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GOVERNMENT POLICY AND AGGREGATE FLUCTUATIONS

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Abstract:

This paper investigates the effect of including fiscal and monetary variables in a simple real business cycle model. The starting point is the stochastic growth model with fuctuations driven by technological shocks. The growth model is first extended to include government spending and taxes on factors of production. A second extension imposes a cash-in-advance constraint on purchases of consumption goods. For the three models, equilibrium decision functions and predictions for second moments are compared.

1. Introduction

One of the main findings of Kydland and Prescott (1982) is that standard growth theory correctly predicts the amplitude of business cycle fluctuations, and the serial and cross-correlations of many aggregate variables. What is perhaps surprising is the fact that the model they use to support this conclusion includes no fiscal or monetary variables. The driving force of the fluctuations is instead a process governing technological change.

There are several recent papers that investigate the role of fiscal and monetary variables in explaining cyclical behavior[1]. This research is motivated in part by the fact that while the model of Kydland and Prescott (1982) does well in accounting for certain salient features of the data, it does not do well along all dimensions[2]. For example, all of the variability of output cannot be explained by a technology shock taken to be the "residual" of the production function. And if the variance on this residual is set sufficiently high to account for variability in output, other aggregates such as consumption and hours still vary too little. Another problem with the standard model is that relative to the data, the correlation between real wages and hours is too high. The technology shock causes fluctuations in the demand for labor with little effect on labor supply. The result is a high and positive correlation between wages and hours. Therefore, one reason for adding the fiscal and monetary factors is to account for the stylized facts not matched by the standard model Given that households are willing to substitute intertemporally and intratemporally, fluctuations in such variables as tax rates can imply a greater variability in consumption and leisure. What is also true is that the fiscal and monetary variables have an effect on labor supply and can therefore potentially account for comovements in wages and hours.

The purpose of this paper is to provide a general framework that can be used to illustrate the aggregate role of monetary and fiscal variables. The framework without taxes or money (the "benchmark" model) is a simple version of the stochastic growth model. This model is extended in two ways. In the first extension, households pay taxes on their labor and capital income with tax revenues used by the government to purchase some fraction of total output. In the second extension, factor taxes are ignored but the government issues currency. The currency must be used by households for the purchase of nonstorable consumption goods[3]. Because the two models with government variables nest the benchmark model, there is a way to quantify the contribution of the fiscal or monetary factors.

To analyze the effect of government policy, decision rules and predictions for second moments are compared. To make the comparison, an equilibrium for each economy is computed. Because the same algorithm is applied to the three economies, a general formulation of the problem and solution are provided. In each case, it is shown how to implement the model within the general framework.

The rest of the paper is organized as follows. In Section 2, a method for constructing the optimal decision functions is described. In Section 3, the algorithm is applied to the benchmark model which is a simple stochastic growth model. The model of Section 4 is an extension of the benchmark model in which households pay taxes on capital and labor income. The model of Section 5 is an extension of the benchmark model in which households are constrained to purchase consumption goods with cash. In Section 6, decision functions and results of time series simulations are reported for alternative parameterizations of the models. Concluding remarks are given in Section 7.

2. A General Framework

In the three sections that follow, competitive equilibria are computed by solving an optimization problem of the form.

$$\max_{ut} E\left[\sum_{t=0}^{\infty} \beta^t r(x_{1t}, x_{2t}, u_t) | x_{10}, x_{20}\right] \tag{1}$$

subject to

$$x_{1t+1} = A_{11}x_{1t} + A_{12}x_{2t} + B_1 u_t + \epsilon_{t+1} \tag{2}$$

where x_{10}, $\{x_{2t}\}_{t=0}^{\infty}$ are given, $E\epsilon_t = 0$, $E\epsilon_t\epsilon'_t = \Sigma$, x_{1t}, x_{2t}, and u_t are real-valued vectors, and r is a scalar, real-valued function. It is assumed that the solution of this optimization satisfies side constraints of the form

$$x_{2t} = \Theta x_{1t} + \Psi u_t \tag{3}$$

and $\lim_{j\to\infty} x_{1t+j} = 0$.

In the case that the return function r is quadratic ($r(x_1, x_2, u) = x'Qx + u'Ru$, $x = [x'_1, x'_2]$)[4] the optimal controls are given by[5]

$$u_t = -\beta R^{-1} B'_1 P (I + \beta(B_1 + A_{12}\Psi)R^{-1}B'_1 P)^{-1} (A_{11} + A_{12}\Theta)x_{1t}. \tag{4}$$

The matrix P is the solution to the associated Riccati difference equation and can be constructed in a non-recursive way as follows. The first order conditions of (1) imply $y_t = Hy_{t+1}$ where $y = [x'_1, \mu']'$, μ is the vector of Lagrange multipliers for the constraints in (2), and

$$H = \begin{bmatrix} \hat{A}^{-1} & \sqrt{\beta}\hat{A}^{-1}\hat{B}R^{-1}B'_1 \\ \hat{Q}\hat{A}^{-1} & \sqrt{\beta}(\hat{Q}\hat{A}^{-1}\hat{B}R^{-1}B'_1 + A'_{11} - Q_{12}\Psi R^{-1}B'_1) \end{bmatrix} \tag{5}$$

where Q has been partitioned so that the dimensions of Q_{12} are the same as that of A_{12} and $\hat{A} = \sqrt{\beta}(A_{11} + A_{12}\Theta)$, $\hat{Q} = Q_{11} + Q_{12}\Theta$, $\hat{B} = \sqrt{\beta}(B_1 + A_2\Psi)$. The matrix H can be diagonalized as $H = V\Lambda V^{-1}$ with the eigenvalues of $\Lambda = \text{diag}(\Lambda_1, \Lambda_2)$ arranged so that those in Λ_1 are outside the unit circle and those in Λ_2 are inside the unit circle. Assuming that $\mu_t = Px_{1t}$, the construction of P follows from the condition $\lim_{t\to\infty} x_{1t}$. That is, $P = V_{21}V_{11}^{-1}$ is the solution to the difference equation in y that shuts down the explosive roots of Λ.

