

PROTECTION, INTERNATIONAL TRADE AND GROWTH*

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Abstract:

The goal of this paper is to investigate the dynamics of a trade liberalization policy by using an optimizing approach. The following questions are raised: How, starting from a highly protective (distorted) situation that is called the statu-quo of disaster, may a small economy be guided along an optimal path? and what properties do the transition paths to the social optimum have? Once the socially optimal solution has been determined, its implementation is constrained to satisfy an income distribution restriction (compensated leveling scheme) in order to reach conditions resembling those of free trade.

Introduction

This lecture and investigation has been conceived in the memory of Miguel Sidrauski, whose scientific activity always revealed a profound connection with the economic reality he so clearly perceived along his short but productive life.

Included in his academic interest was the analysis of the open economy. In his paper originally published in Spanish in the journal *Económica* (1968) almost twenty years ago, Miguel studied the problem of devaluation by means of a general equilibrium macroeconomic model which already included dynamic rules for wage indexation in an inflationary context. Those were the years preceding the popularity of the monetary approach to the balance of payments under fixed exchange rates which temporarily relegated the role of relative commodity prices during the adjustment process. Miguel Sidrauski's

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paper integrated the real and the monetary sectors in the open economy to rationalize the contractive effects of some Argentine devaluations with relative prices, income and monetary effects.

Starting from his important contribution, which shed light over such interesting—and at that time paradoxical—consequences of a devaluation, we asked ourselves, *what* questions would have attracted Miguel's powerful analytic mind today?

Several candidate subjects came to our mind as possible answers. Very soon one of them came to dominate our thoughts, given the passionate tendency he often showed to destroy fallacies and false myths, sometimes with the ardor of a crusader. That subject was protectionism.

The theme of this lecture then belongs to the old debate of Protectionism vs. Free Trade, historically connected with the thirty year long debate over the legendary Corn Laws, the import substitute commodity for England of 1846 (as has been pointed out by Diéguez (1974)).

An erratic, discriminatory, and generalized protectionism historically distorted resource allocation in many developing economies of Latin America. With few exceptions and for a prolonged time those countries had no rational protection policy, that is, one which systematically sought to provide incentives to the development of selected industries in the context of a given, generally accepted policy of economic growth.

The result was merely a system of ad-hoc protectionism, i.e. a set of emergency measures of foreign trade intervention designed to solve chronic and recurrent balance of payment problems, often under the pressure of exogenous factors or in response to demands of special interest groups. For example Macario (1964) gives an illuminating description of protectionism in Latin America; the extreme was often reached of setting high tariffs even on traditional exports—exceeding 200% in 1960!—under the principle of prohibiting all imports of commodities the country was able to produce. Similar observations were made by Myrdal (1956) regarding the complicated and irrational system of restrictions to international trade which often ended up by having unexpected undesired effects on production and trade.

The initial "temporary" protective measures were soon converted into permanent ones, giving rise, in several instances, to import substitution at any cost under extraordinary high protection levels, by using all sorts of protective instruments such as tariffs, import prohibitions, exchange control, quantitative restrictions, multiple exchange rates, export taxes, overvalued exchange rates, etc.

The overall result was that instead of granting a moderate temporary protection to the more efficient import substitute industries, *inefficiency* became generally protected according to a peculiarly "fairness" criterion: "the less able the industry to resist foreign competition the higher protection it deserves" (Macario (1964), pg. 80).

It is the central purpose of the lecture in the memory of Miguel Sidrauski to investigate the dynamics of a trade liberalization policy by using an optimizing approach.

We raise the following question: How, starting from a highly protective (distorted) situation that we call the *statu quo of disaster*, may a small economy be guided along an optimal growth path? and what properties do the transition paths to the social optimum have?

Recent experience of several countries has raised the question of how best to implement liberalization policies.

In order to select an adequate productive structure for our small economy subject to high levels of initial tariffs we discarded the convenient traded-nontraded commodity breakdown because in this case the Hicksian theorem of commodity aggregation is not

valid. The essential feature of ad-valorem protectionism for a small economy is that it changes the domestic terms of trade between exportables and importables *pari passu* with unaltered external terms of trade.

Consequently, the paper is concerned with the problem of implementing the social optimum, by paying attention to the medium run.

Recent literature dealing with trade liberalization has paid attention to its short run macroeconomic effects. For instance, Buffie (1984) undertakes a short run macroeconomic analysis in order to rationalize the recessionary effects that sometimes beset many attempts at liberalization, by using a traded good -nontraded good model with homogeneous labor and sector specific capital.

Aizenman (1983) analyzes some dynamic aspects of trade liberalization policies related to the process of learning that is being triggered by changes in the structure of human capital induced by tariff changes.

Aizenman (1985) investigates the effects of alternative financial restrictions during the trade liberalization process, as possible means of reducing the short run deflationary effects. Calvo (1986) investigates the costs of trade liberalization when it is temporary, not expected to persist over time.

The short run gloomy results so far obtained in the literature when analyzing trade liberalization attempts only suggest in the view of the authors the necessity of devising better policy packages in order to raise the expectations of success of those policies.

This paper can be considered a step in that direction. The policy package derived through an optimization process under income distribution constraints guarantees that the new situation will be a Pareto improvement over the statu-quo. In other words, once the socially optimal solution has been determined for our small economy, we study the problem of how to implement that optimum involving a *compensated leveling scheme* to reach conditions resembling those of free trade.

A policy of *growth liberalization* is proposed which can be considered to be an improved variant of Kaldor's (1964) well known proposal of dual exchange rates to deal with income distribution problems induced by devaluations in a developing economy. Then, the growth liberalization proposal to be analyzed in this paper is a way of overcoming an excessive focusing on the short run costs of liberalization to the neglect of its considerable long run benefits.

Section 1 describes the productive sectors of the economy, which are then analyzed in section 2, where the optimal production programs under free and restricted trade are determined. Section 3 describes the consumption groups in which households are classified in order to analyze problems of income distribution. The consumption programs optimal for the economy are then determined in section 4, which also shows the choice of consumers under restricted trade. Section 5 shows how the social optimum can be implemented, even in the presence of distributive constraints due to the "constitution" of the economy. Section 6 presents a summary of conclusions, whereas the appendix gives in more detail the arguments leading to the propositions cited in the text, where they would interfere with the exposition.

1. Productive sectors

The economy will be assumed to be engaged in four classes of productive activities, performed in four sectors producing traditional exports, T, non-traditional exportable commodities, N, importable goods, M, and dismantling services, D; these sectors will

be indexed by the label j in $\{M, N, T, D\}$. Typical products of these sectors are cattle, automobiles, high-technology equipment, and disinvestment services, respectively.

The output of each sector is produced by the services of two indispensable factors labor, L , and capital, K , under neoclassical conditions of constant returns to scale and diminishing marginal rates of return to any one of the factors given levels of utilization of the other. The production function of the j -th sector is denoted by $F^j(L_j, K_j)$, where the arguments represent the employment of labor and capital by the sector, respectively.

It is today well known that the introduction of the specific sector capital (SSK) model into international trade theory was initially done in order to distinguish carefully the short and the long run effects of several policies. For instance, the SSK model has reversed for the short run the results of the Stolper-Samuelson theorem conceived as it was under the assumption of long run perfect intersectoral capital mobility.

When introducing the dismantling sector, the present paper tries to focus attention not only on the long run, which probably bears no relation whatsoever to any economically relevant time period, but especially on the medium run, a time period nearer to the worries of politicians, policy makers, and private parties (entrepreneurs, trade union leaders, etc.).

The dismantling sector uses both capital and labor in order to produce dismantling services, a non-traded commodity which may be used by the remaining productive sectors of the economy, T , N , and M .

Following Neary (1978) we argue that there is no relevant time horizon over which the assumption of perfect intersectoral capital mobility is relevant or appropriate. On the other hand, for medium run analysis, the assumption of sector specific capital—so convenient for short run analysis—is quite unrealistic, because competitive pressures will sooner or later lead to resource reallocations between sectors when changes in relative prices occur.

In medium run analysis, capital reallocations in practice take the form of a slowing down in the rate of replacement of depreciating capital goods in the declining sectors, coinciding with a rechanneling of new investment towards the expanding sector. At this point the dismantling sector becomes crucial since it allows the declining sector to dispose of excessive capacity by a rate faster than the depreciation rate. The important point is that the absence of any dismantling costs in a dynamic analysis imply the existence of downward discontinuous jumps in the capital stock, so that the equilibrium capacity level is instantaneously attained. On the other hand, if dismantling costs were infinite we would have the SSK model, whose dynamic extensions would imply that capital decumulation cannot exceed the rate of zero gross investment.

The economy evolves over time. In order to simplify the notation, the time-dependence of variables such as the quantities of factors used, output, etc. will usually not be made explicit. Population, P , grows at an exogenously given rate, n , so that its level at time t is $P(t) = P(0) \exp(n t)$, $\exp()$ being the exponential function, where $t = 0$ represents the present moment at which plans are made; the planning horizon will be assumed to extend to infinity.

Labor services are provided by the growing population according to a transformation function $\Lambda(L)$, where $L = (L_T, L_N, L_M, L_D)$ represents the list of man-hours provided by the population to the four sectors. Labor is not homogeneous; different individuals have skills better adapted to one of the sectors or the other. The degree of labor mobility is given by the transformation function, which will be assumed to be linear homogeneous—in order to reflect that population is in demographic equilibrium—to be increasing in its argument, the labor employment distribution $L = (L_T, L_N, L_M, L_D)$, and to exhibit

diminishing marginal rates of labor mobility. Hence Λ is not the sum of labor allocations as would be the case with homogeneous labor: it depends on labor allocations of all sectors.

Following Mussa (1982), Grossman (1983) and Hill and Méndez (1983), it is assumed that there are comparative aptitudinal advantages on the part of workers, labor skills, in producing one commodity or the other. Individual workers will not earn *in equilibrium* the same rate of return, salaries, because they have differing relative and absolute efficiencies in different sectors.

