7 We became aware of work by Sundstrom and David [1988], which is very close to our model, after completing the work on which this paper is based. They consider a model of the family based on non-altruistic parents, and children who work for them. The young support the old in exchange for receiving their property when they die. The terms of the exchange are the result of bargaining between the two generations, implying, as in our model, that the bargaining power of the young is increased by better outside labor market opportunities.
The models are so similar, they should be consider complementary, though there are some differences. Sundstrom and David use a different solution concept and consider marginal childbearing costs. There are two sorts of differences in the results. First, their model stresses sibling rivalry, with parents "playing off" one child against another. Hence, bearing more children may increase a parent's bargaining power, while in our model it unambiguously lowers it because of the Nash bargaining solution. Second, they report that an increase in the outside earning opportunities of the young induces an unambiguous decline in the demand for children in the special case where parents maximize total wealth. Under more general assumptions about preferences, they find an ambiguous effect on the derived desire for children, in contrast to our model.

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CREDIT RATIONING AND THE PERMANENT INCOME HYPOTHESIS*

VICENTE MADRIGAL**
Stern School of Business
New York University

TOMMY TAN
Morgan Stanley, Singapore

DANIEL VICENT
Kellog School of Business
Northwestern University

SERGIO RIBEIRO DA COSTA WERLANG
Fundação Getulio Vargas

Abstract:

We model endogenous credit rationing and its effects on aggregate consumption behavior. We explain two important empirical paradoxes of the consumption literature. First, it is possible that some configurations of income/consumption streams be such that an outside observer could estimate a marginal propensity to consume greater than one. Second, if one tried to estimate an intertemporal substitution model disregarding credit constraints, it might result in a nonconcave utility function, even when consumers' utility functions are concave.

I. Introduction

A persistent goal of empirical economists has been the search for evidence in favour of the permanent income/life-cycle hypothesis (PIHLC). An equally persistent result has been the failure of economic data to support this hypothesis.[1] Strong evidence exists suggesting that a reason for this failure has been the presence of credit constraints.[2] However, there is as yet, no formal framework describing precisely how these market imperfections interact with rational consumer behavior. In order to test a

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revised PIHLC theory of consumption in the presence of credit-constraints, we need to discover the source of these constraints and to examine how they result from optimizing behaviour of lenders and borrowers.

Another goal of this paper involves the recent interest concerning the role of credit rationing in determining the severity of economic downturns.[3] This requires and explanation of how changes in "exogenous" variables worsens credit constraints in an economy. Most of the previous, investment based theories[4] do not provide a direct link between income or other observable variables) and credit rationing. Our model develops a connection between (initial) income, credit rationing and (through the derived marginal propensity to consume) the subsequent feedback to income. This allows one to perform comparative static exercises yielding testable conclusions for the model.

A simple life-cycle model is examined in which consumers desire a particular intertemporal pattern of consumption which is not coincident with their income stream. Consumers need to use the credit market to achieve their desired consumption pattern. Banks must confront the inevitable difficulty of distinguishing among consumers with differing income streams.

The adverse selection problem which results even when banks are allowed to offer any possible form of loan contract leads to the need for credit rationing.[5]

We consider a T-horizon (where T is large and arbitrary) overlapping cohorts world. Consumers live for two periods while lenders exist for T periods. Consumers may differ in the level of their second income (that is, high or low). Consumers borrow to increase first period consumption. Banks are profit maximizers and are allowed to compete for these loans by offering a broad range of contracts. Banks can appropriate payment from consumers (up to a bankruptcy level) provided consumers are able to pay. As long as there are significant proportions of both types of consumers and as long as their income levels are sufficiently different there is a unique subgame perfect equilibrium in pure strategies which entails credit rationing for the high income consumers.

