent's optimization problem can be characterized by the Euler and transversality conditions.

Using the definition of ε_t, equation (14) can be rewritten as

$$\left[\frac{\alpha}{L} (H_{t+1} - G_{t+1}) + (1-\alpha)(h_{t+1} - g_{t+1})\right] = \gamma (1+\rho) \delta \left[\frac{\alpha}{L} (H_t - G_t) + (1-\alpha)(h_t - g_t)\right] \left(\frac{\bar{h}_{t+1}}{h_{t+1}}\right)^{1/\delta - 1} \tag{16}$$

It is immediate from equation (16) that if there is no externality in the education process ($\delta = 1$), income inequality has no effect in the rate of growth. In this case the consumption of all individuals grows at the same rate and there is no income convergence. Before studying the case of income inequality, we consider an economy with homogenous agents.

IV. The Case of Homogenous Agents

If all individuals have the same amount of human capital, adding up equation (14) over agents results in[8]:

$$(H_{t+1} - G_{t+1}) = (1+\rho)\, \delta\, [H_t - G_t] \tag{17}$$

Moreover, using (11) it is possible to rewrite $H_t - G_t$ as

$$\begin{aligned} H_t - G_t &= (1+\rho) \Sigma_u h^u_t + (\bar{h}_{t+1})^{1-1/\delta} \Sigma_u (h^u_{t+1})^{1/\delta} \\ &= (1+\rho) H_t - H_{t+1} \end{aligned} \tag{18}$$

Equation (18) becomes:

$$H_{t+2} - (1+\rho)(1+\delta\gamma) H_{t+1} + (1+\rho)(1+\rho)\delta\gamma H_t = 0 \tag{19}$$

with roots $\lambda_1 = (1+\rho)$ and $\lambda_2 = (1+\rho)\delta\gamma$. If root 1 admits a non-zero constant in the solution to the equation, it will eventually dominate the effect of the second root. But the first root leads to a non-optimal solution because it implies that eventually all human capital is reinvested into producing more human capital and nothing is left to produce consumption goods[9]. This cannot be an individual's optimal solution. More formally it can be seen that the transversality condition is not satisfied.

Therefore the solution to each individual's optimizing problem has a zero constant for the first root. The corresponding growth rate is given by $\lambda_2 - 1$. This is not the maximum feasible rate of growth. In fact, the growth rate of human capital and of the economy is lower than when there is no externality ($\delta = 1$)[10]. It is easy to show that the optimal solution with and without externality is the same[11]. The effects of the externality on the growth rate can be offset and the first best can be achieved by exacting a lump sum tax and subsidizing investment in human capital. Consider a subsidy at a rate of s per unit of human capital invested. The lump sum tax exacts the amount $G_t s/L$ from each agent. This amount is considered to be a fixed constant by the agents. Recalling that all agents consume the same amounts c_{t+1} we may write:

$$c_{t+1} = \left[\frac{\alpha}{L} + (1-\alpha) \frac{(h_t - (1-s)\, g_t)}{H_t - G_t} - \frac{sG_t}{L} \right] \tag{20}$$

Since

$$h_t - (1\text{-}s)g_t = (1 + (1-s)\,\rho)\, h_t + (1-s)\,(h_{t+1})^{1/\delta}\,(h_{t+1})^{1-1/\delta} \tag{21}$$

we have:

$$H_{t+1} - G_{t+1} = [(1 + (1-s)\,\rho)\,\gamma\delta/(1-s)]\,(H_t - G_t) \tag{22}$$

The first best subsidy is the one that achieves the growth rate without externality $(1+\rho)\,\gamma$, that is, when

$$s = (1+\rho)\ (1-\delta)/(1+\rho-\rho\delta). \tag{23}$$

V. Income Convergence in the Heterogenous Case

For the sake of simplicity we consider two consumers who behave as price takers. Agent 1 initially (period 0) has more human capital than agent 2. The externality leads the agents that have above average stocks of human capital to save proportionately less than agents that have a stock of human capital below average (see corollary 3 in appendix). Consequently, the income and consumption of the rich tends to grow slower than that of the poor. We have the following result.