If the return function is not quadratic but the optimal controls are well approximated by linear functions of the states, then $r(x_1, x_2, u)$ can be replaced by an approximate quadratic function. Following Kydland and Prescott (1982), the approximation is found by taking a second-order Taylor expansion of r around the vectors $\bar{x}_1$, $\bar{x}_2$, and $\bar{u}$ that satisfy

$$\begin{aligned} \frac{\partial r(\bar{x}_1, \bar{x}_2, \bar{u})}{\partial \bar{u}} + B'_1(I - \beta A'_{11})^{-1}\beta\frac{\partial r(\bar{x}_1, \bar{x}_2, \bar{u})}{\partial \bar{x}_1} &= 0 \\ \bar{x}_1 - A_{11}\bar{x}_1 - A_{12}\bar{x}_2 - B_1\bar{u} &= 0 \\ \bar{x}_2 - \Theta\bar{x}_1 - \Psi\bar{u} &= 0 \end{aligned} \tag{6}$$

Equation (6) follows from the first order conditions in the nonstochastic case. In some cases, a linearization of constraints may also be necessary. For the examples considered below, it is not.

3. The Benchmark Model

A simple version of Kydland and Prescott (1982) serves as a benchmarck to illustrate the effects of fiscal and monetary variables. Consider a single-good economy in which the representative household maximizes its expected lifetime utility

$$E_0 \left[\sum_{t=0}^{\infty} u(c_t, l_t) \right] \tag{7}$$

by choosing streams of consumption, investment, and leisure, $\{c_t, i_t, l_t\}_{t=0}^{\infty}$, to satisfy the following sequence of budget constraints

$$c_t + i_t \leqslant r(s_t)k_t + w(s_t)n_t, \quad t \geqslant 0 \tag{8}$$

where i_t is date t investment, r and w are price functions for capital and labor, s_t is the economy-wide state at t, k_t is the capital stock at t and n_t is the labor allocation at t. Maximization of (7) is subject to budget constraints (8), constraints on the allocation of the total time

$$n_t + l_t = 1, \quad t \geqslant 0, \tag{9}$$

and the technological constraint for accumulation of capital

$$k_{t+1} = (1-\delta)k_t + i_t, \quad 0 \leqslant \delta \leqslant 1, \quad t \geqslant 0. \tag{10}$$

It is assumed that houscholds behave competitively taking as given the functions r, w and the distribution governing s.

On the firm side, assume that profits are maximized each period with prices taken as given. That is, firms choose output, y_t, and inputs to maximize

$$y_t - r(s_t)k_t - w(s_t)n_t \tag{11}$$

given the technological constraints

$$y_t = f(k_t, n_t, \lambda_t), \quad t \geqslant 0. \tag{12}$$

The production function f is assumed to be a function of physical inputs and a given level of technology, λ_t. The process for the technology shock is assumed to be given by

$$\lambda_{t+1} = a + b\lambda_t + cw_{t+1}, \quad E\omega_t = 0, \ E\omega_t^2 = 1 \quad t \geqslant 0. \tag{13}$$

An equilibrium in this economy consists of functions $c(s)$, $k'(s)$, $n(s)$, $r(s)$, and $w(s)$, for $s = (k, \lambda)$, such that

(i.) if households choose $\hat{c}$, $\hat{n}$, and $\hat{k}'$ that solve

$$v(\hat{k}, s) = \max_{\hat{c}, \hat{n}, \hat{k}'} \left\{ u(\hat{c}, l - \hat{n}) + \beta \int v(\hat{k}', s')\, dG(\lambda' | \lambda) \right\}$$

subject to

$$k' = f(k, n(s), \lambda) + (1-\delta)k - c(s)$$

$$\hat{k}' = (r(s) + 1 - \delta)\hat{k} + w(s)\hat{n} - \hat{c},$$

and conditional distribution function G, then $\hat{c} = c(s)$, $\hat{n} = n(s)$, $\hat{k}' = k'(s)$;

(ii.) if firms choose $\hat{k}$, $\hat{n}$ to maximize the profit function $f(\hat{k}, \hat{n}, \lambda) - r(s)\hat{k} - w(s)\hat{n}$,

then $\hat{k} = k, \hat{n} = n(s)$.