This result has implications for the aggregate consumption function and the way it is affected by changes in income and prices. The empirical implications of the model are discussed in greater detail in Section IV. The most significant result is that there are configurations of income streams for which an outside observer could estimate a marginal propensity to consume of an individual much greater than one. This occurs because changes in income affect both desired consumption (on the demand side) and incentive constraints (on the supply side). A related result is that the consumption function will depend not only on the level of income but on the distribution of incomes as well. The more disparate the streams of income, the greater the adverse selection problem and, so, the greater the likelihood of credit rationing.

Another empirical implication is that an econometrician estimating and intertemporal substitution model disregarding credit constraints may predict a nonconcave utility function even when consumers' utility functions are actually concave.

II. The Model

The dynamic economy lasts for T (T finite) periods and consists of two banks and T–1 generations of consumers. Banks exist for all T periods. Each consumer lives for

two periods; the first is born in period one and each generation is comprised of N consumers.

There exist two types of consumers: a high (H) type and a low (L) type. Both types receive an income y in the first period of their lives. However, H receives and income y_H in the first period while L receives income y_L, $y < y_L < y_H$. The ex-ante probability that a consumer is a H type is π, and an L type is $1-\pi$. The probability for any individual is independent of the rest of the population. A consumer uses his income to purchase consumption goods and both types receive utility from the same underlying preferences represented by $U(c_1, c_2) = \log(a + c_1) + \delta \log(a + c_2)$, $a \geq 0$, $c_i > 0$. The variable a represents a lower limit on current period consumption below which no consumer can go.

There is a government bond which pays a risk free rate of interest in every period. Without loss for generality, we take this rate to be zero. Either the banks or the consumers may buy bonds for redemption in future periods. Only banks may borrow from the government and they may do so at this same rate of zero. Banks borrow from and lend to consumers. They are free to propose any profile of borrowing and lending contracts from the following class. Let b be the amount of consumption of a consumer (is excess of a) in period 1, then b–y represents the amount of a loan (is this is negative, the bank is borrowing from the consumer). A bank may offer in period t to lend to a young consumer i, the quantity b–y and demand repayment $(1 + r(b))(b - y)$ in period t+1. In the event of a loan, a bank has the right to appropriate in the beginning of t+1 all the assets of the consumer up to min $\{(1 + r(b))(b - y), y_i\}$ The bank may never leave a consumer with less than a dollars in any period. Banks offer a profile of contracts simultaneously. A profile can be represented symbolically by the set $\{(b, r(b))\}$, that is, the set of loans, b–y a bank is willing to make and the corresponding interest rate applied to the loan.

An adverse selection problem occurs when a bank has less accurate information about a consumer's future income than the consumer. We consider the extreme case in which the consumer is perfectly informed about his future earning stream while the bank knows only the ex ante probabilities (and whether or not the individual is old or young).

Consumers may borrow from or lend to banks according to the profile of offered contracts. (Only banks can offer contracts). Or consumers may choose not to accept any bank offer at all and either purchase government bonds or simply consume their income. Consumers do not observe the actions of their contemporaries but they know their own type and the profile of contracts offered by the banks in the period.

The model thus defines a finitely 'repeated' game with two banks and (T–1)N consumers. We allow all agents (banks and consumers) to condition actions on all previous observable actions. Strategies and Nash and subgame perfect equilibria of the game are defined in the standard way. A question which immediately arises is whether there exists equilibria in which banks (non–cooperatively) collude. The following section shows that this is not possible in this set-up.

III. The Equilibrium

This section illustrates the fact that when asymmetric information about the ability to pay back a loan exists, credit rationing may occur in a competitive loan market. Proofs of all lemmas and theorems are in the appendix.

Consider the one cohort model as described in Section II. Lemma One shows that in any pure strategy equilibrium, any loan offered is offered at a zero interest rate. If any separating occurs, therefore, it must be through differing loan amounts.

Let $\{(b_1, r(b_1)), (b_2, \rho(b_2))\}$ be a profile of first period consumption/interest rate contracts offered by the banks in some pure strategy equilibrium.