Proposition 1. In an economy with initial heterogeneity, the consumption and human capital stocks of agents converge.

Proof: See appendix.

Let the superindex 1 denote variables corresponding to the wealthy agents. Adding up equation (16) over agents results in:

$$[(H_{t+1} - G_{t+1}) + \tau_t] = (1+\rho)\,\gamma\delta\,[H_t - G_t] \tag{24}$$

where

$$\tau_t = (1-\alpha)\,\Sigma_u\,(h^u_{t+1} - g^u_{t+1}) \left[\left(\frac{h^u_{t+1}}{\bar{h}_{t+1}} \right)^{1/\delta - 1} - 1 \right] - \omega\, \frac{\alpha}{L}\,(H_{t+1} - G_{t+1}) \tag{25}$$

and

$$\omega = \Sigma_u \left(\frac{h^u_{t+1}}{\bar{h}_{t+1}} \right)^{1/\delta - 1} - L \tag{26}$$

The variable τ_t is the key to the study of the effects of the externality. With no externality τ_t equals zero. Note that $H_t - G_t$ represents the amount of human capital which is used to produce the consumption good. Faster growth in $H_t - G_t$ implies faster growth in consumption. Thus, by (24), the effects of income heterogeneity on the rate of growth depend on the sign of τ_t. In the following we discuss the sign of τ_t. Our first result is a simple lemma:

Lemma. The first expression in (25) is always positive.

Proof: In the appendix we show that agent 1 invests less than agent 2 in proportion to their respective stocks of human capital (Corollary 3), i.e.

$$\frac{g^1_t}{h^1_t} < \frac{g^2_t}{h^2_t} \tag{27}$$

It follows that

$$\frac{h^1_t - g^1_t}{h^1_t} > \frac{h^2_t - g^2_t}{h^2_t} \tag{28}$$

On the other hand, as $\delta \le 1$, it is not difficult to show that

$$(h^1_t)^{1/\delta} + (h^2_t)^{1/\delta} - 2(ht)^{1/\delta} \ge 0 \tag{29}$$

Rewritting (29),

$$h^1_{t+1} \left[\left(\frac{h^1_{t+1}}{h_{t+1}} \right)^{1/\delta - 1} - 1 \right] \ge h^2_{t+1} \left[1 - \left(\frac{h^2_{t+1}}{h_{t+1}} \right)^{1/\delta - 1} \right]$$

From (28) and (29) the Lemma follows.

Proposition 2: If the externality is concave, inequality reduces the growth rate of the economy.

Proof: When the externality is concave ($\delta \ge 1/2$), _ is negative in (26). Then the proposition follows from the Lemma, since (25) is positive.

The intuition for this result is as follows. When there is heterogeneity, the externality has two effects on the growth rate. The first effect is to reduce the aggregate savings of the economy. The reason is that the externality lowers the savings rate of the wealthy agents while it increases the savings rate of poor agents. As those that save less are the richer ones, the net effect on aggregate investment is negative. This effect is summarized in the first term in the RHS of equation 25.

The second effect corresponds to the second term in the RHS of (25). The sign of the second effect depends on the shape of the externality function. When the externality is concave, income dispersion has a negative effect on the production of new human capital (by Jensen's inequality). The externality scales down the productivity of investments made by the wealthy agent. This effect, coupled to the reduction in total savings, implies the growth rate of an economy with inequality is lower than that of an homogenous economy. Applying the same arguments to income transfers from the poor to the rich, increases in the degree of inequality lower the growth rate of the economy.

When the externality function is convex, Jensen's inequality makes the second term in (25) negative. The externality scales up the productivity of investments made by the wealthy agent. In this case the two effects described above work in opposite directions and the net effect on the growth rate is ambiguous. It is possible for the growth rate to increase when there is inequality. This possibility is more likely the lower the value of the parameter α, representing the relative importance of unskilled labor in production.