Before implementing the solution procedure outlined in Section 2, consumption and leisure in $u(c, l)$ are replaced by $r(s)k + w(s)n - i$ and $1 - n$. Because the prices of inputs in equilibrium are equal to marginal products, $r(s)$ and $w(s)$ are replaced by $\partial f/\partial k|_{\kappa(s), \eta(s), \lambda}$ and $\partial f/\partial n|_{\kappa(s), \eta(s), \lambda}$, respectively, where κ and η are per-capita quantities of capital and labor[6]. Therefore the optimization problem to be solved to compute an equilibrium is

$$\max_{\{i_t, n_t\}} E \left[\sum_{t=0}^{\infty} u(f_1(\kappa_t, \eta_t, \lambda_t)k_t + f_2(\kappa_t, \eta_t, \lambda_t)n_t - i_t, 1 - n_t) | k_0, \lambda_0, \kappa_0, \eta_0 \right]$$

subject to (10), (13), and $\{\kappa_t, \eta_t\}_{t=0}^{\infty}$ given. For the franmework of Section 2, this implies

$$x_{1t} = [k_t, \lambda_t, 1]', \quad x_{2t} = [\kappa_t, \eta_t]', \quad u_t = [i_t, \eta_t]'$$

and

$$x_{1t+1} = \begin{pmatrix} 1-\delta & 0 & 0 \\ 0 & b & a \\ 0 & 0 & 1 \end{pmatrix} x_{1t} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} x_{2t} + \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} u_t + \begin{pmatrix} 0 \\ c\omega_{t+1} \\ 0 \end{pmatrix} \tag{14}$$

from the transition equation (2). The side constraints in this case are

$$x_{2t} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} x_{1t} + \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} u_t \tag{15}$$

which follow directly from the conditions $\kappa_t = k_t$ and $\eta_t = n_t$ imposed by the definition of equilibrium.

To obtain the matrices Q and R for the approximate objective function, the steady state for $z = [x_1', x_2', u']'$ is computed and a Taylor expansion of u around this vector is constructed. For the benchmark model, the steady state is $\bar{z} = (\bar{k}, a/(1-b), 1, \bar{k}, \bar{n}, \delta\bar{k}, \bar{n})$ where $\bar{k}$, $\bar{n}$ satisfy

$$u_2\left(f(\bar{k},\bar{n},\frac{a}{1-b}) - \delta\bar{k}, 1-\bar{n}\right) - u_1\left(f(\bar{k},\bar{n},\frac{a}{1-b}) - \delta\bar{k}, 1-\bar{n}\right) f_2 \qquad (16)$$

$$(\bar{k},\bar{n},\frac{a}{1-b}) = 0,$$

$$\beta(1-\delta) + \beta f_1(\bar{k},\bar{n},\frac{a}{1-b}) - 1 = 0.$$

The second-order expansion is, therefore,

$$u(f_1(\kappa,\eta,\lambda)k + f_2(\kappa,\eta,\lambda)n - i, 1-n)$$

$$= U(z)$$

$$\approx U(\bar{z}) + \left.\frac{\partial U}{\partial z}\right|_{\bar{z}}(z-\bar{z}) + \frac{1}{2}(z-\bar{z})' \left.\frac{\partial^2 U}{\partial z^2}\right|_{\bar{z}}(z-\bar{z})$$

$$= z'\Bigg[e(U(\bar{z}) - \left.\frac{\partial U}{\partial z}\right|_{\bar{z}}\bar{z} + \frac{1}{2}\bar{z}'\left.\frac{\partial^2 U}{\partial z^2}\right|_{\bar{z}}\bar{z})e'$$

$$+ \frac{1}{2}(e\left.\frac{\partial U}{\partial z}\right|_{\bar{z}}' + \left.\frac{\partial U}{\partial z}\right|_{\bar{z}}e' - e\bar{z}'\left.\frac{\partial^2 U}{\partial z^2}\right|_{\bar{z}} - \left.\frac{\partial^2 U}{\partial z^2}\right|_{\bar{z}}\bar{z}e' + \left.\frac{\partial^2 U}{\partial z^2}\right|_{z})\Bigg] z$$

$$= z'\begin{pmatrix} Q & W \\ W' & R \end{pmatrix} z, \qquad z = [x', u'] \qquad (17)$$

where e is a vector of zeros with the exception of element three which is equal to one.

With a specification for the transition equation (A_{11}, A_{12}, B_1), the equilibrium conditions (Θ, Ψ), and the approximate objetive function (Q, R, W) found in (14), (15) and (17), the optimal investment and labor supply functions can be computed as in Section 2. After rewriting the problem without cross-products, the matrix H is constructed and diagonalized. With the eigenvectors of H, P is constructed. The rules for investment and hours at work are then given by (4)[7].

Because the competitive equilibrium and the social optimum are equivalent for this model, it turns out that the decision functions can also be found by first substituting (15) into the objective function and rewriting the problem without per-capita variables κ and η. In fact. H is identical in the two cases.

4. Taxation on Factors of Production

If taxes are imposed on factors of production in the benchmark economy, the households face budget constraints

$$c_t + i_t \leqslant (1-\tau_t)r_t k_t + (1-\varphi_t)w_t n_t + \delta\tau_t k_t + \zeta_t, \quad t \geqslant 0 \qquad (18)$$

where the tax rate on capital and labor income are τ and φ, respectively. Adding the term $\delta\tau_t k_t$ assumes that households are not taxed on the depreciated part of capital. The term ζ_t is a government transfer. To keep the problem simple, the government is assumed to transfer to the households all revenues not used for the purchases of goods. Therefore, the government budget constraint in t is

$$\alpha_t y_t + \zeta_t = \tau_t(r_t - \delta)k_t + \varphi_t w_t n_t \qquad (19)$$

where α_t is the fraction of output consumed by the government in t. Let $\nu_t = (\lambda_t, \alpha_t, \tau_t, \varphi_t)$ be the vector of exogenous state variables. Assuming that this vector process is governed by a first-order autoregressive process, the law of motion for ν is given by

$$\nu_t = a + b\nu_{t-1} + c\omega_t, \quad E\omega_t = 0, \; E\omega_t\omega_t' = I. \qquad (20)$$