The labor transformation function $\Lambda(L)$, is similar to a production transformation function, such that families can "produce" labor services for one sector or the other. They do it in proportions determined by the relative wage differentials w_i/w_j , so as to equalize them to the marginal rate of labor mobility, the ratio Λ_j / Λ_i . Here Λ_j , the derivative of $\Lambda(L)$ with respect to labor employment in sector j , can be called *the marginal population requirement of j* and interpreted as *the increase in population required to be able to employ in the margin one additional worker with the skills used by sector j* .

Workers are perfectly free to move between sectors. Nevertheless they are economically motivated not to use that freedom.

Notice that the existence of relative wage differentials does not represent any economic distortion, since it is determined by the technology of producing labor by each family: there is no perfect substitution in the labor services produced.

It should be remarked that some assumption such as the present one of imperfect labor mobility, is needed in the present dynamic context. If for example labor were homogeneous, then there would be only one non-producible factor in the long run. In that case the economy would exhibit constant costs and would specialize completely, in the long run, in a single commodity following the Ricardian theory of comparative advantage.

Since $\Lambda(L)$ is linear homogeneous, its derivatives with respect to labor employment are homogeneous of degree zero, so that the ratios Λ_j / Λ_i depend only on the three independent proportions of workers employed in the four sectors. Thus in the present case one obtains three independent equations which allow the determination of those proportions.

"Capital" is really an index of productive capacity; the services of capital goods such as machines, cattle, etc. are used in certain amounts to provide capacity according to a neoclassical production function, under constant returns to scale and diminishing returns to the increases in the employment of one capital good. Note that this includes as a special case capital goods used in fixed proportions in a fashion similar to the Leontief interindustry model.

The rationale for this definition of capacity stems from the fact that the final output of each sector is used: 1) for consumption purposes, 2) for exports to the rest of the world and 3) for capital accumulation. The accumulation in turn may take place in the same sector which produces the output or in the remaining sectors of the economy.

Corresponding to the production of capital there exists a dual unit cost function giving the domestic cost q_j of investment in a unit of capacity expansion of sector j , where j is one of the four productive sectors, in terms of domestic output prices p_M , p_N , and p_T —not p_D , since services are not used as capital goods. The share of capital good j in the price of the composite investment good of sector i is a_{ij} . This share is the capacity elasticity of the corresponding capital good, and also the elasticity of the price

of the capacity of the sector with respect to the prices of the different capital goods required to provide such capacity, so that

$$(1.1) \quad \hat{q}_i = \sum a_{ij} \hat{p}_j,$$

where the caret ($\hat{\cdot}$) indicates the relative rate of change of the variable under it, e.g. $\hat{x} \equiv d \log x$.

Domestic prices of the three traded goods $j = T, N, M$, are linked to international prices by the well-known formula

$$p_j = E (1+t_j) p_j^*,$$

where E is the nominal exchange rate, t_j the ad-valorem tariff rate, and p_j^* the given international price. Of course, it is usual to see positive tariffs in the case of importables and negative ones in the case of exportables, since those are the signs which correspond to taxes, not to subsidies.

Employment is restricted by the labor transformation function $\Lambda(L)$; the labor availability constraint can then be expressed as

$$(1.2) \quad \Lambda(L) \leq P$$

The international interest rate is fixed at i .

The rate of capital depreciation is d , so that the restriction on gross investment I_j of sector j becomes

$$(1.3) \quad I_j \geq DK_j + dK_j$$

where the operator D represents differentiation with respect to time t , e.g. $Dx = dx / dt$.

Dismantling services are asymmetric, and their quantity is defined as negative gross investment if not positive,

$$(1.4) \quad D_j \equiv \max \{0, -I_j\}.$$

The asymmetry between positive and negative gross investment means that when a sector acquires investment goods, the corresponding installation costs are included in the cost function of the capacity of the sector. The situation is not the same as that in which a sector has to contract—in accordance with the social optimum—beyond the rate of depreciation. If it has to contract at a faster pace than that given by the depreciation rate, it is not always possible to sell used capital goods immediately, as it is very likely that they have to be reconverted in order to be useful in other sectors of the economy; at the least the installation cost will be lost, so that the price of used capacity most certainly will not be the same as that of new capacity. This reversion and reinstallation process is what here is called dismantling activity.

The definition, together with the condition that investment goods are measured in units which require similar dismantling services in case disinvestment is called for, allows the presentation of the dismantling capacity restriction

$$(1.5) \quad D_M + \dots + D_D \leq F^D(K_D, L_D).$$

The left hand side of this inequality is a measure of the services rendered by the sector, which must satisfy the demand for dismantling from all productive sectors.

It is now possible to write down the net present value of output of the private sector to the rest of the world, the foreign sector. The public sector will be disregarded in this first approximation to the problem. With labor and dismantling services internalized, it is

$$(1.6) \quad O(p) \equiv \int \exp(-it) \left[\sum_{S \setminus D} p_j F^j(L_j, K_j) - \sum_S q_j I_j \right]$$

The value of the output of the dismantling services sector does not enter the first term of this equation, since it does not produce tradeable commodities. On the other hand the value of investment in the sector does enter the second sum in (1.6) because it uses investment goods which are tradeable.

The agents of the private sectors maximize their profits; this means that competition will lead the economy to maximize the net present value of aggregate output, subject to the employment constraint, the investment constraints, the definition of dismantling, and the dismantling services constraint, as the appendix to the paper shows in detail.

The shadow prices related to these constraints are the income of the average worker family, y ; the shadow value of capacity to sector j , m_j ; the value of dismantling services to the user, sector j , s_j , and the value of dismantling services to the producer of D , p_D .

2. Production programs under free and restricted trade

The open economy being analyzed is a price taker in the world markets. Consequently it is legitimate to separate the decisions on optimal production from the corresponding decisions on optimal consumption.

Since prices are exogenously determined, in order to attain the welfare optimum—main goal of the paper—the problem can be divided into two consecutive stages: 1) the maximization of income, and 2) the maximization of utility subject to that income. This implies separating the problem of production from that of distribution.

When analyzing the short run, the Lagrangean provides the optimality conditions valid not only in the present instant, time 0, but also at any future instant. Since these conditions are rather complicated, the usual procedure is to analyze first the behavior of the system in the long run, once the economy has had time to adapt its structure to the optimal one. This leads to the analysis of what Hicks (1965) calls *regularly progressive* economies, in which all quantities grow at the same rate, and at the pace of the *silvery path*—phrase coined by Morishima (1969) to oppose to the *golden path* of maximal instantaneous consumption—along which the economy is guided by the maximization of the prospective utility of the consumption program extending from the present to the infinite future.

a) Short run optimality conditions

The short run conditions that have to be satisfied by the production and consumption programs of the agents of the economy corresponding to the optimization problem presented in section 1, are derived in section 2 of the appendix from the Kuhn-Tucker theorem on convex programming. After some algebraic manipulations one obtains the following marginal pricing relations needed to sustain an optimum, valid for the four productive activities at any moment within the planning period.

The wage required to retain workers in sector j is defined by the equation

$$w_j \equiv y \Lambda_j,$$

which can be interpreted as follows. The marginal increase in population due to an increase in labor employed in sector j , is given by the partial derivative Λ_j . The average income of the family is y ; therefore the product of these two quantities is the amount paid by sector j for one additional unit of labor, i.e. the wage rate. With this definition one can write the marginal condition on the labor productivity by the inequality

$$(2a.1) \quad p_j F_L^j \leq w_j \equiv y \Lambda_j,$$

where the equality will be binding if $L_j > 0$, that is to say if the sector employs positive quantities of labor. The inequality says that the value of the marginal product of labor should not exceed the wage rate.

For capacity in sector j one has a similar inequality

$$(2a.2) \quad p_j F_K^j \leq [(i+d) - \hat{m}_j] m_j,$$

with the equality holding if $K_j > 0$, i.e. if the sector uses capacity in positive quantities. According to it, the value of the marginal product of the marginal product of additional capacity should not exceed its price, which in relation to the shadow value of capacity m_j is given by the opportunity cost of interest foregone plus the depreciation cost minus any capital gains due to the rate of increase in price, $\hat{m}_j$, over time.

The price of investment goods equals their value to the sector demanding them plus the value of the eventual dismantling services employed in disinvesting, so that

$$(2a.3) \quad q_j = m_j + s_j,$$

where of course the value of these services can be positive only in the case that dismantling equals negative investment, so that the latter must be negative —positive investment implies zero dismantling services.

The value of dismantling services for the sector demanding them cannot exceed their price, so that

$$(2a.4) \quad s_j \leq p_D,$$

with equality if $D_j > 0$, that is to say if sector j demands positive quantities of these services.

b) Long run regularly progressive solution

In the long run regularly progressive solution net investment will have been adapted to just maintain the capital-labor ratio unchanged. Since the population, and so also employment in each of the sectors, grows at the exogenous rate n , this will also be true of installed capacity, giving the equations

$$DK_j = n K_j.$$

From the investment constraint (1.3) one obtains then the inequality

$$(2b.1) \quad I_j \geq (d+n) K_j.$$

This relation implies that $I_j \geq 0$; hence there will be no demand at all for dismantling services in the long run regularly progressive solution —dismantling is required only in the short run, when the relative sizes of the sectors given by the past evolution of the system did not have yet the time to adapt to the requirements of the social optimum.

It is a consequence of the absence of dismantling services that

$$m_j = q_j,$$

i.e. the value of capacity to sector j coincides with its cost. This implies that for the productive sectors, which still are producing in the long run, the condition on the value of the marginal product of capacity simplifies to

$$(2b.4) \quad p_j F_K^j = (i+d) q_j.$$

This means that the capital-labor ratio is directly determined from external conditions —the exogenously given prices, p and q , interest rate, i , and rate of depreciation, d —, since it is the only unknown in this equation for all sectors producing goods.

Consequently, once the capital labor ratio is known, the wage rate w_j of sector j can be determined on the basis of the marginal product of labor and the known price of output. From their definitions (2b.3) one obtains, taking the wage in the traditional exports sector T as numeraire, for $j = N, M$ the equalities between the relative wage differentials and the marginal rate of labor mobility,

$$w_j / w_T = \Lambda_j / \Lambda_T.$$

Taking into account that the dismantling sector does not exist in the long run, one obtains a system of two equations in two unknowns which under the assumptions made can be solved for the ratios l_j of labor employed in sector j to labor employed in sector T . The scale can be determined from the employment restriction.

