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INTERTEMPORAL RESOURCE ALLOCATION AND INCOME TAX EVASION

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Abstract:

The discrimination against saving, in favour of present consumption, due to the income tax has been studied, at least, since the late thirties and was mentioned by John Stuart Mill more than a century ago.
This paper is concerned with the effect of evading such a tax on the discrimination against savings and capital accumulation. In particular, we want to study a situation in which the probability of detecting an evader is an increasing function of his accumulated evasion in the past. This is consistent with the tax authorities being stricter in the control of tax payers with a relatively high net wealth, with respect to the incomes declared.

1. Introduction

The discrimination against savings, in favour of present consumption, due to the income tax has been studied, at least, since the late thirties and was mentioned by John Stuart Mill more than a century ago[1].

Moreover, it is well known that such a tax distorts the labor market, as it reduces the equilibrium quantity below what it would have been in the presence of a lump sum tax of equal revenue[2]. The possibility of evading the income tax could compensate this substitution effect on the labor supply and demand, thus improving resource allocation and welfare[3].

This paper is concerned with the effect of the possibility of evasion on the discrimination against savings and capital accumulation due to the income tax. In

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particular, we study a situation in which the probability of detection at every point in time is an increasing function of the evader's accumulated evasion in the past.

Needless to say, we follow the standard procedure in excess burden analysis of ignoring the income effect (presumably positive) of evading the tax, to concentrate on the substitution effect of evasion on the path of capital accumulation. Accordingly, this paper assumes that there exist neutral taxes, whose revenues are adjusted to keep fiscal revenue constant independently of income tax evasion.

The literature on tax evasion is relatively scarce on the determinants of the probability of detection. The simplest procedure, of course, is to assume that this probability is an exogenous parameter. Allingham and Sandmo (1972) made the assumption that this probability is a decreasing function of the income declared. However, in the section of their paper devoted to the dynamic analysis of evasion they adopt the assumption of an exogenous probability.

We assume, instead, that the probability of detection at every point in time is an increasing function of the evader's accumulated evasion. This is consistent with a behaviour of the tax authority enforcing a stricter control on tax payers with a high wealth, relative to the incomes declared. Our assumption makes the probability of detection endogenous in a dynamic model; that is, the probability of detection becomes a state variable which depends on the trajectories of the control variables.

2. The Model

Following the literature on this topic[4], we assume that the objective to be maximized is the expected value of utility; that is, the economy behaves so as to maximize

$$\int_0^\infty E\,[U\,(C\,(t))]\,e^{-\delta t}\,dt \tag{1}$$

where U [C (t)] denotes utility at every point in time as a function of consumption, C (t), such that

$$C\,(t) = f\,[K\,(t)] - \underline{K}. \tag{2}$$

$$K\,(0) = K_0.$$

Also, δ is the rate of time preference (which is assumed constant over time and exogenously given, for simplicity); f [K (t)] is total output at time t, as a linearly homogeneous function of capital at time t, K (t). An underlined variable denotes its derivative with respect to time; in particular $\underline{K}\,(t) = dK\,(t)/dt$.

It is also assumed that:

$$f' > 0, \qquad f'' < 0, \qquad U' = dU/dC > 0, \qquad U'' = d^2U/dC^2 < 0.$$

The boundary conditions are:

$$\lim_{c\,(t)\to 0} U\,[C\,(t)] = -\infty$$

$$\lim_{c\,(t)\to 0} U'\,[C\,(t)] = \infty$$

$$\lim_{c\,(t)\to \infty} U'\,[C\,(t)] = 0$$

The constraints to the maximization problem above are as follows: Let Π (t) be the probability of detection in period t. If the evasion is detected the evader must pay the tax evaded in t plus a penalty equal to P times the tax evaded in t (P > 1). Therefore, we can rewrite (1) and set the problem as follows:

$$\text{Max} \int_0^\infty [(1 - \Pi\,(t))\,U^s\,(C^s\,(t)) + \Pi\,(t)\,U^f\,(C^f\,(t))]\,e^{-\delta t}\,dt \tag{4}$$

subject to equations (3), (5), (6) and (7):

$$C^s\,(t) = f\,(K\,(t))\,[1 - T\,(1 - \alpha\,(t))] - \underline{K}^s\,(t) + Z^s\,(t). \tag{5}$$

$$C^f\,(t) = f\,(K\,(t))\,[1 - T\,(1 + P\alpha\,(t))] - \underline{K}^f\,(t) + Z^f\,(t). \tag{6}$$