With these extensions, the definition of equilibrium is given as follows. An equilibrium consists of functions $c(s)$, $k'(s)$, $n(s)$, $r(s)$, and $w(s)$, for $s = (k, \lambda, \alpha, \tau, \varphi)$, such that

(*i.*) if households choose $\hat{c}$, $\hat{n}$, and $\hat{k}'$ that solve

$$v(\hat{k}, s) = \max_{\hat{c},\hat{n},\hat{k}'} \left\{u(\hat{c}, 1-\hat{n}) + \beta \int v(\hat{k}', s')\, dG(\nu'|\nu)\right\}$$

subject to

$$k' = (1-\alpha)f(k, n(s), \lambda) + (1-\delta)k - c(s)$$

$$\hat{k}' = (r(s)-\delta)(\hat{k} - \tau\hat{k} + \tau k) + w(s)(\hat{n} - \varphi\hat{n} + \varphi n) +$$

$$\hat{k} - \alpha f(k, n(s), \lambda) - \hat{c},$$

and conditional distribution function G for $\nu = (\lambda, \alpha, \tau, \varphi)$, then $\hat{c} = c(s)$, $\hat{n} = n(s)$, $\hat{k}' = k'(s)$;

(*ii.*) if firms choose $\hat{k}$, $\hat{n}$ to maximize the profit function $f(\hat{k}, \hat{n}, \lambda) - r(s)\hat{k} - w(s)\hat{n}$, then $\hat{k} = k$, $\hat{n} = n(s)$.

With marginal products substituted in for the prices on inputs, the household's optimization problem is

$$\max_{\{i_t, n_t\}} E\Bigg[\sum_{t=0}^{\infty} u\Big(f_1(\kappa_t,\eta_t,\lambda_t)(k_t - \tau_t k_t + \tau_t\kappa_t) + f_2(\kappa_t,\eta_t,\lambda_t) \qquad (21)$$

$$(n_t - \varphi_t n_t + \varphi_t\eta_t) - i_t - \alpha_t f(\kappa_t,\eta_t,\lambda_t) + \delta\tau_t(k_t - \kappa_t), 1 - n_t\Big)$$

$$k_0, \nu_0, \kappa_0, \eta_0\Bigg]$$

subject to (10), (20), and the sequence $\{\kappa_t, \eta_t\}_{t=0}^{\infty}$. An application of the framework of Section 2 implies

$$x_{1t} = [k_t, \lambda_t, \alpha_t, \tau_t, \varphi_t, 1]', \quad x_{2t} = [\kappa_t, \eta_t]', \quad u_t = [i_t, n_t]'$$

and

$$x_{1t+1} = \begin{pmatrix} 1-\delta & 0 & 0 & 0 & 0 & 0 \\ 0 & b_{11} & b_{12} & b_{13} & b_{14} & a_1 \\ 0 & b_{21} & b_{22} & b_{23} & b_{24} & a_2 \\ 0 & b_{31} & b_{32} & b_{33} & b_{34} & a_3 \\ 0 & b_{41} & b_{42} & b_{43} & b_{44} & a_4 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} x_{1t} + \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} u_t + \epsilon_{t+1}$$

where A_{12} is a matrix of zeros and $\epsilon_t = [0, \omega_t' c', 0]'$. The matrices for the side constraints, (Θ, Ψ), are again given by (15) since $k_t = \kappa_t$ and $n_t = \eta_t$ in equilibrium.

To obtain the matrices Q and R for the approximate objective function, the steady state for $z = [x_1', x_2', u']'$ is computed and a Taylor expansion of u around this vector is constructed. For the model with taxes, the steady state is $\bar{z} = (\bar{k}, \bar{\lambda}, \bar{\alpha}, \bar{\tau}, \bar{\varphi}, 1, \bar{k}, \bar{n}, \delta\bar{k}, \bar{n})$, where $\bar{v} = (I - b)^{-1}\, a$ and $\bar{k}, \bar{n}$ satisfy

$$u_2\,((1 - \bar{\alpha})\, f(\bar{k}, \bar{n}, \bar{\lambda}) - \delta\bar{k},\ 1 - \bar{n}) - (1 - \bar{\varphi})\, u_1\,((1 - \bar{\alpha}) f(\bar{k}, \bar{n}, \bar{\lambda}) -$$

$$\delta\bar{k},\ 1 - \bar{n})\, f_2(\bar{k}, \bar{n}, \bar{\lambda}) = 0$$

$$\beta(1 - \delta + \delta\bar{\tau}) + \beta(1 - \bar{\tau}) f_1(\bar{k}, \bar{n}, \bar{\lambda}) - 1 = 0$$

Note that this is equivalent to (15) if $\alpha = 0$. As in the benchmark case, the coefficient matrices for the quadratic objective function can be found by the approximation in (17). In this case, $U(z_t)$ is the period t return of preferences specified in equation (21).

As a test case, consider setting a and b so that the tax rates and government purchases are zero and the technology shock is parameterized as in the benchmark case. The decision rules and predictions for time series in this example are equivalent to those of the benchmark model.