It is interesting to note that in the long run the economy behaves as if at any instant it were maximizing its net income

$$\sum_{S \setminus D} [p_j F^j(L_j, K_j) - q_j (d+i) K_j],$$

—sales of the outputs of the sectors minus depreciation allowance and the interest foregone on investment in capacity— subject to the labor employment constraint (1.2). That this should be so is intuitively clear, since labor services are the only ones which cannot be produced within the system; the scarcity of capacity and the magnitude of the foreign debt is relevant only in the short run and can be overcome with accumulation.

c) Long run comparative dynamics: the problem

Once the long run solution for the growing economy has been obtained, the following natural question arises.

What is the effect of tariff changes, in particular in the case of trade liberalization, on the relative rates of change in relative commodity prices, wage rates, employment, and other relevant targets?

To answer this question a long run comparative dynamic analysis has to be made, comparing the long run quasi-stationary solutions corresponding to the two cases in which the present study focuses, free trade and tariff restricted trade.

First the meaning of trade liberalization has to be described. By *trade liberalization* one understands a policy pursuing the equality between relative domestic and foreign traded commodity prices by equating nominal (and effective) tariff rates in all sectors. This implies that when these uniform tariff rates are positive, imports are taxed and exports are subsidized.

This admits the possibility that in case the nominal prices are sticky downwards, prices may adjust upwards to the highest tariff rate in the first place, and thereafter the common rate can be decreased in the same proportion as the nominal exchange rate is increased to compensate. In other words, all traded good prices can be freed (tariff cut) followed by a devaluation, in the proportion of the highest tariff rate.

In order to carry out the proposed comparative analysis it is necessary to differentiate logarithmically the equations for the long run regularly progressive solution of section b.

This has been done in the appendix where one can find the formulae giving the relative changes in the variables in terms of relative price changes $\hat{p}_j$ and of the parameters of the system. These are a_{ij} be the elasticity of the cost of the i -th sector capacity with respect to the price of the j -th capital good, s_L^j and s_K^j the shares of labor and capacity in the output of sector j , σ_{ij} the elasticity of substitution between labor and capacity in sector j , S_{ij} the degree of labor mobility between sectors i and j , and λ_{mj} the elasticity of population with respect to employment.

The effects of change in the tariffs can then easily be seen, since the constancy of international prices and of the exchange rate implies that the relative price change equals the relative change in the price distortion. In terms of changes in the tariff of sector j this amounts to $\hat{p}_j = D\tau_j / (1 + \tau_j)$.

It is reasonable to assume that no other sector uses the output of the traditional exports sector as capital goods; in that case the matrix A has the following structure:

$$A = \begin{matrix} & \begin{matrix} T & N & M \end{matrix} \\ \begin{pmatrix} ** & * & ** \\ 0 & * & ** \\ 0 & * & ** \\ 0 & * & ** \end{pmatrix} & \begin{matrix} T \\ N \\ M \\ D \end{matrix} \end{matrix}$$

where the asteriks indicate possibly non-zero elements. The columns give the shares of each of the three investment goods in the capacity of the sector using them; the rows, one for each sector, indicate who uses the capital goods. It is also reasonable to assume that most capacity comes from the import substituting sector, so that the last column of this matrix strictly dominates the second one; the elements assumed to be relatively large have therefore been indicated with double asterisks.

d) Long run comparative dynamics: an example

In order to be able to analyze the effects of changes in domestic prices, it will be assumed that the elements of the central column of matrix A are negligible, hence it will be replaced by a column of zeros. Thus

$$(2d.1) \quad A = \begin{pmatrix} x & 0 & 1-x \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix},$$

where the last row of the matrix, corresponding to the dismantling sector, has been omitted since that sector is not in operation in the long run.

In order to simplify the following computations take the import substitute good, M , as numeraire, so that its relative rate of change with respect to the tariffs is $\hat{p}_M = 0$. Equivalently, assume that the nominal exchange rate compensates for the change in the import tariff—the result is the same, the domestic price of imports remains constant.

Premultiplying the vector $\hat{p} \equiv (\hat{p}_T, \hat{p}_N, 0)$ of rates of change in commodity prices by the matrix A described in (2d.1) one finds the corresponding rates of change in the prices of the composite investment good for each sector from (1.1),

$$(2d.2) \quad \hat{q} \equiv (x \hat{p}_T, 0, 0).$$

From (2c.4) and (2c.5) (any reference to a formula in section c correspond to the appendix) one then obtains the effect on the capital-labor ratios,

$$(2d.3) \quad \hat{k} = [(\text{sig}_T/s_L^T) (1-x) \hat{p}_T; (\text{sig}_N/s_L^N) \hat{p}_N; 0].$$

In this relation the effects of prices, seemingly different, are nevertheless the same if one takes into account that the numeraire does not change, and that a fraction x of T 's output price increase is offset by the increase in the value of its capacity, since x is the share of goods produced by the sector in the value of its own capacity.

Similarly one obtains the effects on the wage rates of the sectors,

$$(2d.4) \quad \hat{w} = [(1/s_L^T) (1 - x s_K^T) \hat{p}_T; (1/s_L^N) \hat{p}_N; 0].$$

In this case part of the output price increase is lost due to the fraction s_K^T of the capacity price increase, which has just been seen to share a fraction x of sector T 's output price increase.

It should be noted that when it is expressed in terms of the sectoral wage rates, the equation (2d.3) for the capital labor ratios simplifies to

$$(2d.5) \quad \hat{k} = [\text{sig}_T g \hat{w}_T; \text{sig}_N \hat{w}_N; 0]$$

where $g = (1 - x) / (1 - s_K^T x)$ is a number between zero and one.

The income of the average wage earner changes, as can be inferred from (2c.8) and (2d.4), by

$$(2d.6) \quad \hat{y} = (\text{lam}_T/s_L^T) (1 - x s_K^T) \hat{p}_T + (\text{lam}_N/s_L^N) \hat{p}_N;$$

it is seen to increase if prices increase in relation to the price of the import substitute M.

In order to analyze changes in real terms and not only relative to the numeraire as up to here, one must compute the change in the consumer price index due to the price changes induced by the new commercial policy. This gives, differentiating the price index,

$$(2d.7) \quad \hat{v} = \text{alf}_T \hat{p}_T + \text{alf}_N \hat{p}_N,$$

where the alf_j are the shares of consumer expenditures on good j in total consumption expenditure. The relative change in real average income of wage earners is therefore the difference

$$(2d.8) \quad \hat{y} - \hat{v} = (1/s_L^T) \left\{ \text{lam}_T (1 - s_K^T x) - \text{alf}_T s_L^T \right\} \hat{p}_T + (1/s_L^N) \left\{ \text{lam}_N - \text{alf}_N s_L^N \right\} \hat{p}_N;$$

the coefficients of this expression are such that it may be negative if prices increase when the export sectors are labor intensive (have large s_L^j 's) but small employers (small lam_j 's), and their outputs face a large domestic demand (large alf_j 's).

The analysis of employment will be simplified by assuming that labor services are produced by households following the CES functional form, so that the substitution matrix can be expressed in terms of the shares of the workers of the different sectors in aggregate labor income and the common elasticity of labor output substitution—the degree of labor mobility ρ —as follows,

$$(2d.9) \quad \begin{aligned} S_{ij} &= -\rho && \text{for } i \neq j; \\ S_{jj} &= \rho (1/\text{lam}_j - 1) && \text{otherwise.} \end{aligned}$$

Thus (2c.6) simplifies to

$$(2d.10) \quad \hat{L}_j = \rho (\hat{w}_j - \hat{y})$$

where use has been made of (2c.8), $\hat{y}$ being the average of sectoral relative wage rate changes.

This formula gives a first result; it states that labor employment in a sector expands as a consequence of a tariff reduction if, and only if, the sectoral wage rate increases more than the average worker's income.

In particular—since as has been shown before the wage rate in M is fixed in terms of the price of the output of that sector—this implies that employment in the import substitute sector M falls if and only if average labor income rises relative to the price of M. The magnitude of the shifts in employment depend—as can be seen from the formula—on the degree of labor mobility ρ , which for convex functions $\Lambda(\cdot)$ must be positive. Large values of ρ correspond to a high mobility, with workers responding drastically to the slightest wage differences, in the limit tending to the usual homogeneous, perfectly mobile labor case. In the other extreme, where ρ is close to zero, labor becomes specific to the sector employing it, and even large differences in wages will not induce workers to abandon one sector and move into another.

Replacing for the relative change in average labor income in terms of the changes in the wage rates and writing out the matrix of coefficients in full one has

$$(2d.11) \quad (1/\rho) \hat{L} = \begin{pmatrix} 1-\lambda_T & -\lambda_N \\ -\lambda_T & 1-\lambda_N \\ -\lambda_T & -\lambda_N \end{pmatrix} \begin{pmatrix} \hat{w}_T \\ \hat{w}_N \end{pmatrix}$$

so that positive price changes in the other prices, $\hat{p}_T$ and $\hat{p}_N$ —that is relative prices; a tariff reduction inducing a fall in the price of M would produce similar real effects—, would induce by (b.4) increases in the wage rates of the same sectors, hence employment would fall in sector M. The question arises whether such relative price increases can also cause employment to decrease in both T and N.

It is easy to show that this can only happen if the upper 2x2 submatrix in (2d.11) has a negative inverse, so that its determinant

$$(1-\lambda_N)(1-\lambda_T) - \lambda_M \lambda_T$$

must be negative. But this is equivalent, as a little algebra will show, to the requirement that the shares of the workers of the two sectors T and N add up to more than unity, which is an absurdity.

Equivalently, that this cannot happen can be seen by multiplying the first row of the matrix by λ_M , the second by λ_N , and adding. This gives

$$(2d.12) \quad \lambda_M \hat{L}_T + \lambda_N \hat{L}_N = \rho \lambda_M [\lambda_T \hat{w}_T + \lambda_N \hat{w}_N]$$

The right hand member of this equation is positive, hence employment must increase in at least one of the two exportables sectors.