where T denotes the income tax rate, and the index s denotes the value of the variable if the evasion is succesful (undetected), and f denotes the value of the variable if the evasion fails (i.e., it is detected). We assume that risk aversion prevails, which implies $U^{s\prime} < U^{f\prime}$. Moreover, α (t) stands for the fraction of income over which the tax is evaded. Accordingly, α (t) is an additional control variable since we now assume that the economy maximizes utility with respect to capital accumulation and tax evaded in each period. Also Z (t) denotes neutral transfers which compensate for the income effect of the tax; that is, $Z^s\,(t) = T\,(1 - \alpha\,(t))\,f\,(K\,(t))$ and $Z^f\,(t) = T\,(1 + P\alpha\,(t))\,f\,(K\,(t))$, although from the point of view of the maximizing entity Z^f (t) or Z^s (t) are considered exogenous constants.

Finally, we assume that the probability of detection in period t, Π (t), depends on the accumulated evasion up to this period, E (t); that is,

$$\Pi\,(t) = \Pi\,[E\,(t)] = \Pi\left[\int_0^t TfK\,(i)\,a\,(i)\,di\right],\ \Pi' > 0. \tag{7}$$

The state variables in this problem are K (t) and Π (t) and the control variables are $\underline{K}$ (t) and $\underline{\Pi}$ (t) or, equivalently, $\underline{K}$ (t) and α (t) since $\underline{\Pi}\,(t) = \Pi'\,\alpha\,(t)\,f\,(K)\,T$.

Replacing (5), (6) and (7) in (4) and calling H(t) the objective function we write the problem as:

$$\text{Max} \int_0^\infty H\,(t)\,dt$$

The first order conditions are[5]:

$$\partial H/\partial K = d\,(\partial H/\partial \underline{K})/dt. \tag{8}$$

$$\partial H/\partial \alpha - d\,(\partial H/\underline{\partial\partial})/dt. \tag{9}$$

To simplify notation we will omit the arguments of the functions; thus we will write f in the understanding that it really means f [K (t)], or Π which means Π [E (t)], etc.

From (8) and (9) we get,

$$T\alpha f' \{\Pi' (U^f - U^s) + (1 + T) [U^{s\prime} (1 - \Pi) + \Pi U^{f\prime}]/\alpha T + U^{s\prime} (1 - \Pi) - U^{f\prime} P \Pi\} = \quad (10)$$

$$= \delta [U^{s\prime} - \Pi (U^{s\prime} - U^{f\prime})] + [\Pi' f \alpha T (U^{s\prime} - U^{f\prime}) + \Pi (\underline{U}^{s\prime} - \underline{U}^{f\prime}) - \underline{U}^{s\prime}],$$

$$\Pi' (U^f - U^s) + U^{s\prime} (1 - \Pi) - U^{f\prime} P \Pi = 0. \quad (11)$$

Equation (11), of course, indicates that the equilibrium level of evasion requires the expected value of the marginal utility per dollar evaded, $U^{s\prime} (1 - \Pi)$, to be equal to the "marginal cost" per dollar evaded. Such a "marginal cost" has two components: $U^{f\prime} P \Pi$, the marginal disutility of the expected value of the penalty; and $\Pi' (U^f - U^s)$, the increase in the probability of being detected and losing $(U^s - U^f)$ in the future.

Equation (11) allows us to determine the conditions for tax evasion to exist; that is, the conditions under which:

$$U^f < U^s, \quad (12)$$

$$U^{s\prime} < U^{f\prime}. \quad (13)$$

Given (12) and (13), let:

$$U^s - U^f = \gamma > 0,$$

$$U^{f\prime}/U^{s\prime} = \beta > 1.$$

Therefore (11) can be written as:

$$-\Pi' (\gamma/U^{f\prime}) + (U^{s\prime}/U^{s\prime}) (1 - \Pi) = \Pi P,$$

$$-(\Pi'/\Pi) (\gamma/U^{f\prime}) + (1/\beta) (1 - \Pi)/\Pi = P$$

$$(1 - \Pi)/\Pi > P$$

$$\Pi < 1/(1 + P).$$

This result coincides with Allingham and Sandmo's equation (6') and indicates that evasion will take place (i.e., α (t) > 0) as long as Π (t) remains below $1/(1 + P)$. Likewise, we see from equation (11) that evasion disappears; i.e., $U^s = U^f$ and $U^{s\prime} = U^{f\prime}$, when $\Pi = 1/1 + P$.