Lemma One: For any $b > 0$, $r(b) = 0 = \rho(b)$. That is, all loans offered and accepted in equilibrium are at zero interest rate.

The next step is to show that for a wide enough second period income spread, $(Y_H - Y_L)$, high types must be credit rationed.

First, we compute the optimal loan contract for either type under full information. This yields $5\ c_1^i = \frac{1-\delta}{1+\delta}\, a + \frac{1}{1+\delta}\,(y + y_i)$, for $i = L, H$. The optimal contract results in a first period consumption which is linear in total income. Note that the marginal propensity to consume out of income is the same for both types and is less than $(1/(1+\delta))$. Also, the response of consumption to a transitory shock (only y is affected) is less than to a permanent shock (both y and y_i are affected). For example, a shock which results in an ε rise in y only, induces a $2\varepsilon/(1+\delta)$ rise in first period consumption.

Consider now the situation where only the borrower knows his future income. If the full information, low income contract is available, low-income types gain utility $U^* = \log\{(1/(1+\delta)^{1-\delta}\,\delta^{\delta}\} + (1+\delta)\log(2a + y + y_L)$. If the high type loan $c^H_1 - y$ is less than y_L and L chooses the $(0, c^H_1)$ contract, then L would not default in the second period and would gain a strictly lower utility than his optimal contract. Since $c^H_1 - c^L_1 = (y_H - y_L)/(1+\delta)$, if y_H is close enough to y_L, both the high and the low types' optimal contracts can be offered in equilibrium with no fear of default.

Suppose, instead, that y_H, is large enough that $c^H_1 - y > y_L$ and that the utility L gets from borrowing $c^H_1 - y$ and then defaulting exceeds what he would gain from his optimal, full information contract $U^* < \log(a + c^H_1) + \delta\log a$. In this case, offering a profile of contracts $\{(0, c^L_1), (0, c^H_1)\}$ would necessarily lose money since low types would also choose $(0, c^H_1)$ and then default. Define b^* such that

$$U^* = \log(a + b^*) + \delta\log a. \qquad \text{(IC)}$$

Let $b^{**} = \min(b^*, c^H_1)$.

Lemma Two: There exists a $\pi^* < 1$ such that for all $\pi < \pi^*$, the unique pure strategy equilibrium is a profile of contracts $\{(0, c^L_1), (0, b^{**})\}$ where L types accept the first contract and high types accept the second.[6]

Consider now the full model with overlapping cohorts. Recall that our economy consists of T generations and 2T+1 periods. Theorem 1 states that the one generation –two period model has a unique Nash equilibrium (call it the one–shot equilibrium). Backward induction implies that the unique subgame perfect equilibrium in the period 2T (the penultimate period), banks will offer to the young agents the loan contracts specified by the one-shot equilibrium. That is, the one-shot equilibrium is the unique equilibrium of the subgame comprised of the last two periods. But this implies that in

period 2T–1, banks will also offer the one-shot equilibrium contracts. Since the contract offered by a bank does not affect its profits in the final two periods, a bank will offer the contract which is a best-response to the contract offered by the other bank. This best response pair is by the above proof, again, unique. Proceeding backward, we can establish that the unique subgame perfect equilibrium involves offering the one-shot equilibrium contracts to the young in periods 1, 2, 3, ..., T. In any period, on average, the aggregate consumption function is given by

$$\pi N(b^{**}) + (1-\pi)\,N(c^L_1) + \pi N(y_H - b^{**}) + (1-\pi)\,N(y_L - (c^L_1)$$

Comments: Note that when y_H is significantly larger than y_L, the incentive compatibility constraint on L types binds and H types are credit constrained in that the marginal utility of first period consumption is strictly greater than the marginal utility of second period consumption at zero interest rate. The actual amount of credit rationing is given by $c^H_1 - b^{**}$. The consequences for measured marginal propensity to consume are discussed in the next section.

IV. **Implications**

An economy in which credit constraints limit agents' abilities to smooth consumption will exhibit some distinctive features. Some of these features involve testable implications, others have strong implications for government policy.