A third and simplest way of proving the result is obtained by considering the employment restriction. Since the function $f(\cdot)$ is increasing in its arguments and must equal at any instant the exogenously given population, it is impossible that all sectors move in the same direction without violating the full employment condition, one of the consequences of optimality.

In short, whenever the domestic relative prices—and hence also wages—move against the import substitute sector, it must necessarily contract its labor force, whereas at least one of the other sectors will expand it, as long as the degree of labor mobility is not zero (the case of specific labor).

The effect on real wages can be obtained by subtracting the change in the consumer price index from each of the wage rate changes. Then one has

$$(2d.13) \quad \hat{w} - e \hat{v} = \begin{pmatrix} (1/s_L^T)(1-x) + x & -\alpha f_T; & -\alpha f_N \\ -\alpha f_T; & (1/s_L^N) & -\alpha f_N \\ -\alpha f_T; & -\alpha f_N \end{pmatrix} \begin{pmatrix} p_T \\ p_N \end{pmatrix}$$

It is worth noticing that the first element in the first row and the second element in the second row of the matrix of coefficients are both positive. This is so because in both cases the positive term exceeds unity and the negative term does not—all the numbers involved are shares of one sort or another and therefore are positive and less than one—. It is therefore clear that increases in the other prices relative to that of the import substitute will affect the real wage of that sector adversely, while one cannot predict the direc-

tion of the effect on the other sectors without knowledge on the relative size of their price increases.

If the relative prices increase approximately equally in the two exportables sectors, then the real wage in both will increase since the sum of the coefficients of the matrix along any one of the first two rows contains a positive term which is greater than unity and two negative terms whose sum is one minus the share of import substitutes in consumption, hence less than unity.

If on the other hand the price increases in only one of the export sectors, then the real wage in the other sector will be less in the long run, though it will fall by less than in the import substitute sector.

In spite of these relative falls in some of the wage rates, there is no need for workers to be worse off since as will be seen later on the government can utilize the tax rate as a policy instrument to compensate for the losses.

The relative changes in the real wage bill can be obtained by adding (2d.10) and (2d.13) to obtain

$$(2d.14) \quad \hat{L} + \hat{w} - e \hat{v} = \rho (\hat{w} - e \hat{y}) + \hat{w} - e \hat{v};$$

from where it is obvious that the wage bills of the various sectors move in the same directions as the real wage rates when ρ is small, and in the same directions as employment in case that labor mobility ρ is high.

3. Consumption groups

The assumption of a small country producing internationally traded goods —traditional and nontraditional exports plus import substitutes— used in the previous section allows us to separate neatly the problem of production efficiency from the problem of distribution of income among several social groups as will be considered below.

In the present section we introduce the separability property in order to focus the attention on a concrete proposal for growth or trade liberalization, which implements the social optimum determined in the previous pages.

In order to minimize the political opposition on the part of the declining import substitute sectors, use will be made of the compensation principle, designed so as to maintain the income distribution of the four social groups —or income recipients— considered proportional to that of the statu quo.

The statu quo —identified as the pre-liberalization production equilibrium obtained in the first sections of the paper— only provides a floor to the final welfare of society, since it only shows that it is possible to improve the efficiency of the system without harming anyone.

Once the benefits of a greater efficiency have been attained, it is always possible to improve the resulting income distribution in order to attain more desirable social goals, since the statu quo by itself does have no moral merit. Once the solution to the optimal production problem has been determined, the question can be asked, how can one proceed so that no social group suffers a loss?

For this it is necessary to know the preferences of consumers and the functional distribution of income.

The preferences of the representative consumer are of the Ramsey type, with marginal utilities of commodities consumed in different periods of time independent of each other —in the words of Koopmans (1960), one has limited noncomplementary over time

for any three time periods, implying that consumption in one period is independent of that of another one—. Prospective utility—that is, the utility of the households' consumption program extending over its planning horizon which extends from the present time zero to the openended future—is additive, with terms composed of independent instantaneous utility functions. In other words, consumers are not—in Morishima (1969) words—“momentalists”, they go beyond the present instant, but the time structure of their preferences follow Ramsey's simplified form. Thus—see for example the explanation in Morishima (1969)—it can be represented as a discounted sum of instantaneous utilities $u(c)$, each referring to the bundle of consumption rates c of the current instant of time, and exhibiting diminishing marginal utility and homogeneity of positive degree less than one. The discount factor for time t is $\exp(-rt)$, with r being the rate of time preference.

The triple $c=(c_T, c_N, c_M)$ is the household's current consumption bundle. The corresponding consumer price index is $v(p) \equiv \min \{ p \cdot c | u(c) \geq u_0 \}$, i.e. the cheapest way to provide a certain given level of utility u_0 .

All households determine their demand for consumption goods by maximizing their utility subject to their budget constraint, which is of the form given by Fisher, expressing that the present value of its consumption stream should not exceed that of the net present value of its wealth, that is to say the sum of its net income stream plus any initial net wealth, the rate of discount being the given interest rate i .

The solution exists whenever the ratio r/i of the rate of time preference to the interest rate exceeds the degree of homogeneity of the instantaneous utility function. Since the latter is less than unity, this will certainly be true if the rate of time preference r of the representative consumer exceeds or equals that of the rest of the world implicit in the international interest rate i . It will also be true if the degree of homogeneity is not positive, so that the elasticity of substitution of present vs. future consumption is at most equal to unity. The only possibilities of non-existence of a solution arise when r is less than i and the intertemporal substitution elasticity exceeds unity by a large amount.

In order to clarify the meaning of the preceding paragraph, let us compare two economies with the same production plan and different consumption plans, as follows.

Consider the possibility of domestic residents being less impatient than residents in the rest of the world, so that the rate of time preference r falls short of the international interest rate i and their ratio is less than the elasticity x of the instantaneous utility function. For any rate of growth of consumption expenditure $g < i$ there exists a level of expenditure so that the budget restriction is satisfied, since the net present value is finite—it equals $z(0)/(i-g)$, where $z(0)$ is the initial consumption expenditure—. Instantaneous utility grows at the rate xg , which becomes $xg-r$ after discounting with the rate of time preference. If g is chosen large enough without exceeding i —the argument started by assuming that this is just the case in which this is possible—it is obvious that the discounted value of instantaneous utility does not decrease over time, so that prospective utility is infinite. Therefore a case such as this, in which consumers can attain the point of bliss, will be discarded in the present investigation for being uninteresting—there is no economic problem left to be solved—. The main reason for this is that households feel so strongly about the wellbeing of future generations, that their sheer number outweigh any satisfaction to be derived from present consumption, so that in the end society will restrict consumption at all times in order to save for the generations which never will come into existence.

It is shown in the appendix that the prospective utility attained by the household is an increasing function of its real wealth, defined as $y/v(p)$ —wealth deflated by the consumer price index.

Households will be classified into four categories, depending on the origin of their income. Three of these groups are the owners of the firms of the three productive sectors producing goods T, N, and M; the fourth corresponds to workers, irrespective of their place of employment. The set of households, or consumers, or income perceiving groups will be denoted by $H \equiv \{T, N, M, L\}$. The dismantling sector, though having from the technological point of view a distinct production function, has no separate owners; whichever sector requires dismantling services sets up its own dismantling facilities, as a special department of the firm —of course this is possible without loss in efficiency because of constant returns to scale, and everybody has access to the technology.

At the present time, $t=0$, the income distribution —the aggregate net present value of each of the four income perceiving groups— is given by Y_i^0 for i one of the capitalists of sectors, T, N, and M, or labor, respectively. The corresponding relative income distribution represents the statu-quo which it is deemed from the political point of view to preserve. The following analysis will seek to preserve the relative income distribution corresponding to the statu-quo.

Aggregate welfare will be assumed to be of the Bergson-Samuelson type and egalitarian, in the sense that the country's welfare function is symmetric in its arguments. Welfare optimization will be carried out restricted to preserving the relative income distribution prevailing at the statu-quo, in order to separate the economic problem of resource allocation from the political problem of income distribution. The aim of the present paper is to show that it is possible to increase every individual's wellbeing, how much of that increase will be taken from one group to give it to another is a negotiation between the parties which cannot be analyzed from the point of view of economic efficiency. In fact, in order to show the possibility of increasing social welfare it is sufficient to show that an increase is possible given this constraint, since an unconstrained optimum will always be at least as good as a constrained one.

As a consequence of keeping the income distribution constant, all welfare functions will lead to the same optimal consumption and production programs. Thus, given this restriction, it is allowable to consider the case in which social welfare just consists of a sum of the individual's utility functions; since as has been seen above, these are increasing function of real income, so that after certain convenient increasing transformation they are equal to real income. Social welfare is then just aggregate real income, defined on the basis of the consumer price index. Given the prices, consumers then maximize their prospective utilities, given the relative income distribution, by maximizing wealth, that is to say, the net present value of the private productive sector. This observation justifies having introduced the model starting with the production side and maximizing that present value.

4. Consumption programs under free and restricted trade

a) Restricted trade

Let the vector $p = (p_T, p_N, p_M)$ denote the domestic price list of the three goods produced in the country; that is, traditional exportables, non-traditional exportables, and importables —dismantling services are not included here because they are not tradeable—, and let p^* denote the international prices of the same goods. Given those domestic prices, on the basis of which the private sector arrives at its consumption and production decisions, there corresponds a certain trajectory of quantities produced and

invested in each of the four sectors, including dismantling services. Let $F(p)$ be the vector of quantities produced by the three goods producing industries at the instant t , excluding the dismantling sector because its output is not traded, when domestic prices are fixed at p .

Discounting these quantities to the present at the going international interest rate i , one obtains the three-dimensional vector

$$(4a.1) \quad G(p) \equiv \int \exp(-it) F(p),$$

where one should note that the vector of current outputs of the three productive sectors, F , depends not only on the explicit argument p but also, in general, on time.

A similar reasoning gives the corresponding discounted stream of investment goods purchases,

$$(4a.2) \quad V(p) \equiv \int \exp(-it) I(p),$$

where $I(p)$ is the four-dimensional vector of current levels of capacity installed—gross investment—by the four sectors, including dismantling services—this is so since even though services are not traded, the capital goods used to produce them are—. Therefore, the net present value of output minus investment purchases can be computed by pricing them out using their prices p and $q=c(p)$, respectively, as

$$(4a.3) \quad O(p) \equiv p \cdot G(p) - q \cdot V(p),$$

and is assumed to be maximized by the private sector as a whole, since competitive conditions prevail there.