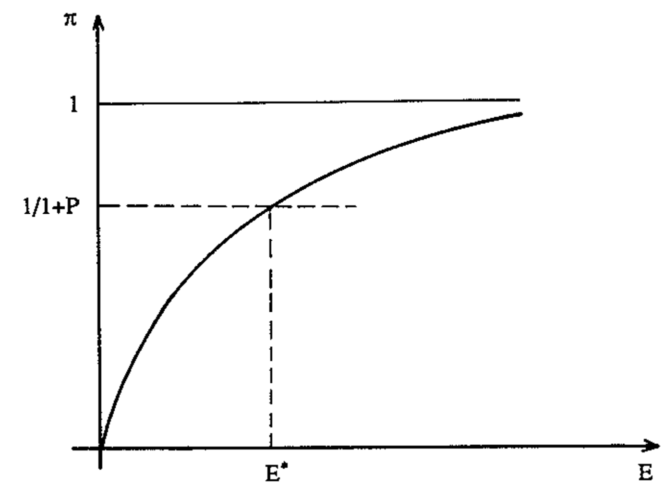
Figure 1 shows the behaviour of evasion for the function $\Pi [E (t)] = E (t)/1 + E(t)$.

Equation (11) indicates that evasion will take place as long as accumulated evasion remains below E^*.

Moreover, we can show that optimal evasion decreases as Π increases[6]: Let us call J the left hand side of equation (11). We calculate:

$$d\gamma/d\Pi = -(\partial J/\partial \Pi)/(\partial J/\partial \gamma) = -(U^{s\prime} + U^{f\prime} P)/\Pi' < 0.$$

FIGURE 1



Or, equivalently,

$$d\beta/d\Pi = -(\partial J/\partial \Pi)/(\partial J/\partial \beta) = -(P + \beta^{-1})/(1 - \Pi)\beta^{-2} < 0.$$

These results are consistent with the observation that tax evasion falls percentage-wise as countries grow richer and approach their steady states[7]. But one could also argue that this is rather a result of better control procedures, which cannot be afforded by poorer countries. It is, of course, possible that the observed behaviour of evasion may be due to both, the phenomenon described by this model, and the fact that richer countries can afford better evasion control systems.

If we replace equation (11) into equation (10), we get:

$$[\delta - (1 - T)f'] [U^{s\prime} (1 - \Pi) + \Pi U^{f\prime}] = \underline{\Pi} (U^f - U^{s\prime}) + \Pi (\underline{U}^{f\prime} - \underline{U}^{s\prime}) + \underline{U}^{s\prime}. \quad (14)$$

We also know that

$$\Pi = \Pi' \alpha f T. \quad (15)$$

From equation (5) we get[8]:

$$\underline{K} = f (K) - g (U^{s\prime}), \text{ where } g (U^{s\prime}) = C^s. \quad (16)$$

The steady state requires $\underline{K} = 0 = \Pi$. This implies, from equation (15) that $\alpha^* = 0$; i.e., no evasion takes place in the steady state. Therefore, $U^f = U^s$, $U^{f\prime} = U^{s\prime}$, $\underline{U}^{f\prime} = \underline{U}^{s\prime}$, in the steady state.

Moreover, from equation (16), $g(U^{s\prime}) = f(K^{*TE})$, where K^{*TE} denotes the capital stock in the steady state in the presence of both an income tax at rate T and the possibility of evading it. Therefore, consumption remains constant in the steady state, equal to $f(K^{*TE})$, hence $\underline{U}^{s\prime} = 0$. Therefore, the right hand side of equation (14) is zero in the steady state. This, in turn, shows that the economy converges to the same steady state stock of capital, with or without tax evasion.

$$(1 - T)\, f'(K^{*T}) = \delta = (1 - T)\, f'(K^{*TE}),$$

that is,

$$K^{*T} = K^{*TE},$$

where K^{*T} is the steady state stock of capital in the presence of an income tax at rate T without the possibility of evasion.

This result as well as the absence of evasion in the steady state, $\alpha^* = 0$, depend critically on the assumption that the accumulated evasion does not depreciate over time as a determinant of Π. If the tax payers could count on a certain degree of forgetfulness by the tax authority, then α^* would be positive and K^{*T} would be lower than K^{*TE}.