5. Money via a Cash-In-Advance Technology

In the monetary economy[8], households enter period t with nominal money balances of m_{t-1}. At the beginning of the period, these money balances are augmented by a lump-sum government transfer of $M_t - M_{t-1}$, where M_t is the per-capita money supply in period t. Assuming that households must purchase the nonstorable consumption goods (c_t) with previously acquired real money balances, the following "cash-in-advance" constraint must hold

$$p_t c_t \leqslant m_{t-1} + (\tilde{g}_t - 1) M_{t-1} \qquad (22)$$

where p_t is the price level at t and $\tilde{g}_t$ is the growth rate of the money stock in t (*i, e,* $M_t/M_{t-1} = \tilde{g}_t$)[9]. Assuming that investment and leisure are "credit" goods, the budget constrainsts in (8) are replaced by

$$c_t + i_t + m_t/p_t \leqslant r(s_t) k_t + w(s_t) n_t + (m_{t-1} + (\tilde{g}_t - 1) M_{t-1})/p_t,$$
$$t \geqslant 0. \qquad (23)$$

In this case, households maximize (7) subject to budget constraints (23), cash-in-advance constraints (22), technological constraints (9) and (10), and the transition functions for λ and $\tilde{g}$. The functions r, w, and the distribution for s are assumed to be given.

As in the benchmark and tax models, the exogenous states are assumed to follow a first-order autoregressive process. With these specifications, an equilibrium can be defined. An equilibrium in the monetary economy consists of functions $c(s)$, $k'(s)$, $n(s)$, $r(s)$, $w(s)$, $\mu'(s)$, and $\rho(s)$ for s $= (k, \lambda, g)$, such that

(*i.*) if households choose $\hat{c}, \hat{n}, \hat{k}'$, and $\hat{\mu}'$ that solve

$$v(\hat{k}, \hat{\mu}, s) = \max_{\hat{c}, \hat{n}, \hat{k}', \hat{\mu}'} \left\{ u(\hat{c}, 1 - \hat{n}) + \beta \int v(\hat{k}', \hat{\mu}', s')\, dG(\nu' | \nu) \right\}, \quad \nu = (\lambda, g)$$

subject to

$$k' = f(k, n(s), \lambda) + (1 - \delta) k - c(s)$$

$$\hat{k}' = r(s)\hat{k} + w(s)\hat{n} + (1 - \delta)\hat{k} - \hat{\mu}'/\rho(s)$$

$$\hat{c} = \frac{\hat{\mu} + \exp(g) - 1}{\exp(g)\, \rho(s)}$$

and conditional distribution function G for $\nu = (\lambda, g)$, then $\hat{c} = c(s)$, $\hat{n} = n(s)$, $\hat{k}' = k'(s)$, $\hat{\mu}' = \mu'(s)$;

(*ii.*) if firms choose $\hat{k}, \hat{n}$ to maximize the profit function $f(\hat{k}, \hat{n}, \lambda) - r(s)\hat{k} - w(s)\hat{n}$, then $\hat{k} = k, \hat{n} = n(s)$; and

(*iii.*) $\mu(s) = 1$.

Note that m_t/M_t and p_t/M_t have been replaced by μ and ρ in the above definition. Therefore, the condition $\mu_t = 1$ implies that the money balances of the representative household is equal to the per-capita level.

Using the definition of equilibrium, the states and controls can be defined as follows

$$x_{1t} = [k_t, \mu_{t-1}, \lambda_t, g_t, 1, \kappa_t]', \quad x_{2t} = [\rho_t, \iota_t]', \quad u_t = [i_t, \mu_t]'$$

where ι_t is per-capita investment at date t and $g_t = \ln(\tilde{g}_t)$. Assuming that λ_t and g_t are described by a first-order autoregressive process, the law of motion for x_1 is given by

$$x_{1t+1} = \begin{pmatrix} 1-\delta & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & b_{11} & b_{12} & a_1 & 0 \\ 0 & 0 & b_{21} & b_{22} & a_2 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1-\delta \end{pmatrix} x_{1t} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} x_{2t} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} u_t - \epsilon_{t+1}$$

where $\epsilon_t = [0, 0, \omega'_t c', 0, 0]'$. As the problem is specified, elements of the matrix Θ corresponding to the price equation cannot be set *a priori*. Therefore, an iterative approach to the problem of constructing P is followed instead. In equilibrium, ρ and ι are functions of $s = (\lambda, g, \kappa)$. For the linearized model, this relationship is of the form $x_{2t} = \Omega x_{1t}$ with all elements in the first and second columns of Ω equal to zero. For some guess of Ω the law of motion for x_2 is given by

$$x_{2t+1} = \Omega(A_{11}x_{1t} + A_{12}x_{2t} + \epsilon_{t-1})$$

With these equations explicitly specified, the optimization problem can be solved without side constraints using one of the methods described in McGrattan (1990a)[10]. The result is a decision function of the form $u_t = -F[x'_{1t}, x'_{2t}]$. If the conditions $k_t = \kappa_t, i_t = \iota_t$, and $\mu_t = 1$ are imposed on this solution, ρ_t and ι_t can be written as a function of λ_t, g_t, and κ_t which in turn implies some linear relationship between x_{2t} and x_{1t}. The new function relating x_2 and x_1 may or may not be equal to Ω. If it is, the algorithm is converged. If it is not, the new function is used for the next iteration.