The same net output program, evaluated at international prices,—compare with (1.6) becomes

$$(4a.4) \quad O^*(p) = p^* \cdot G(p) - q^* \cdot V(p)$$

where of course q^* is determined on the basis of the international prices p^* using the unit cost function, i.e. $q^*=c(p^*)$.

If the net external indebtedness or borrowing from abroad at the initial time 0 is denoted by B , then the net present value of the aggregate income available for the consumption programs of the private households, given domestic prices p but again evaluated at international prices, is

$$(4a.5) \quad Y^*(p) \equiv O^*(p) - B,$$

which provides the balance of payments equilibrium condition, expressed in terms of stocks, as soon as the left hand side is set equal to the net present value of consumption expenditures at international prices. It is then exactly Fisher's equation, stating that over the period of time between the present and the planning horizon, the present value of the consumption program plus the initial debt should equal the net present value of the income stream, represented in this case by the net effect of the production and investment streams.

The net present value of aggregate consumption expenditures induced by domestic prices p evaluated at international prices can be calculated, recalling equation (3.4), by pricing out the discounted consumption stream using p^* , so that

$$(4a.6) \quad Z^*(p) = Y(p) \cdot p^* \cdot c(p).$$

After equating its left hand side with that of (4a.5), this gives a relation which permits the computation of the net present value of the aggregate income stream $Y(p)$. As a check, substituting domestic prices p instead of p^* in this formula to evaluate consumption, one obtains

$$(4a.7) \quad Z(p) = Y(p)$$

so that consumption expenditure equals the income earmarked for consumption. Thus one arrives at the value of $Y(p)$,

$$(4a.8) \quad Y(p) = Y^*(p) / p^* \cdot c(p).$$

The left hand side is $Y(p) / p \cdot c(p)$ —by definition the denominator of this expression is equal to one—, so that this equality just says that external balance requires that income used for consumption stands in the same relation to the value of a typical consumption basket, no matter whether domestic or international prices are used to evaluate both concepts.

It is assumed that in the initial situation the foreign debt is paid out of a value added tax. For the initial aggregate budget of the private sector to balance, that initial tax rate must equal the ratio

$$(4a.9) \quad \theta^0 = [O(p) - Z(p)] / O(p)$$

of the excess of the present value of before tax earning over the present value of consumption expenditures to the former.

b) Free trade

i) *Equilibrium under free trade, and optimality*

Social welfare is an increasing function of aggregate income $Y(p)$, deflated by the consumer price index $v(p)$. This means that a prerequisite for social welfare maximization, given the restriction that the relative income distribution is to be held constant, is that real income so defined be maximal.

Since the initial foreign debt is given, this means that the aggregate net present value of the private productive sectors should be maximized, of course using the international prices and interest rate, which are the only prices which from the point of view of the planner are given.

The usual argument for showing the optimality of free trade for a small country can be extended from its comparative statics framework to the present case of comparative dynamics. The main content of the balance of payments equation (4a.5) is that even though domestic prices may have been distorted so that their ratios—and therefore also the domestic marginal rate of productive transformation—do not coincide with relative international prices, the plans of consumers and producers must be consistent with external balance. Of course also the domestic production and factor supply restrictions cannot be exceeded no matter how distorted the price system is. This means that no matter how the economy is organized, the joint production, investment, consumption and employment program must be attainable. Thus in particular the *statu quo* must

represent a feasible starting point. Since it is practically impossible for the *statu quo* to be a social optimum, the latter will certainly offer a better aggregate consumption plan, and therefore it is obvious that the socially optimal program will in general provide a higher level of welfare. It is then the responsibility of the government to compensate the losers.

A slightly more formal argument can be easily provided in the present situation. At the given, distorted domestic prices p , the discounted production and investment plans $G(p)$ and $I(p)$ are jointly feasible—there is no way one can cheat on resource availability or technological knowledge—. Therefore, the net present value of this aggregate production and investment plan at international prices $O^*(p) = p^* \cdot G(p) - c(p^*)$. $V(p)$ is attainable under free trade by the profit maximizing private sector; in fact since the previous program was not decided by taking international prices into account, entrepreneurs can do better, so that the optimal plan under free trade has a value $O(p^*) > O^*(p)$. This in turn implies that the net discounted value of output available for consumption once the initial foreign debt has been deducted, again valued at international prices, satisfies the same inequality, that is to say, $Y(p^*) = O(p^*) - B > O^*(p) - B = Y^*(p)$.

It only remains to show that real income satisfies the same inequality, so that

$$(4b.1) \quad Y(p^*)/v(p^*) > Y(p)/v(p) = Y^*(p)/[v(p) p^* \cdot c(p)],$$

where the last equality is obtained from (4a.8) by dividing both members of the equation by the consumer price index $v(p)$. The foregoing argument has shown that the numerators of the two extremes of this relation have the correct magnitudes. In order to show that the denominators are also ordered correctly, consider the consumption bundle $v(p) c(p)$, obtained by multiplying the bundle $c(p)$ corresponding to unit expenditure at prices p by the minimum expenditure $v(p)$ required to attain the unit level of utility at those prices. Since the expenditure required by the bundle is $p \cdot [v(p) c(p)] = v(p) p \cdot c(p) = v(p)$, it is clear from the definitions that the utility level the typical households derives from it is unity. Thus the bundle gives the required satisfaction of one util; but since it is not adapted to the international prices p^* , its value at those prices, $p^* \cdot c(p) v(p)$, must exceed the minimum expenditure needed to attain the same level of utility, which by definition is $v(p^*)$. This shows that the denominator in the left hand side of (4b.1) falls short of the denominator in the far right hand side, so that the two sources of inequality combine in the right direction to prove that the assumed inequality in fact holds true.

ii) *The shadow foreign exchange rate*

The model has been formulated so as to show explicitly the shadow exchange rate. It corresponds to the Lagrangean of the Fisherian balance of payments condition in (4a.5) and measures the value to the country of an additional unit of foreign exchange in terms of consumption or "utils". Its level is not really relevant, since only relative prices matter when there is no money illusion. What is important is that it should be constant if the domestic interest rate is held equal to the international rate, otherwise foreign exchange should be devalued or revalued so as to maintain the equality of the real earnings of all assets. Thus for example a rate of devaluation of the domestic currency of ϕ would require a domestic interest rate of $\phi + i$ in order to keep the foreign accounts in line—note that for simplicity it is assumed that there is no foreign inflation; the foreign currency just is quoted so that its value is constant in real terms, otherwise its rate of devaluation should be subtracted from the previous formula.

It should also be mentioned that an alternative presentation of the external restriction could have been given by the differential inequality

$$p^*.c + (q^*.I - p^*.F) + iB \leq DB; B(0) \text{ given,}$$

stating that the net trade deficit—consumption plus investment minus output—plus interest on the debt go into additional debt (or debt reduction in case that this sum is negative). This form is of course equivalent to the one used in this paper, though it would require a slightly longer computation for the shadow exchange rate.

5. Implementing a social optimum under distributive constraints

In the preceding section it has been shown that a social optimum requires that domestic prices be proportional to international prices; such a solution has been designated with the name "free trade". In fact, it is really not necessary that trade be free; what is required is that the marginal conditions coincide with those of free trade. This is independent of the institutional setup of the economy.

In the present section it will be shown how such marginal conditions can be satisfied, always with the restriction of a constant relative income distribution binding. It is a different problem to attain a more egalitarian distribution; the problem of a better income distribution is already present before a better trade policy is adopted. It is the stand of the present paper that this problem either has been solved before, so that the relative income distribution already is the desired one, or that it will be solved later, once the benefits of a more rational foreign trade policy have been reaped. Only the feasibility of a policy that will produce a Pareto-improvement will be discussed, since it is deemed that such a policy would not be resisted by the different sectors once it is understood that everybody will gain from it.

It is obvious from general considerations, and it has been shown in the previous section, that efficiency in the allocation of resources requires that all productive activities be priced equally in the margin. On the other hand consumption expenditure of household i should be proportional to its statu quo consumption expenditure Z_i^0 . How can this be reconciled with the marginal pricing requirement? The usual answer of welfare theory is that income redistribution should be based on lump sum taxes, which are independent of the activity of economic agents and so are not taken into account in their decisions.

But the inevitable question arises, how can such lump sum taxes be implemented? In any country there exists a constitution—by it we mean not the usual legal concept, but the set of property rights guaranteed, be it by the constitution or by some other means of social consensus—which in general will impede the adoption of economic policies which may hurt some of the income classes. Usually such property rights will have the consequence of preserving the statu quo, so as to maintain the income redistributions within more or less strict limits.

In the sequel it will be shown that it is not necessary to change the constitution in order to improve trade policies. Of course the present proposal is not the only possible one, but it is the belief of the authors that it is the simplest one that may achieve the purpose of bringing domestic prices in line with international prices which at the same time will not be resisted by the country's pressure groups.

Let E_i be the earnings of group i according to the social optimal solution. Let Z_i^0 be the present value of the group's consumption expenditures. Then if $Z(p^*)$ is the new

aggregate expenditure, whereas $Z(p)$ is the corresponding concept in the initial situation, then the amount it is entitled to is $Z_i(p^*) = [Z_i^0 / Z(p)] Z(p^*)$.

If the workers are the lower income group, a value added tax with a rate equal to $\theta \equiv (1 - Z_L/E_L)$ will be just high enough to absorb their excess earnings. It is recommended that such a value added tax be imposed, so as to free this group from the administrative problems associated with the rest of the policy proposal. In order to compare this tax rate with the initial one, $0 -^0$, determined in (4a.8), it should be taken into account that the aggregate before tax earnings of the private sector amount to

$$\sum_H E_i = O(p^*),$$

so that the formula for θ just described is based on the same type of concepts, a ratio of unspent earnings to before tax earnings. If workers are the lower income classes, then it is to be expected that the new value added tax will be lower than the old one.