The path towards the steady state, on the contrary, does depend on the existence of evasion:

The right hand side of equation (14) is $\underline{E(U')}$ and we know that $\delta - (1 - T) f'$ is the rate of change, along the optimal path, of the marginal utility in the case without evasion and of the expected marginal utility in the case with evasion. Also notice that

$$\lim_{\Pi \to 0} E(U') = U^{s\prime},$$

$$\lim_{\Pi \to 1/(1+P)} E(U') = U'.$$

But $U^{s\prime}$ and U' are the marginal utilities associated to C^s and C^T; i.e., the consumption levels that prevail without evasion and with taxes at rates $T(1 - \alpha)$ and T respectively.

That is to say, the path of E (U') approaches, for low levels of accumulated evasion, the path of $U^{s\prime}$ and consequently, the path of consumption approaches the path of consumption which would exist in the absence of evasion and with a tax rate of $T(1 - \alpha)$. As evasion accumulates (i. e., as Π approaches $1/1 + P$), the path of E(U') approaches the path of U' and consequently, the path of consumption approaches the path of consumption which would prevail in the absence of evasion and with a tax rate of T.

The intuitive explanation of the above results is as follows: When accumulated evasion is insignificant, Π approaches 0, thus economic agents behave as if the tax rate were $T(1 - \alpha)$ as they evade a fraction α of their taxes. As accumulated evasion increases Π increases, thus α decreases and the economic agents, who behave as if the tax rate were $T(1 - \alpha)$, perceive this phenomenon as an increase in the tax rate. This process continues up until Π reaches the value $1/1 + P$, when evasion stops, $\alpha = 0$, and the economic agents finally find themselves subject to the tax rate T.

More formally, the possibility of evasion induces the economic agents to think of the tax rate effectively paid as a random variable of expected value $E(T) = (1 - \Pi)(1 - \alpha)T + \Pi(1 + \alpha P)T$). Then

$$\lim_{\Pi \to 0} E(T) = (1 - \alpha)\, T$$

$$\lim_{\Pi \to 1/1+P} E(T) = T$$

As E (T) approaches T, the paths of consumption and capital accumulation shift according to the successive changes in the expected value of the tax rate. This phenomenon can be depicted as successive shifts of the line XY in Figure 2, until this line finally reaches the position of line ZW. Likewise the paths EF and AB set the limits within which the optimal paths will be located according to the successive optimal values of evasion.

Capital accumulation and consumption follow paths that shift over time from the path EF towards the path AB. These shifts occur always in the same direction because $\underline{\Pi} \geq 0$, thus it is never optimum to evade a fraction of taxes greater than the fraction evaded in the preceding period.

Figure 3 depicts the consumption path under three alternative assumptions: The full line indicates the path of consumption in the absence of income tax, C (t); the broken line denotes the consumption path under an income tax at rate T without evasion, $C^T(t)$; and the dotted line represents the path of consumption under an income tax at rate T with evasion as described in this paper, $C^{TE}(t)$.

Given a low initial value of Π, the economy will initially follow a path close to EF, hence initial consumption is less than C^T0. Given that the economy converges to a steady state independent of the existence of evasion, it follows that the path $C^{TE}(t)$ increases over time at a rate higher than the growth rate of the path without evasion, $C^T(t)$.

FIGURE 2

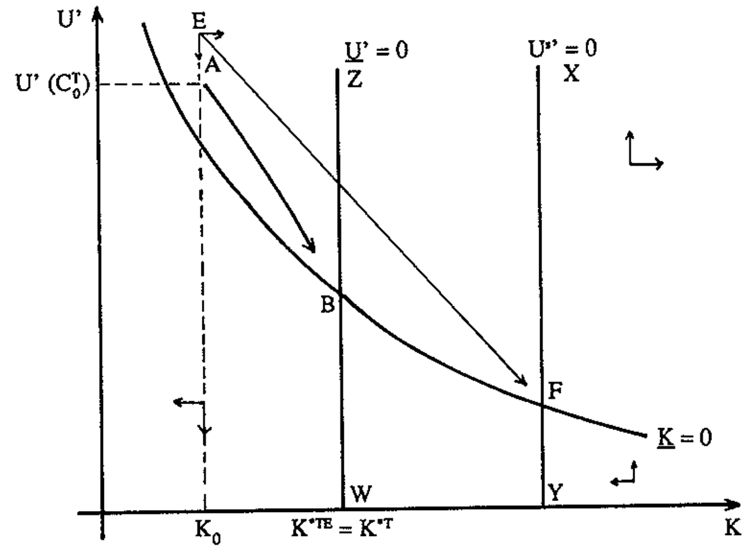
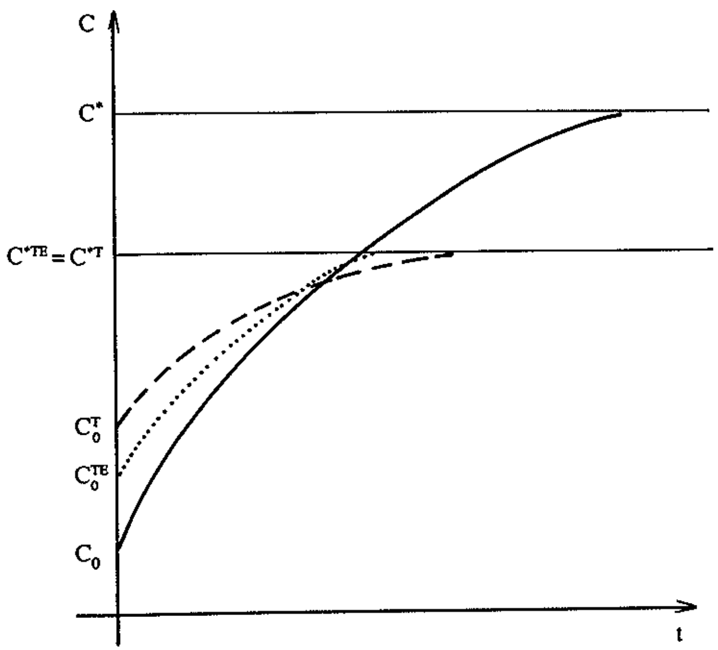