Low income economies may be assumed to have values of α close to 1, reflecting the fact that the goods produced are not highly sophisticated. For these economies the first term in the RHS of equation (25) is close to zero. For high income economies the human capital-labor ratio must be high, *i.e.*, α must be close to 1. In high income economies, the first term in (25) dominates even when the externality function is convex and the growth rate falls when there is inequality. In low income economies, α is close to zero, so that when the externality is convex the second term in (25) could be dominant, resulting in a higher growth rate in economies with inequality.

Appendix

Lemma 1. Suppose that initially (period 0) agent 1 has more human capital than agent 2, then in all subsequent periods he has more human capital than agent 2.

Proof: First we show –by contradiction– that in the first period agent 1 has more human capital than agent 2. Assume the opposite, i.e., that in the first period agent 2 has more human capital than agent 1, that is to say $h_1^2 \geq h_1^1$. The Euler condition (14) implies that $\varepsilon_1^2 / \varepsilon_0^2 \leq \varepsilon_1^1 / \varepsilon_0^1$. From the definition of ε_0^1, it follows that $\varepsilon_0^2 < \varepsilon_0^1$. Gathering these results we obtain:

$$\varepsilon_1^2 < \varepsilon_0^2 < \varepsilon_0^1 < \varepsilon_1^1 \quad \text{(A1)}$$

Given that in period 1 agent 2 has more human capital ($h_1^2 \geq h_1^1$), but consumes less than agent 2 ($\varepsilon_1^2 < \varepsilon_1^1$), it follows that in period 2 agent two has more human capital ($h_2^2 > h_2^1$),). By recursive reasoning we have that (i) ($h_t^2 > h_t^1$), and (ii) $\varepsilon_t^2 < \varepsilon_t^1$, for $t \geq 1$. From these relations we obtain that

$$\varepsilon_{t+1}^2 < \varepsilon_t^2 < \varepsilon_t^1 < \varepsilon_{t+1}^1 \quad \text{(A2)}$$

Thus, from period 1 onwards we have that agents 1 consumes more than agent 2 ($\varepsilon_t^2 < \varepsilon_t^1$), even thought the latter is wealthier from period 1 onwards ($h_t^2 > h_t^1$). This is a contradiction, because it means that agent 2 is not maximizing utility. We conclude that agent 2 has less human capital in the first period than agent 2, and thereby in each following period.

Q.E.D.

Lemma 2. Suppose that initially agent 1 has more human capital than agent 2. Then the former consumes more in all periods.

Proof: Assume that initially $\varepsilon_0^2 > \varepsilon_0^1$. From Lemma 1 we know that $h_1^2 < h_1^1$. This result, in conjunction with the Euler equation (14), implies that

$$\varepsilon_1^2/\varepsilon_0^2 > \varepsilon_1^1/\varepsilon_0^1 \quad \text{(A1)}$$

It follows that $\varepsilon_1^2 > \varepsilon_1^1$. By a recursive procedure analogous to the one used in Lemma 1 we have that $\varepsilon_t^2 > \varepsilon_t^1$, $t \geq 1$. Therefore, even agent 1 is wealthier than agent 2, he always consume less. This is a contradiction to the idea that agent 1 maximizes utility. Thus, agent 1 consumes more in period 0. From Lemma 1 we know that agent 1 has more human capital than agent 2 in all periods, it follows that he consumes more in all periods.

Q.E.D.

Corollary 1: The stock of human capital and consumption of both agents converge in the long run.