In order to make up for the difference remaining to balance the governments' budget, an additional lump sum tax would be in order, with a magnitude equal to

$$(5.1) \quad (1 - \theta) E_i - Z_i,$$

for each of the other income groups, if the social optimum is to be achieved. But as has been argued before, such a measure is not feasible.

If it is not possible to impose a lump sum tax directly, a differential exchange rate regime might be used as a substitute. But since taxes affecting the marginal conditions are known to create incentives for inefficient behavior, it is the proposal of the authors that these instruments be applied only on existing trade, not on the new trade created by the opening up of the economy! Thus, exporters would have to sell a certain fixed—though possibly varying over time—amount of foreign exchange to the Central Bank at the given official exchange rate; they would be allowed to sell all incremental foreign exchange earnings at par in the free market together with the other two sectors.

For a simplified example, if the rate at which sector i was earning foreign exchange in the past was equivalent to G_i in the domestic currency during the latest instant—or perhaps during some average past period in case there are important cyclical irregularities such as are usual in the agricultural sector due to climatic problems—then it can be inferred that with the economy growing at the long run rate n the amount $G_i \exp(nt)$ would be provided by that sector at time t . In that case, one possible arrangement would be to force it should sell that amount forever at an exchange rate related to the free market rate by a certain factor of h_i . The net present value of the corresponding income stream, given the international interest rate i , would of course be equivalent to a lump sum tax of

$$(5.2) \quad \int (1 - h_i) G_i \exp[(n - i)t] = (1 - h_i) G_i / (i - n),$$

which when compared with the expression in (5.1) yields the equation

$$(1 - \theta) E_i - Z_i = (1 - h_i) G_i / (i - n),$$

from which the needed exchange rate par-factor can be computed as shown by the following formula,

$$(5.3) \quad h_i = 1 - (i - n) [(1 - \theta) E_i - Z_i] / G_i$$

Some observations on this formula are in order. If the value added tax just wipes out the excess earnings of workers, as suggested before, and if—as is probable—this leaves all other income groups with additional excess earnings, then the expression within square brackets in (5.3) will be positive, as will be the first parenthesis, especially in the case of countries with slow population growth such as Argentina. The rates of past foreign exchange earnings presumably can be ordered by size as follows, $G_T \gg G_N > 0 > G_M$, meaning that the largest foreign exchange earnings stem from traditional exports, followed far behind by the non-traditional export sector, whereas the import substitutes sector uses foreign exchange. This means that for the exporting sectors the corresponding official exchange rate should be set below the par, whereas it should be set above the par for the import substitutes sector. This of course is a burden—a tax—for all the sectors; in the case of the latter because they have to buy at an official rate which is above the market rate, whereas the former have to sell under par.

More explicitly: In the case of the *differential non distorting exchange rates* the harmed import substituting sector turns out to be compensated by the trade opening, since in each period it obtains a certain amount of foreign exchange from the Central Bank at a price that is lower than that of the foreign exchange free market. The exportables sectors, on the other hand, must surrender to the Central Bank a certain amount from its net flow of exports in each period, representing its sales of foreign exchange to the market, at a price which also is lower than that in the free foreign exchange market. In other words, for each dollar it is forced to sell to the Central Bank it ends up paying the difference between said price and that of the market. The gap between the exchange rates is transferred from one sector to the other by the Central Bank, making explicit the subsidy that before was implicitly paid by Society as a whole.

In the case of the non distorting sales or value added tax, the economic meaning of the tax-subsidy mix is equivalent to the previous case. Import substituting sectors will perceive after the opening of the economy a compensation in the form of a subsidy equivalent to the percentage on the flow of sales it had as a producer of the import competing goods. In other words, during any post-opening future period it will continue to obtain that subsidy independently from the output level after the opening. As it is to be expected, due to the change in relative prices against importables this sector will be induced to reduce its output, even though the subsidy would permit it to continue acting as up to the present without incurring in lower net income. The policy has certain similarities with that carried out for their agricultural sectors by the European Economic Community and the United States of North America, the latter through their Reduced Acreage Program (RAP). The effect is to *dissociate the subsidies from current output*.

The subsidy is the lump sum compensation that can smoothen the *reconversion of the unutilized physical and human capital* toward the sectors producing exportables. More concisely: if they maintain the productive levels corresponding to the *statu quo* they will be better off than before if tariffs are eliminated *and* they are compensated, though if they move to the free trade equilibrium of maximum profits they will be better off still.

In the case of exportable sectors, the meaning of the retrospective sales or value added tax is the tax that they will have to pay on the flow of its historically given sales at domestic prices, so as to absorb the positive surplus that the opening will generate in these sectors which endured in the past a considerable anti-export bias, in some cases having to survive under heavily negative effective rates of protection.

It means that the opening of trade increases the efficiency of the economy to such a degree that the producers of exportables are able to pay this lump sum tax, computed

on their historically given sales. Furthermore it is important to point out that there exists no possibility of market substitution (internal or external) for the exporters, since the tax base belongs exclusively to the past and is perfectly identifiable. There are not two prices coexisting simultaneously that would induce substitution, since the past price has been written down in the accounting books and that of the future requires no additional controls, not being subject to new taxes.

It is important to point out that the amounts of foreign exchange to be exchanged in the official market must be independent of economic conditions if they are to serve the purpose of correcting the income distribution —or as in the present investigation maintaining the statu quo— so as not to interfere with the marginal conditions for a social optimum; any additional trade should be allowed to be carried out at the free market rate.

6. Conclusions

The paper has shown that it is possible to go behind the gloomy results so far attained when investigating the short run macroeconomic effects of trade liberalization. The paper sets up a simple dynamic growth model for an open three traded goods economy, price taker in international commodity and asset markets, which incorporates into the analysis a nontraded sector providing *dismantling* (or disinvestment) services to those sectors of the economy which have to contract.

Every sector uses non-homogeneous labor skills and productive capacity, and the final production flows of the goods producing sectors are utilized for consumption, exports, or capital accumulation purposes.

From the point of view of consumption, households were classified into four income groups, depending on the origin of their income, i.e., three capitalist groups corresponding to the three productive sectors, and one labor income group, irrespective of the place of employment.

The socio-political objective of the citizenry is conceived so as to preserve the relative income distribution corresponding to the initial statu quo or pre-liberalization productive equilibrium.

Using an optimizing analytical framework —a Bergson-Samuelson egalitarian social welfare function— the paper formalizes the application of a policy package consisting of a system of backward looking differential exchange rates or value added tax rates *on existing trade and sales flows* in order to deal with the medium run income distribution problems of implementing the optimal (extended) free trade solution. The tools formalizing the compensation have been selected so as to be easily implementable and not to interfere with economic incentives.

Simultaneously, the growth liberalization policy proposal being advanced is designed so as to deal with the problem of efficiency in resource allocation *in the new trade flows* created by the opening up to the economy, without inducing any political resistance on the part of the country's harmed income groups. It is shown that it is possible to increase every income group's wellbeing; the final solution is a strict Pareto improvement, so that it should be relatively easy to attain a social consensus for its adoption, leaving the political discussions for the problem of the distribution of the gains. The essay shows that society is perfectly able to compensate the losers and at the same time improve general welfare in terms of the chose index, i.e., the consumption level in *every* future period for *each* of the social groups considered.

Our investigation reveals that for a small open developing economy the optimal long run policy is undoubtedly the application of the principle of the leveled compensation—which could be termed extended free trade.

The principle admits the following optimal variants—compare with Mantel and Martirena-Mantel (1986)—all of which imply the fulfillment of the condition of equality between the domestic resource cost (DRC) of saving foreign exchange through import substitution and the DRC of earning foreign exchange through export activities. In other words, the principle simply means the desirability of an equal treatment for sales of commodities for domestic and foreign markets.

The variants are the following.

a) Outright free trade policy—although not necessarily accompanied by domestic laissez faire if there are domestic distortions (see Martirena-Mantel (1986)).

b) In a three sector economy with traditional exports, nontraditional exports and import substitutes, if the economy is a price taker regarding only the last two sectors, the compensated leveling scheme means that when a tariff on the importable sector is *optimally* applied, then there should also be subsidies on nontraditional exports.

c) Mutatis mutandis, whenever subsidies are not politically feasible, the optimal policy consists of taxing imports and simultaneously, taxing traditional exports.

Undoubtedly and outright application of the *extended free trade principle* in some economies immediately raises the problem of implementing the social long run optimum, which is the focal contribution of the present proposal.

According to Kaldor (1964), the remedy to this dilemma lies in the adoption of a dual nominal exchange rate regime, in line with the Pigovian medicine of adjusting relative prices in accordance with marginal social opportunity costs. In a developing economy—the argument goes on—there is no unique exchange rate capable of equilibrating internal and external costs of traded commodities.

According to the results in the present paper instead, the growth liberalization proposal for short run implementation of the social optimum maintains the statu quo income distribution by a system of backward-looking differential exchange rates which work as a lump sum tax system, i.e., they do not affect marginal decisions. In addition the optimal scheme absorbs all windfall gains of the more efficient sectors. Hence all problems of income distribution are taken care of by the proposal.

Gaining external competitiveness under unequal income distribution conditions does not necessarily end up with a reduced standard of living.

The present proposal, furthermore, clearly disposes of the so called *false trade-off* or *quid pro quo* between inward or outward industrialization policies of some Latin American structuralists as Pinto (1975) or French-Davis (1986).

Since the proposal refers to the move from a highly distorted initial situation to the social optimum, it can be made perfectly compatible with the apparently heterodox export policies of some real life economies such as South Korea, Brazil or Japan. As a matter of fact, the final result of the proposed backward looking differential exchange rate system of “protection” would be the absence of any conflict or dilemma of the sort caused by the implicit equalization of incentives in the margin. “Natural” import substitution sectors under the extended free trade system will replace “forced” import substitution industries under the old distorted system of inefficient and discriminatory incentives.

The growth liberalization here proposed, to be implemented during the transition path, implies equilibrium exchange rates, once the income redistribution impacts of the initial dismantling stage have been solved in the manner suggested.