6. Results of Simulations

For the three models described above, investment is computed as a linear function of the states. If equilibrium conditions hold, investment can be written as a function of the vector s, which in the three cases implies.

$$i_t = f_0 + f_1 k_t + f_2 \nu_t, \tag{24}$$

for some computed coefficients f_0, f_1, and f_2. The vector ν is set equal to λ for the benchmark case, $(\lambda, \alpha, \tau, \varphi)$ in the model with taxes, and (λ, g) in the model with money. Substituting (24) into (9) then implies a law of motion for $s = (k, \nu'\ 1)$ of the form

$$s_{t+1} = \begin{pmatrix} 1-\delta+f_1 & f_2 & f_0 \\ 0 & b & a \\ 0 & 0 & 1 \end{pmatrix} s_t + \epsilon_{t+1} \tag{25}$$

where $\epsilon_t = [0, \omega'_t c', 0]'$. With the computed coefficients of (24), an initial condition s_0, and a realization of ω_t, the equation (25) can be simulated. If this is done many times, time series statistics and their standard deviations can be computed.

The first step towards simulating the economies is to choose functional forms and parameter values. For all of the reported results, the following choices of preferences and technologies are used:

$$\begin{aligned} v(c, l) &= \ln(c) + \gamma l, & \gamma &= 2.86 \\ f(k, n, \lambda) &= e\lambda k^{\theta} n^{1-\theta}, & \theta &= 0.36 \\ k' &= (1-\delta)k + i, & \delta &= 0.025 \end{aligned} \tag{26}$$

The choice of utility function is the same as that used by Hansen (1985)[11] to model indivisible labor choice. It is used here because it implies greater variability of hours in the benchmark case than can be captured by a Cobb-Douglas specification[12].

In Table 1, various specifications of a, b, and c are used to demonstrate the different effects that can be achieved by adding the tax rates, the fraction of government spending to output, or the growth of the money stock[13]. All cases reported assume the functional forms in (26) and that ω_t is normally distributed. At the end of the table, parameterizations of a, b, c are given. All elements not provided are set to zero. For example in case (1), only the technology shock is nonzero and has law of motion $\lambda_{t+1} = .95\lambda_t + \epsilon_{t+1}$ where ϵ is normally distributed with mean 0 and variance $(.0073)^2$.

In each panel the numbers in parentheses (first column) correspond to the parameterization reported at the end of the table. Case (1) is the benchmark case since all government variables are set to zero. Case (2) is an example with the technology shocks and tax rates set to zero but the fraction of government spending in output is assumed to be on average 20% and serially correlated. For this case, the expenditures are financed by lump-sum taxes ($\zeta = \alpha_t y_t$). Case (3) assumes that the technology shock, government spending, and the tax rate on labor are zero. The tax rate on capitalis approximately 40% and serially correlated. Case (4) is like case (3) but with taxes on labor being positive. The tax rate on labor is set so as to have a mean of 20%. For case (5), all of the state variables enter the decision rule since all of the diagonal elements of b and c are nonzero.

The rows marked "US" are standard deviations and correlations of aggregate US time series that are first logged and detrended[14]. Some of the US statistics reported here differ from those reported in Kydland and Prescott (1982) and Cooley and Hansen (1989) because of choices of data. However, for all choices of data, a comparison of the benchmark case (1) with the US statistics reveals that consumption varies too little and the correlation between productivity (real wages) and output is too high.

The decision rules for investment are given in the first panel of Table 1. The response of investment to changes in capital, the technology shock, and tax rates are as expected. In the first case, the decision function is exactly that obtained for the model without taxes or government spending. The standard deviations and correlations with output are reported in the second and third panels. The simulated series are also logged and detrended and are, therefore, percentage deviations. In all cases, the elements of c are chosen to make the variance of output for the artificial economy close to that of the US economy. Notice how drastically different are the implications for the standard deviations of consumption, investment, capital, and hours and the correlation between wages and output. When α is the only shock, the variability in consumption and hours increases significantly, the variability in investment and capital decreases significantly, and the wages are perfectly negatively correlated with output. In this case, the labor supply function is shifting with changes in government spending while the labor demand function is fixed. What cases (1) and (2) indicate is the *potential* of the technology shock and the government spending shock to match the cleven statistics reported. For the case that only τ is nonzero., there is a large and significant increase in all standard derivations. It is clear that there is a lot of intertemporal subsistution in response to the capital tax. As with the shock to α, the correlation between productivity and output is

TABLE 1

DECISION FUNCTIONS, STANDARD DEVIATIONS IN PERCENT, AND CORRELATIONS WITH OUTPUT FOR US AND MODEL WITH TAXES

Investment Rule: $i_t = f_0 + f_1 k_t + f_2 v_t,\ v = (\lambda, \alpha, \tau, \varphi)$

	f_0	f_1	f_2			
(1)	0.67	−0.033	1.78			
(2)	0.88	−0.033		0.12		
(3)	0.45	−0.017			−0.26	
(4)	0.76	−0.033				−1.14
(5)	0.69	−0.017	1.25	−0.066	−0.27	−0.99

Standard Deviations

	y	c	i	k	n	y/n
US	1.81	0.91	5.11	0.45	1.52	1.32
(1)	1.81	0.52	5.90	0.49	1.38	0.52
(2)	1.81	2.26	0.57	0.05	2.83	1.02
(3)	1.83	1.14	14.41	1.10	2.85	1.13
(4)	1.80	0.52	5.80	0.48	2.81	1.03
(5)	1.81	0.94	5.39	0.46	1.65	0.61