Proof: From Lemma 1 we know that $h_{t+1}^1 > h_{t+1}^2$. Furthermore equation (14) implies that $\varepsilon_{t+1}^2/\varepsilon_t^2 < \varepsilon_{t+1}^1/\varepsilon_t^1$.
From Lemma 2 we know that $\varepsilon_t^1 > \varepsilon_t^2$. Thus, given that $\varepsilon_t^1 + \varepsilon_t^2 = 1$, for all t

$$\varepsilon_t^2 < \varepsilon_{t+1}^2 < \varepsilon_{t+1}^1 < \varepsilon_t^1 \quad \text{(A4)}$$

Thus consumption of both agents converges overtime. Now equation A4, in conjunction with equation 14, implies that the stocks of human capital also become closer.

Corollary 2: $(h_t^1 - g_t^1) > (h_t^2 - g_t^2)$, $t \geq 0$.

Proof: It follows from Lemma 2 and the definition of ε_t.

Corollary 3: Agent 1 invests less than agent 2 in proportion to their respective stocks of human capital.

Proof: From corollary 1 it follows that

$$\frac{h_{t+1}^1}{h_t^1} < \frac{h_{t+1}^2}{h_t^2} \quad \text{(A5)}$$

replacing (10) in (A5) results in,

$$\frac{(\bar{h}_{t+1})^{1-\delta} (\rho h_t^1 + g_t^1)^\delta}{h_t^1} < \frac{(\bar{h}_{t+1})^{1-\delta} (\rho h_t^2 + g_t^2)^\delta}{h_t^2} \quad \text{(A6)}$$

Equation (A6) implies that

$$\frac{g_t^1}{h_t^1} < \frac{g_t^2}{h_t^2} \tag{A5}$$

Q.E.D.

Notes

1 Clarke (1992) has shown that the negative relation between income inequality and growth is robust in the sense of Extreme Bounds analysis. Clarke regressed growth on the so called Barro variables, inequality indexes and combinations of other variables. Even though the measures of inequality are not completelly satisfactory, the consistency of the results accross multiple specifications is convincing.

2 Guatemala is not included in the averages because of apparent data inconsistencies.

3 In Glomm and Ravikumar (1992), the externality appears because under the public schooling system, the quality of education is common to all. This externality leads to fast convergence of incomes in the case of public schooling.

4 Azariadis and Drazen (1991) have a human capital accumulation function that shows some similarities.

5 Bill Easterly has suggested that a displaced Cobb-Douglas utility function would lead to wealthy agents that save a higher proportion of their income. So long as the increased savings do not overwhelm the effects of the externality in human capital accumulation, our main results probably would survive. We have not attempted this specification, because the model becomes very complex when income is heterogenous.

6 Furthermore there is some casual evidence supporting the idea that the rich proportionally invest less in education. For instance, the education of the middle class is not obviously inferior to that of the wealthy.

7 Function u satisfies the conditions of Theorem 4.15 in Stokey, Lucas and Prescott (1989).

8 The same formulas apply when there is no externality in the production of human capital, *i.e.*, when δ equals 1.

9 Since in (18), $H_t - G_t = 0$ when the growth rate is $(1 + \rho)$.

10 The value function is increasing in δ. To see this, use the solution to (19) to obtain the value function as (assume L = 1)

$$V(h_t) = \sum_{t=0}^{\infty} \gamma^t (1-\alpha) \log (H_t - G_t) + C_1 = \sum_{t=0}^{\infty} (1-\alpha)\gamma^t \log \left[(H_0 - G_0)(1+\delta\rho)^t\right] + C_2$$

were C_i, $i = 1, 2$ are constants. This expression is maximized when $\delta = 1$.

11 Consider the problem of a social planer. The planner is aware that $ht = \bar{h}t$. In this case

$u(c_t) = a \log [(1+\rho) h_t - h_{t+1}]$

and the Euler equations satisfies:

$$\frac{1}{\varepsilon_t (H_t - G_t)} = \frac{\gamma(1+\rho)}{\varepsilon_{t+1}(H_{t+1} - G_{t+1})}$$

Therefore the economy's rate of growth is $(1+\rho)\gamma - 1$.

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