7. References

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Appendix

Note: due to typographical limitations some symbols have two meanings:

- $c(p)$ cost of capacity
 $c(p)$ consumption for unit income
 i subscript for individual or household
 i international interest rate
 E nominal exchange rate
 E_i earnings of group i
 t time
 t_j tariff rate of sector j

1. Productive sectors

The set of productive sectors is $S = \{T, N, M, D\}$, where

traditional exportables	(T) — Catle
non-traditional exportables	(N) — Cars
importables	(M) — High-Tech Machines
dismantling (disinvestment)	(D) — Services

Time will be represented by t . Though the variables of the system depend on time, this dependence will in general not be made explicit unless strictly necessary for clarity.

The factors of production are labor (L) and capital (K); their quantities employed in sector j are L_j and K_j , respectively. The capital/labor ratio is k_j .

The unit cost function $c()$ of capacity —the cost of a unit of capacity— is

$$q_j = c^j(p) \quad \text{for } j \text{ in } S,$$

and relates output domestic prices

$$p = (p_T, p_N, p_M),$$

to investment goods domestic prices

$$q = (q_T, q_N, q_M, q_D),$$

where

$$p_j = E (1+t_j) p^*_j \quad \text{for } j \text{ in } S \setminus D,$$

and E is the nominal exchange rate, t_j the ad-valorem tariff rate, and p^*_j the given international price.

The share of good j in the price of the composite investment good of sector i is, for i in S and j in $S \setminus D$, a_{ij} . This share is the capacity elasticity of the corresponding capital goods; it is also the elasticity of the price of the capacity of the sector with respect to the prices of the different capital goods, so that

$$(1.1) \quad \hat{q} = A \hat{p},$$

where the caret ($\hat{\cdot}$) indicates the relative rate of change of the variable below it, e.g. $\hat{x} \equiv d \log x$.

The production function of sector j is

$$F^j(I_j, K_j) \equiv f_j(k_j)L_j,$$

where $F' > 0$; $F'' < 0$; $f > kf'$; $f(0) = 0$; $\lim_{L \rightarrow 0^+} f(1/L)L = 0$.

Employment is restricted by the labor transformation function $\Lambda(L)$, which is strictly quasi-convex, linear homogeneous, and increasing in its argument, the labor employment distribution

$$L = (L_T, L_N, L_M, L_D).$$

The derivative Λ_j of this function with respect to employment in sector j is the marginal population requirement of sector j . The labor availability constraint can then be expressed as follows, with $P = P(0) \exp(n t)$.

$$(1.2) \quad \Lambda(L) \leq P$$

The international interest rate is fixed at i .

The rate of capital depreciation is d , so that the restriction on gross investment I_j of sector j becomes

$$(1.3) \quad I_j \geq DK_j + dK_j.$$

Dismantling services are asymmetric, and their quantity is defined as negative gross investment if not positive,

$$(1.4) \quad D_j \equiv \max \{ 0, -I_j \}.$$

This definition, together with the condition that investment goods are measured in units which require similar dismantling services, allows the presentation of the dismantling capacity restriction

$$(1.5) \quad D_M + \dots + D_D \leq F^D(K_D, L_D)$$

In the sequel, since all integrals refer to the infinite time planning horizon, the symbol

$$\int_0^\infty dt$$

will be represented throughout just by $\int$.

The net present value of output of the private sector to other sectors, with labor and dismantling services internalized, is

$$(1.6) \quad O(p) \equiv \int \exp(-it) \left[\sum_{S \setminus D} p_j F_j^I(L_j, K_j) - \sum_S q_j I_j \right]$$

and is to be maximal subject to the following restrictions, each followed by the corresponding non-negative shadow price (Lagrange multiplier),

- employment constraint (1.2) y
- investment constraints (1.3) m_j
- dismantling definition (1.4) s_j
- dismantling services constraint (1.5) p_D

and the non-negativity conditions $(L, K, D) \geq 0$.

The economic interpretation of these shadow prices is the following.

- y — income of the average worker family
- m_j — shadow value of capacity to sector j
- s_j — value of dismantling services to the user, sector j
- p_D — value of dismantling services to the producer of D .

2. Production programs under free and restricted trade

a) Short run optimality (Kuhn-Tucker) conditions

The short run optimality (Kuhn-Tucker) conditions corresponding to the optimization problem presented in section 1 of this appendix, valid for all sectors j in S and all moments t , are the following.

$$(2a.1) \quad \exp(it) \partial L / \partial L_j = p_j F_L^I - y \Lambda_j \leq 0 \quad [=0 \text{ if } L_j > 0]$$

$$(2a.2) \quad \exp(it) \partial L / \partial K_j = p_j F_K^I - (i+d) m_j + D m_j \leq 0 \quad [=0 \text{ if } K_j > 0]$$

$$(2a.3) \quad \exp(it) \partial L / \partial I_j = -q_j + m_j + s_j = 0$$

$$(2a.4) \quad \exp(it) \partial L / \partial D_j = -p_D + s_j \leq 0 \quad [=0 \text{ if } D_j > 0]$$

The validity of the Kuhn-Tucker theorem using Lagrange multipliers in the present continuous-time optimization case has been established by Hurwicz (1958). A different approach, based on the Hamiltonian, is used for example by Arrow and Kurtz (1970), but is fully equivalent.

b) Long run regularly progressive solution

In the long run, for all j in S

$$DK_j = n K_j,$$

so that from the investment constraint (1.3) one obtains

$$(2b.1) \quad I_j \geq (d+n) K_j.$$

Consider now any sector producing some output. Since both capacity and labor are indispensable, it will be using some quantities of both of them. Therefore gross investment, I_j , will be positive, as will be the sum of gross investment and dismantling, D_j , since the latter is non-negative. This implies due to constraint (1.4) that the shadow value of dismantling to the sector, s_j , is zero.

The derivative of the Lagrangean with respect to I_j shown in (2a.3) simplifies to

$$m_j = q_j,$$

and therefore the shadow value of capacity is constant, with a zero time-derivative.

Since the Kuhn-Tucker conditions for K and L will be satisfied with equality, one obtains from (2a.2) the two usual conditions on their marginal products. In the case of labor it is

$$(2b.2) \quad p_j F_L^j = w_j,$$

where the shadow wage rate of sector j has been defined as

$$(2b.3) \quad w_j \equiv y \Lambda_j,$$

and for capacity one has

$$(2b.4) \quad p_j F_K^j = (i+d) q_j.$$

In the case of the productive sectors j in $S \setminus D$, equation (2b.4) determines then the capital-labor ratio, k_j , in terms of the exogenously given prices, p and q , interest rate, i , and rate of depreciation, d . Consequently the left hand member of equation (2b.2) gives the wage rate, w_j , of sector j .

Suppose now that it is the dismantling sector D that produces some output. The preceding reasoning then applies also, except that the price of its output is not given exogenously. Nevertheless, the condition (2b.4) on the marginal product of capital insures that the shadow price, p_D , of dismantling services is strictly positive.

For all sectors j in S , (2b.1) implies that gross investment cannot be negative in the long run; therefore, if sector j demands dismantling services, so that $D_j > 0$, the shadow value s_j of these services, being associated with constraint (1.4), must be zero for this sector. From the derivative (2b.4) of the Lagrangean w.r.t. D_j one can then infer that no sector demands these services, since

$$0 = s_j < p_D$$

implies that $D_j = 0$. Furthermore a positive shadow price for dismantling services guarantees that the corresponding constraint (1.5) should be satisfied with equality, and this of course means that no such services will be produced in the long run regularly progressive solution.

Having disposed of the dismantling sector, the rest is quite straightforward. Since the wage rates, w_j , of the sectors j in $S \setminus D$ are known, from their definitions (2b.3) one obtains for j in N, M the ratios

$$w_j / w_T = \Lambda_j (1, l_N, l_M, 0) / \Lambda (1, l_N, l_N, 0),$$

where use has been made of the homogeneity of Λ to divide the arguments by employment of sector T and defining for j in N, M the ratios $l_j \equiv L_j / L^T$.

One obtains a system of two equations in two unknowns which under the assumptions made can be solved for the ratios l_j . Writing the employment restriction (1.2) as

$$L_T = P / \Lambda(1, l_N, l_M, 0),$$

which is again possible due to homogeneity, one deduces the level of employment in sector T, and from it and the ratios l_j the employment levels in the remaining sectors.

To summarize, it has been shown how to solve the system consisting of the following sets of equations, where j is in $S \setminus D$, giving a total of ten equations in ten unknowns:

$$(2b.5) \quad p_j F_L^j(L_j, K_j) = w_j$$

$$(2b.6) \quad w_j \equiv y \Lambda_j(L_T, L_N, L_M, 0)$$

$$(2b.7) \quad p_j F_K^j(L_j, K_j) = (i+d) q_j$$

$$(2b.8) \quad \Lambda(L_T, L_N, L_M, 0) = P$$

In order to check the reasonableness of the existence of a solution of these equations, note that they correspond to the Kuhn-Tucker conditions for the following maximization problem:

$$\max \quad \sum_{S \setminus D} [p_j F^j(L_j, K_j) - q_j (d+i) K_j]$$

$$\text{s.t.} \quad \Lambda(L_T, L_N, L_M, 0) \leq P$$

c) Long run comparative dynamics

In order to obtain relations between the relative rates of change in the variables induced by changes in tariff rates it is necessary to differentiate logarithmically the equations for the long run regularly progressive solution of section b. The following elasticities will be required.

A = matrix of sector capacity/capital goods elasticities

$$= (a_{ij}), i \text{ in } S, j \text{ in } S \setminus D$$

s_L = vector of sector output/employment

$$= (\partial \log F^j / \partial \log L_j), j \text{ in } S$$

s_K = vector of sector output/capacity

$$= (\partial \log F^j / \partial \log K_j), j \text{ in } S$$

sig = vector of sector elasticities of L/K substitution

$$= \partial \log (K_j/L_j) / \partial \log (F_L^j/F_K^j), j \text{ in } S$$

S = matrix of labor mobility substitution

$$= 4 \times 4 \text{ submatrix of the inverse of the bordered Hessian matrix of } \Lambda$$

lam = vector of population/employment elasticities

$$= \partial \log \Lambda / \partial \log L_j, j \text{ in } S$$

Then, if e is a vector with all its coordinates equal to unity, the differentiated relations are as follows.