FIGURE 3



3. Summary and Conclusions

We start out from the well known result that the income tax distorts capital accumulation and present consumption. The effects of the possibility of evasion upon that result is the topic of this paper. Thus, we assume that evasion comes out of a maximization process over time, and that the probability of detection in time t is an increasing function of accumulated evasion previous to t. If evasion is detected in t, the penalty consists of having to pay the tax evaded in t plus P times such amount (P > 1). There is no penalty for taxes evaded in the past; the only role of past evasion is to determine the probability of being caught in the present or in the future[9]. This model can represent a behaviour of the tax authorities being stricter on tax payers with a relatively high net wealth despite consistently low declared incomes.

The results of the model are: A) Evasion will take place as long as the expected value of marginal utility per dollar evaded exceeds (1) The expected value of marginal disutility of the current penalty, plus (2) the increase of the expected value of marginal disutility of future penalties. This condition is fulfilled as long as Π remains below 1/1 + P. B) The fraction of income that attempts to evade the tax is non-increasing over time and disappears as Π equals 1/1 + P. C) This is equivalent to impose an income tax at a non-decreasing rate, which approaches a stable value as evasion ceases. D) In the steady state there is no evasion and the economy converges to a stock of capital equal

to the one in the model without evasion. E) Therefore, in the steady state, the kind of evasion studied in this paper does not affect the income tax discrimination against savings. F) On the other hand, the paths of consumption and capital accumulation are twisted by the kind of evasion we studied. It creates an incentive to accumulate more capital, *vis-à-vis* the case without evasion. Such an incentive vanishes as the economy approaches the steady state. G) As a consequence of the previous results it follows that the economy approaches the steady state faster with evasion than without it.

Notes

1 Irving Fisher (1939), John Stuart Mill (1965).
2 Ian M. D. Little (1951), Arnold C. Harberger (1964).
3 Laurence Weiss (1976).
4 See, for instance, M. G. Allingham and Sandmo (1972); V. Christiansen (1980) and S. C. Kolm (1973).
5 The fulfillment of the second order conditions is assured by the concavity of functions U and f and by the fulfillment of the transversality conditions.

$$\lim_{t\to\infty} e^{-\delta t} U'(C(T))\,k(t) = 0$$

$$\lim_{t\to\infty} e^{-\delta t} U'(C(t))\,\Pi(t) = 0.$$

6 This result coincides with Allingham and Sandmo's. They conclude, on the basis of a rather different model that an optimizing evader will gradually reduce the fraction of tax evaded.
7 The author is grateful to an anonymous referee for this comment.
8 It is a matter of indifference to use equation (5) or (6), since in case of taking equation (5) $Z^a(t) = f[K(t)]\,T(1-\alpha(t))$ and in case of taking equation (6) $Z^f(t) = f[K(t)]\,T[1 + P\alpha(t)]$. In any case $C(t) = f[K(t)] - \underline{K}(t)$.
9 This is the opposite of what Allingham and Sandmo (1972) call "myopic Behaviour" which ignores that present evasion involves mortgaging the future.

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