1) The obvious empirical implication of a credit rationing model is its effects on the marginal propensity to consume (MPC). For a large class of instantaneous utility functions in the pure life-cycle model, the MPC out of permanent income is a constant less than one. The MPC out of 'transitory' incomes as noted by many authors should be substantially lower yet. With credit rationing restricting the ability to smooth consumption, however, these conclusions no longer hold. Observe, first, that in the model of Section II, the MPC of a person with higher permanent income should also be higher. Further, this MPC could potentially be much higher, in fact, possibly even greater than one. In addition, the effects of 'transitory' and 'permanent' income shocks on consumption for these people can in some cases be expected to be exactly the same.

To derive these conclusions explicitly, consider the effects of the following income shock: an economy-wide shock increases current period income by y. Low income consumers are not credit-constrained. Their consumption is a linear function of income.

$$c^1_L = \frac{1-\delta}{1+\delta}\, a + \frac{1}{1+\delta}\,(y + y_L)$$

Consumption in the first period rises by $1/(1+\delta)\ y$.[7]

The high income consumers may be credit-constrained, however. The marginal utility of first period consumption strictly exceeds the marginal utility of second period consumption. Given this, we would certainly expect their MPC to be at least one. To see that it could exceed one, note that the level of first period consumption of high-types under credit-constraints is completely determined by IC, the incentive compatibility constraint. This is, after manipulation,

$$c^H_1 = \left(2 + \frac{y + y_L}{a} \right)^{\delta} \delta (1+\delta)^{\delta} - a \quad \text{so that}$$

$$\text{MPC} = \left\{ \delta (1+\delta) \left(2 + \frac{y + y_L}{a} \right) \right\}^{\delta}$$

This exceeds one if $\delta > a/(a + y + y_L)$. The reason why consumption may rise more than one for one with income is that the income of low types affects the constraint binding the high-types' ability to borrow. An economy-wide rise in income relaxes that constraint as well as providing direct liquidity to the high-income consumers. Of course, if the income shock affects only the high-types we would expect the IC constraints to remain the same and the expected MPC = 1 would obtain.

The differing effects of permanent versus transitory income shocks also depends on whether the shock is personal or economy-wide. In this model, a transitory shock is implied by a change in this period's income only, y becomes y + h and the high type is that given above. A permanent shock is reflected by an expected change in future income as well: for example, $y + y_L$ becomes $h + y + y_L + h$, say. For an economy-wide permanent shock, an observed current increase of y would lead to an MPC (with respect to y) of $2/(1+\delta)$ for the low-types and $2 \left\{ \left[\frac{\delta}{1+\delta} \right] \left[2 + \frac{y + y_L}{a} \right] \right\}^{\delta}$ for the high types. An economy wide permanent shock leads to a stronger MPC than an economy-wide transitory shock.

On the other hand, personal income shocks to high-types, whether transitory or permanent, will generate an MPC of one in either case –the only effect due to the direct increase in liquidity.

These conclusions point to potential tests for the presence of credit constraints. Life-cycle data which indicate significant differences in MPCs between future high-income earners and future low-income earners should be observed in the presence of these constraints. Differing reactions to economy-wide and personal shocks would also be evidence either in terms of its effects on permanent income or on transitory income.[8]

A recent paper by Campbell (1988) suggests another test of the presence of credit-constraints. Campbell notes that an implication of the PIH is that current savings behavior is a good predictor of future income streams –the feature he terms, 'saving for a rainy day'. Lower future income was signalled by an increase in saving and higher income by current dissaving. Our model suggests that there may be an asymmetry of response if dissaving is made up in part by borrowing. Credit constrained agents may not increase their borrowing if their expected increase in income cannot be verified. The reserve behaviour, of course, is not similarly constrained and we might expect longitudinal panel data to reflect this asymmetric response. In the aggregate, one might expect this presence to weaken the predictive power of savings. Disaggregated data might, then, lead Campbell's conclusions to be even stronger.