Correlations with Output

	c	i	k	n	y/n
US	0.81	0.64	0.65	0.70	0.57
(1)	0.86	0.99	0.06	0.98	0.87
(2)	1.00	0.99	0.05	1.00	−1.00
(3)	−0.84	0.95	0.12	0.98	−0.84
(4)	0.88	0.99	0.07	1.00	−0.96
(5)	0.94	0.97	0.05	0.94	0.41

(1): $a = [0,0,0,0],\ b_{11} = .95,\ c_{11} = .0073$
(2): $a = [0,.01,0,0],\ b_{22} = .95,\ c_{22} = .0145$
(3): $a = [0,0,.02,0],\ b_{33} = .95,\ c_{33} = .073$
(4): $a = [0,0,0,.01],\ b_{44} = .95,\ c_{44} = .0085$
(5): $a = [0,.01,.02,.01],\ b = .95I,\ c_{11} = .0068,\ c_{22} = .003,\ c_{33} = .003,\ c_{44} = .003.$

near-1 since the taxes affect labor supply. In case (4), with only the tax on labor nonzero, the statistics are very close to that of the benchmark case. The only differences are in the variability of hours and productivity, and the correlation between wages and output. It may be the case that with a different utility function, the tax on labor could imply a "fit" to the data comparable to that of the technology shock. The main difference, however, is the effect on the correlation of hours and wages. What cases (1) and (4) indicate is that neither variable can explain all of the second moments.

For case (5), all of the variables in v are nonzero. The point of this case is to show that the fit can be improved when all four variables are considered. Notice in particular, that relative to case (1), consumption is more variable and the productivity-output correlation is not as high. Note also that the parameters are not chosen to maximize any metric. They are chosen only so as to illustrate the *potential* importance of the different variables.

A similar exercise is performed for the monetary economy. Three of the cases considered by Cooley and Hansen (1989) were run and the results are reported in Table 2. Cases (1) and (2) assume a constant growth rate. For $\bar{g} = log\ (.99)$, the result is the benchmark case (*i.e.* deflate at the optimal rate). Therefore, the decision rule in (1) corresponds exactly to the decision rule in (1) of Table 1. The second case assumes a higher rate of growth of the money stock. Notice however that the higher rate does not affect greatly the time series statistics. There is only a small (but significant) change in consumption. This result also holds for the case that g is serially correlated. It also holds for cases with cross-correlations in λ and g (not reported here).

For case (4), the technology shock is set to zero and the variance of g is set so that the artificial output series is as variable as that in the data. In this case there is far too much variation in the other aggregate series. However, this case implies that the model has the potential to fit the correlation between real wages and hours. Case (5) attempts to exploit this by reducing the variance in the technology shock and increasing the variance in the growth of money. What the result of (5) implies is that the variation of consumption is greatly affected by the growth of the money stock but the productivity and output correlation is not. Therefore, although the monetary economy described here makes some improvement on the benchmark model, the improvements are small relative to that found with the tax model.

7. Conclusion

This paper illustrates how government policy variables can potentially affect business cycle activity. The message on the side of fiscal policy is that taxes and government spending can have a big effect on fluctuations. However, like technology shocks, no one variable can explain all of the second moments and correlations computed. The fiscal variables primarily affect the supply of labor and capital while technological shocks affect demand. Therefore, to understand why real wages do not fluctuate very much over the cycle while hours do, it is necessary to consider both demand and supply shocks. The message on the monetary side is that the addition of the currency has an effect on variation of consumption but is not much different than the benchmark model in most of its predictions of cyclical properties.

There are a number of issues not considered here. There is no attempt to obtain a good overall fit to the data and ask whether or not these models are useful for empirical work. However, the exercise performed here (and originally by Kydland an Prescott (1982)) gives a good indication of what stylized facts can be resolved with these models

TABLE 2

DECISION FUNCTIONS, STANDARD DEVIATIONS IN PERCENT, AND CORRELATIONS WITH OUTPUT FOR US AND MODEL WITH MONEY

Rule: $i_t = f_0 + f_1 k_t + f_2 (\lambda, g)_t$

	f_0	f_1	f_2	
(1)	0.67	−0.033	1.78	
(2)	0.57	−0.033	1.53	
(3)	0.54	−0.033	1.53	0.27
(4)	0.54	−0.033		0.27
(5)	0.54	−0.033	1.53	0.27

Standard Deviations

	y	c	i	k	n	y/n
US	1.81	0.91	5.11	0.45	1.52	1.32
(1)	1.85	0.54	6.01	0.52	1.42	0.54
(2)	1.85	0.54	6.02	0.52	1.42	0.54
(3)	1.85	0.69	6.11	0.52	1.42	0.54
(4)	1.86	10.05	26.83	1.22	2.90	1.17
(5)	1.81	1.52	6.83	0.53	1.40	0.54

Correlations with Output

	c	i	k	n	y/n
US	0.81	0.64	0.65	0.70	0.57
(1)	0.87	0.99	0.07	0.98	0.86
(2)	0.87	0.99	0.07	0.98	0.86
(3)	0.69	0.97	0.07	0.98	0.86
(4)	0.82	−0.89	0.14	0.97	0.81
(5)	0.38	0.79	0.09	0.97	0.82

(1): $a = [0, \log(.99)]'$, $b_{11} = .95$, $c_{11} = .0073$
(2): $a = [0, \log(1.15)]'$, $b_{11} = .95$, $c_{11} = .0073$
(3): $a = [0, .52 \log(1.15)]'$, $b_{11} = .95$, $b_{22} = .48$, $c_{11} = .0073$, $c_{22} = .009$
(4); $a = [0, .52 \log(1.15)]'$, $b_{22} = .48$, $c_{22} = .21$
(5): $a = [0, .52 \log(1.15)]'$, $b_{11} = .95$, $b_{22} = .48$, $c_{11} = .0071$, $c_{22} = .03$

and what cannot. Thus, the calibration exercise can be thought of as a first step towards formal estimation[15].