A final test is suggested by the qualitative features of the model. A society with a relatively equal distribution of income will be expected to encounter fewer problems of credit-constraints than one with greater disparity of income. The same is true for a high-income country vis-a-vis a low-income country. This is simply because the IC constraints are only binding if the first-best solution for the high-types involves a

substantially higher degree of consumption shifting than for the low-types. If high-types are relatively similar to low-types the latter always prefer their first-best, full-information outcome to borrowing like a high-type and suffering the penalties of default. The institutional restrictions on interstate banking might lead this effect to be discernible among different states in the U.S. Drawing on the earlier analysis, we might expect that those states with more unequal distributions of income to exhibit great MPCs in the aggregate.

The model also has implications concerning the possible divergence of observed with predicted intertemporal substitution. Consider the following very simple case. Let P_1 be the current period price for consumption goods and let P_2 be the perfectly predicted price for consumption goods tomorrow. If we ignore borrowing constraints, consumers will consume according to

$$\delta (a + c_2) / (a + c_1) = P_1/P_2, \; P_1c_1 + P_2c_2 = y + y_i, \qquad (1)$$

where c_1 is consumption today and c_2 is consumption tomorrow. Consider an increase in tomorrow's price to $P'_2 = P_2 + K$. Neoclassical demand theory predicts a shift from future to current consumption expenditure by the amount (normalizing $P_1 = 1$).

$$c'_1 - c_1 = P_2c_2 - P'_2c_2 = [(a + y) / \delta D - aP_2/D] - [(a + y) / \delta D - aP_2/D)]$$

$$= a (P_2 - P) / D, \text{ where } D = \delta (1 + \delta).$$

However, if consumers are credit constrained under the configuration of prices (P_1, P_2) then under (P_1, P'_2) a first period consumer will not be able to raise first period expenditure (he is already at the boundary of his expenditure possibilities set, given the credit constraints). Thus with credit constraints, $(c'_1 - c_1) = 0$.

This result has two empirical implications. First, tests of the equations in (1), assuming (correctly) an intertemporal, logarithmic utility function may reject the intertemporal substitution hypothesis. Second, if using equation (1) we try to estimate the value of γ in the utility function

$$U (c_1, c_2) = c_1^{1-\gamma} / (1 - \gamma) + \delta c_2^{1-\gamma} / (1 - \gamma)$$

we will predict that γ is non-positive; that is, we will predict that U is not concave. This observation is relevant to the results presented in Mankiw, Rotemberg and Summers (1986).

2) Policy Implications: The policy implications of the constraints on consumption shifting are well-known and are mentioned here only for completeness. The obvious point is that an economy with credit-constraints of the type described above will exhibit higher MPCs than a pure life-cycle/permanent income hypothesis model suggests, either out of permanent income or out of transitory income. As a result, the possibilities for demand management via consumption-directed policies are greater. Furthermore, Ricardian equivalence arguments which claim that deficit spending has no real effect since in has no effect on permanent income may no longer hold. Deficit spending which is successful in shifting income from future periods to the current period will have a real and positive effect insofar as it allows high-types. Permanent income for either types may remain unchanged: the deficit spending, by altering the income stream, reduces the need for consumption shifting.

The larger MPCs implied by the presence of credit constraints also have implications for the stability of an economy with respect to real shocks. The analysis above has shown that the MPC of consumers may be very large. A sudden fall in per capita income reduces immediate liquidity and tightens access to credit markets as well. The breakdown in credit markets in the early 1930's may have been a reflection of this effect. In any economy where demand shocks are important, because of sticky prices for example, the resulting fall in consumption could have reinforcing effects.