While the paper considers the effect of fiscal and monetary variables on cyclical behavior, the models are silent on the issue of optimal policy. The government policy in the tax and monetary economies considered here is assumed fixed at date 0 for all time and is not the result of optimizing social welfare. Some recent work by Aiyagari (1991) investigates the effect of government policy when the government along with the households is optimizing. Although the purpose of Aiyagari is to see how well such a model can fit the stylized facts, his framework is probably better than those described here for addressing pertinent issues of policy. The models described here provide only a first step to understanding the effects that tax and monetary variables can have.

Notes:

1 Braun (1990), Cassou (1990), Christiano and Eichenbaum (1988), Greenwood and Huffman (1990), McGrattan (1990b) are among those who have considered the importance of fiscal variables. Cooley and Hansen (1989), Kydland (1987), and Lucas (1987) are among those who have considered monetary factors.

2 The research has also been motivated by an interest in knowing the welfare effects of tax and money policy. For a discussion of these issues, see Cooley and Hansen (1989), Greenwood and Huffman, and McGrattan (1990).

3 The benchmark model is a simple version of Kydland and Prescott (1982) that does not include time-nonseparabilities in preferences or gestation lags in building capital. The model with factor taxation is a simple version of that analyzed in McGrattan (1990). The model with money is the same as that studied in Cooley and Hansen (1989).

4 If the objective function has cross-product terms, $2x'Wu$ replace $(Q, [A_{11}, A_{12}], [\Theta, \Psi])$ by $(Q - WR^{-1}W', [A_{11}, A_{12}] - B_1R^{-1}W', (I + \Psi R^{-1}W_2')^{-1} [\Theta - \Psi R^{-1}W_1', \Psi])$ where $W = [W_1', W_2']'$ and proceed as if there are no cross-products. If the solution to the problem without cross-products is $u_t = -Fx_{1t}$, then the solution to the problem with cross-products is $u_t = -(R + W_2'\Psi)^{-1}(RF + W_1' + W_2'\Theta)x_{1t}$ where Θ and Ψ are the matrices of the original specification.

5 See McGrattan (1991) for the derivation.

6 This follows directly from the firm's problem. Since the social and private optima are equal in this example, there is no need to distinguish between per-capita and individual levels of capital and labor. This is done for illustration because the equilibrium and social optimum is not equivalent in the extensions of Sections 4 and 5.

7 In this case, terms that include W must be added to the solution to obtain the solution for the original specification with cross-product terms.

8 The economy of this section is that described in Cooley and Hansen (1989).

9 Cooley and Hansen (1989) show that with certain restrictions on the growth process, this constraint will always be binding. This assumption is important for the solution procedure.

10 Alternatively, use the framework of Section 2 with no x_2 variables.

11 See also Rogerson (1988).

12 With a Cobb-Douglas specification of utility, the variability of hours is too low. Here, the assumption of indivisible labor implies that a fixed number of hours are worked and people move in and out of the workforce. If there is one agent adjusting the number of hours worked, the variability of hours is increased by introducing taxes and spending. Therefore, other specifications of utility can be used but the technology shock cannot be the only driving force.

13 *Matlab* programs used to generate these results are available from the author upon request.

14 The series used for the U.S. are consumption of durables, nondurables, and services plus fixed private investment plus government consumption from NIPA (y), consumption nondurables and services from NIPA (c), fixed private investment from NIPA (i), end of period net private capital stock from NIA (k), and total manhours for all workers from the BLS Household Survey (n). All series can be found in the Citibase databank with the exception of the capital series found from various issues of the *Survey of Current Business*. The annual capital series was log-linearly interpolated to obtain a quarterly series. Numbers used are per-capita with the population statistics obtained from Citibase. See Prescott (1986) for the method of detrending.

15 See Altug (1989), McGrattan (1990b).

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SECTORAL SHIFTS AND CYCLICAL FLUCTUATIONS*

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Abstract:

This paper studies a two sector real business cycle model in which the sectors experience different trend rates of growth and labor mobility is costly. Predictions are derived concerning the correlation between sectoral reallocation of workers and the cycle. This correlation may be positive or negative depending upon whether the growing sector displays larger or smaller fluctuations than the shrinking sector. The post-World War II period has witnessed two major patterns of sectoral change in industrialized countries: movement out of agriculture and movement out of the industrial sector. The model's basic prediction is shown to be consistent with the observed pattern of reallocation.

1. Introduction

Virtually all industrialized countries have experienced substantial secular changes in the sectoral distribution of labor in the past thirty years. The main pattern is a decrease in the share of employment going to agriculture and industry with an increase in the share of employment in the service sector. Although these shifts have continued over a long period, they have not always proceeded at an even pace. In a paper that has stimulated much work, Lilien (1982) argued that a substantial part of cyclical fluctuations in the post WWII US economy can be viewed as the economy responding to sectoral shocks that affect the desired allocation of resources, and labor in particular, across sectors. The underlying factors causing sectoral shifts do not change smoothly over time, but rather evolve stochastically, resulting in a stochastic process for the

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