Appendix

Prof: Note that any point in the (r,b) the slope of the indifference curve of a high type which passes through (r, b), $\frac{dr}{db} \Big|_{H} (r, b)$ is greater than that of the low type for all (r,b) such that $(1+r)(b-y) < y_L$. (No default). From the profile of contracts, choose the contract which yields the highest profits per high type. Call it $A = (r_A, b_A)$. Suppose $r_A > 0$.

Case I: $(1 + r_A)(b_A - y) \leq y_L$. Since

$$\frac{dr}{db} \Big|_{H} (r, b) > \frac{dr}{db} \Big|_{H} (r_A, b_A),$$

there is a point A' in the neighbourhood of A which H types prefer to A and L types do not. All can be chosen arbitrarily close to A to yield profits arbitrarily close to those of high type. Consider the firm which is attracting at most half of the H types at A. Suppose it continues to offer the same contract profile as before but also offers A'. By assumption, all types weakly prefer their current contracts to A and only H types strictly prefer A' to A. The firm, by offering and attracting all H types, increases is profits on those types and, thus, its total profits. This would break the equilibrium.

Case II: $(1 + r_A)(b - y) > y_L$. If at the current profile L types strictly prefer other contracts to A, a deviating firm only needs to offer r' slightly lower than r_A to attract all the H types to its loans. So suppose some L types would buy A (and hence default). Note that at A, L types indifference curves are locally flat in r.

That is, $\frac{dr}{db} \Big|_{L} (r, b) = \infty$. (For any b, a higher r is irrelevant since they will default anyway). For H types, $\frac{dr}{db} \Big|_{H} (r,b)$ is finite so there exists a slightly lower b and r which H types strictly prefer to A and again the same argument as in Case I holds. Cases I and II show that there is no contract which can make a positive profit from the H types, that is H types gain loans at an interest rate of zero. Suppose some contract makes a positive profit on the L type. Standard undercutting arguments apply to show that this can not be an equilibrium outcome either (the lowest profiting firm would offer the same loan at a slightly lower rate). Thus, loans for each type must make zero profits which implies that each type must gain loans with an interest rate of zero.

Proof:

Define $P = \{(r, b) \log (a + b) + \delta \log (a + y_H - (1 + r)(b - y) > \log (a + b^{**}) + \delta \log(a + y_H - (b^{**} - y)$ and $(1 + r)(b - y) \leq y_H\}$, the set of (r, b) pairs which are weakly preferred by H to $(0, b^{**})$. Define π^* so that

$$\pi^* = \inf \{\pi \mid \pi\, r' (b' - y) - (1 - \pi)(y_L - (b' - y)) < 0, \text{ for all } (r', b') \in {}^{-}P\}.$$

Note that if $b^{**} < c^H_1$, then P is non-empty and that for any $r' > 0$, since $b' - y > y_L$, there always exists a π' such that $\pi'\, r' (b' - y) - (1 - \pi')(y_L - (b - y) < 0$. Therefore, $\pi^* 0 <$ is always defined. (Note that for $b^{**} < c^H_1$, $\pi^* < 1$ can also be shown). Observe also that if $b^{**} c^H_1$, $P = \emptyset$.

Assume a pure strategy equilibrium exists. Lemma One shows that it must be of the form (b,0), *i.e.*, all loans must offer an interest rate equal to the government rate.

Step One: Case I: $b^{**} = c^H_1$. Suppose one bank offers a profile for contracts which does not include c^L_1. Find the contract offered which is closest to, say, c^L_1 call it η, $|c^L_1 - \eta| > 0$. Since c^L_1 represents a unique maximum, for L types, the rival bank could offer a loan with a slightly positive interest rate and a consumption amount between η and c^L_1 and make strictly positive profits. But this violates Lemma One. The similar argument is made for c^H_1.

Case II: $b^{**} = b^*$. Suppose a bank offers a profile which does not include $(0 , b^*)$. (The case for c^L_1 is the same as in Case I). If it offers a contract with $1 > b^*$, both L and H types accept. Since L types will default, this contract loses money and is sub-optimal. If the highest loan contract is at a consumption amount $\eta < b^*$, the rival company can always offer a pair (r', η') with $r' > 0$ such that H are better off than at $(0, \eta)$ and L types still prefer at least the $(0, \eta)$ pair. Such a contract would be profitable and thus violates Lemma One.

Step Two: Step One shows that if a pure-strategy profile of contracts exist, the profile must include $\{(0, c^L_1), (0, b^{**})\}$. If $b^{**} = b^*$, then no consumption amount greater than b^* is offered. We now show that for $\pi < \pi^*$, any such profile of contracts offered by both firms is, in fact, an equilibrium.

Suppose a rival bank is offering $\{(0, c^L_1), (0, b^{**})\}$ as part of his profile of contracts. If a deviating bank offers some other contract which is weakly preferred (r', b') then

i) If $b^{**} = c^H_1$, then the only reason any type would weakly prefer the contract to the current profile is because they would default and so the contract would lose money. Therefore suppose $b^{**} = b^*$ (*i.e.*, H is constrained).

ii) If L weakly prefers any contract to c^L_1, it is because L defaults, therefore all such contracts lose money on L types.

iii) If $b^{**} = b^*$, so that L types are just indifferent between c^L_1 and choosing the default loan b^*, any (r', b') which is weakly preferred by the H types to $(0, b^*)$ is strictly preferred by the L types. (Since $b' > b^*$ and because they are defaulting anyway, the interest rate r' is irrelevant). Therefore any deviating contract which attracts high types always attracts all the low types.

Points i), ii), and iii) imply that any deviating contract (r', b') attracts H and L types in proportion $\rho : 1 - p$ with $\pi < \pi^*$ and the contract loses money on L types. For the deviation to be profitable, then some H types must be attracted, or in other words, (r', b'), must be an element of P.

But π satisfies $\pi r' (b'-y) - (1-\pi)(y_L - (b'-y)) < 0$. Since the left-side is increasing in π and since $\pi < \pi^*$, any deviating contract must lose money. Therefore the profile $\{(0, c^L_1), (0,b^{**})\}$ is a best response to itself and any profile which includes those pairs of contracts and any other contracts (r, b) such that no type prefers (r,b) to either $(0, c^L_1)$ or $(0, b^{**})$ is an equilibrium profile.

Notes

1 See, for example, Hayashi (1982, 1985), Hall and Mishkin (1982), Mankiw, Rotemberg and Summers (1985), Flavin (1981), Sargent (1978), Mariger (1987), Campbell (1987).

2 For example, Hayashi (1982, 1985), Hall and Mishkin (1982), Mariger (1987) and Flavin (1981).

3 See Bernanke (1983), Blinder and Stiglitz (1983), Jackling (1985), Scheinkman (1984) and Scheinkman and Weiss (1986), for example.

4 See Stiglitz and Weiss (1981), Jaffe and Russell (1976), and Mankiw (1987), for example. See Bernanke and Gergler (1987), Jacklin (1985), and Greenwald and Stiglitz (1987) for further discussions on alternative approaches to this problem.

5 The idea that adverse selection in the credit market could lead to credit rationing is, of course, not a new one. Stiglitz and Weiss (1981), Bernanke and Gertler (1986), Mankiw, Jaffe and Trussell, incorporate this idea in their models. To our knowledge, however, no study has yet examined the explicit connection between adverse selection, consumption behaviour, and income.

6 Note that this equation is relatively robust to the choice of the instantaneous utility function. For any u (•) such that u (•) is homogenous of any degree γ not equal to zero (positive or negative, first period consumption is a linear function of life-time income).

7 The requirement $\pi < \pi^*$ is needed for the same reason that a pure strategy equilibrium may not exist in the Rothschid-Stiglitz model. Even when this condition is violated, an application of a result by Dasgupta/Maskin (1986) can be used to show that a mixed strategy equilibrium with similar features exists.

8 It is straightfoward to show that L accept only $(0, c^L_1)$ and H accept only.

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