From the definition of the matrix A of sector capacity/capital goods elasticities one has

$$(2c.1) \quad \hat{q} = A \hat{p},$$

which just repeats equation (1.1).

The capital/labor ratio depends on the wage/rental ratio,

$$(2c.2) \quad \hat{k} = \text{diag}(\text{sig}) (\hat{w} - \hat{q}),$$

where $\text{diag}(v)$ represents a diagonal matrix with the coordinates of the vector v along its main diagonal. This follows from the definition of elasticity of substitution, taking into account that the rental of capacity is in a constant proportion—given by the exogenous factor $r(i+d)$ —to its price.

Output prices depend on factor prices,

$$(2c.3) \quad \begin{aligned} \hat{p} &= \text{diag}(s_L) \hat{w} + \text{diag}(s_K) \hat{q}, \\ &= \hat{q} + \text{diag}(s_L) (\hat{w} - \hat{q}). \end{aligned}$$

This follows from the fact that s_L^j and s_K^j are also the shares in total output of labor and capital in sector j , respectively. This implies that they sum to unity.

From the previous two relations (2c.1, 2), eliminating wages and rentals, one obtains

$$(2c.4) \quad \hat{k} = \text{diag}(s_L)^{-1} \text{diag}(\text{sig}) (\hat{p} - \hat{q}),$$

whereas (2c.2) can be rewritten as follows

$$(2c.5) \quad \hat{w} = \hat{q} + \text{diag}(\text{sig})^{-1} \hat{k}.$$

The relation between sectoral wage rates and employment is

$$(2c.6) \quad \hat{L} = S \text{diag}(\text{lam}) \hat{w},$$

and follows from the definition of labor mobility substitution in the usual Hicks-Allen way—see Uzawa (1962).

The relative shift in sectoral capacity obviously can be obtained from the definition of the capital/labor ratio and is

$$(2c.7) \quad \hat{K} = \hat{L} + \hat{k}$$

Finally, the rate of change in worker households' average income is

$$(2c.8) \quad \hat{y} = \text{lam} \hat{w},$$

a weighted average of the relative sectoral wage increases weighted by the share of the sector's wage bill to total labor income. This follows from the fact that lam_j is also the share in aggregate labor income of labor employed in sector j .

It is now easy to work out the solution for given tariff changes. These imply relative commodity price changes—in fact, the relation between the change in the tariff of sector

j and the corresponding domestic price is $Dt_j = (1+t_j) \hat{p}_j$. Following in order the equations (2c.1, 4-8) one can then obtain directly the relative rates of change of q , k , w , L , K , and y . Thus one obtains in succession:

$$\begin{aligned}
 (2c.1') \quad \hat{q} &= A \hat{p} \\
 (2c.4') \quad \hat{k} &= \text{diag}(s_L)^{-1} \text{diag}(\text{sig}) (I - A) \hat{p} \\
 (2c.5') \quad \hat{w} &= \text{diag}(s_L)^{-1} [I - \text{diag}(s_K) A] \hat{p} \\
 (2c.6') \quad \hat{L} &= S \text{diag}(\text{lam}) \text{diag}(s_L)^{-1} [I - \text{diag}(s_K) A] \hat{p} \\
 (2c.7') \quad \hat{K} &= \{ [S \text{diag}(\text{lam}) + \text{diag}(\text{sig})] \text{diag}(s_L)^{-1} (I - A) \\
 &\quad + S \text{diag}(\text{lam}) A \} \hat{p} \\
 (2c.8') \quad \hat{y} &= \text{lam} \text{diag}(s_L)^{-1} [I - \text{diag}(s_K) A] \hat{p}
 \end{aligned}$$

3. Consumption groups

The preferences of the representative consumer are of the Ramsey type. Prospective utility is additive, and is defined over the set of admissible consumption programs—bounded, measurable functions on the non-negative real line to Euclidean n -dimensional space—which extend from the present initial time 0 to the infinite planning horizon. They are convex, homothetic, and independent from one time interval to another, so that they can be represented by an instantaneous utility function $u(c)$ —which is increasing, strictly concave, homogeneous, and with marginal utilities tending to infinity as consumption rates tend to zero (this just to sidestep irrelevant corner solutions)—multiplied for each period by a discount factor $\exp(-rt)$ — r being the rate of time preference—and integrated. The triple $c=(c_T, c_N, c_M)$ is the household's current consumption bundle. The corresponding consumer price index is $v(p) \equiv \min \{ p.c | u(c) \geq u_0 \}$, i.e. the cheapest way to provide a certain given level, u_0 , of utility. If the degree of homogeneity of $u(\cdot)$ is denoted by $1-b$, where $0 < b$, then one has from the Sheppard duality theorem that¹

$$\max \{ u(c) \mid p.c \leq 1 \} \equiv [v(p)]^{(b-1)} / (1-b).$$

The optimization problem for any household can be expressed as that of maximizing prospective utility

$$\int \exp(-rt) u(c)$$

subject to the budget constraint

$$\text{s.t. } \int \exp(-it) p.c \leq y,$$

where y represents the net present value of its income. This is equivalent to maximizing

$$[v(p)]^{(b-1)} \int \exp(-rt) z^{(1-b)}$$

¹ The limiting case $b=1$ corresponds to the log-homogeneous utility function $u(tc)=u(c)+\log(t)$, where the indirect utility function equals the logarithm of the reciprocal of the price index, i.e., $-\log(v(p))$.

subject to the constraint on consumption expenditure

$$\int \exp(-it) z \leq y,$$

where z represents the household's current consumption expenditure.

In the foregoing formulation of the utility function in terms of consumption expenditure it can be seen at first sight that $1/b$ is the intertemporal elasticity of substitution of the consumption expenditure of any one instant for that of another instant.

It is easy to see that the optimal consumption expenditure program satisfies

$$\exp(-rt) [z / z(0)]^{-b} = \exp(-it)$$

or, equivalently,

$$z = z(0) \exp[-(r-i)t/b].$$

From the budget constraint one obtains the value for $z(0)$,

$$z(0) = y [r - (1-b)i] / b.$$

The solution exists whenever $r > (1-b)i$; this will certainly be true if $b \geq 1$, or if the rate of time preference r of the representative consumer is close to that of the rest of the world implicit in the international interest rate i .

The total discounted utility attained by the household is proportional to

$$(3.1) \quad U(p, y) \equiv [y/v(p)]^{(1-b)}$$

where the factor of proportionality is the constant $b^b/[r - (1-b)i]^b$. It will be ignored in the sequel since rescaling the utilities by a positive constant has no effect on the consumers' preferences. It is easily seen that the prospective utility is an increasing function of income deflated by the consumer price index, which will be called real income. The elasticity of prospective utility with respect to real income is seen to be $1-b$.

Households will be classified into four categories, depending on the origin of their income. Three of these groups are the owners of the firms of the three productive sectors producing goods T , N , and M ; the fourth corresponds to workers, irrespective of their place of employment. The set of income perceiving groups will be denoted by $H \equiv \{T, N, M, L\}$. The dismantling sector has no separate owners; whichever sector requires dismantling services sets up its own dismantling facilities —of course this is possible without loss in efficiency because of constant returns to scale.

At the present time, $t=0$, the income distribution —the aggregate net present value of each of the four income perceiving groups— is given by $Y_T^0, Y_N^0, Y_M^0, Y_L^0$, for the capitalists of sectors T , N , and M , and labor, respectively. The corresponding relative income distribution represents the statu-quo which it is deemed from the political point of view to preserve. The following analysis will seek to preserve the relative income distribution. Since all households are equal, this means —taking into account the assumptions made in the text— that aggregate welfare is

$$W = \sum_H N_j U(p, y_j)$$

where N_j represents the number of factor owners in sector j for j in H —in particular $N_L \equiv P$, the worker population—, and y_j represents the corresponding mean income,

i.e. income Y_j divided by the number of households in the group. The population is assumed to be in demographic equilibrium, so that all income groups grow at the same exogenous rate n .

Using equation (3.1) and the definition of mean class income one arrives then at the following expression for social welfare,

$$W = [v(p)]^{(b-1)} \sum_H (N_j)^b [Y_j]^{(1-b)}$$

where H is the set of income perceiving groups defined above. This expression can be further simplified because of the restriction that the relative income distribution is to be preserved, since in that case one has $Y_j = (Y_j^0/Y^0) Y$, where Y and Y^0 refer to final and initial aggregate income, respectively. Then one has that welfare is proportional to

$$(3.2) \quad W(p, Y) \equiv [Y/v(p)]^{(1-b)}$$

with the constant of proportionality

$$\sum_H (N_j)^b [Y_j^0/Y^0]^{(1-b)}$$

which again will be neglected in the sequel since only welfare changes preserving the relative income distribution of the statu-quo will be considered.

For use in a later section, it should be noted that due to the homogeneity of the instantaneous utility function $u()$ it is relatively simple to determine the present value of any consumer's optimal program. Let $c(p)$ be the consumption bundle which maximizes instantaneous utility when current expenditure is equal to one. Then is clear that at times, any household will consume in proportion to the coordinates of this vector, and therefore also the quantities consumed over the whole planning horizon, when discounted to the present and added up over time and over households, will be in the same proportions. Let $C(p)$ stand for this discounted aggregate consumption program, i.e.,

$$(3.3) \quad C(p) \equiv \int \exp(-it) c(p) z = h c(p),$$

since $c(p)$ is independent of time. The constant h can be determined directly, without need to compute the integral, by noting that the discounted consumption expenditure will equal the household's discounted income available for consumption purposes, at the given market prices p , so that $p.C(p)=Y$. By definition, $p.c(p)=1$, hence $h=Y$. This allows one to express (3.3), when Y depends on p , as

$$(3.4) \quad C(p) = Y(p) c(p).$$

5. Implementing a social optimum under distributive constraints

Efficiency requires that all sectors be taxed equally in the margin. Consumption expenditure of household i should be proportional to its statu quo expenditure Z_i^0 .

Let E_i be the before tax earnings of group i , and $Z_i = (Z_i^0 / Z^0) Z$ the amount it is entitled to, then a value added tax with a rate equal to $\theta \equiv 1 - Z_L/E_L$ will be just high enough to absorb the excess earnings of workers.

The aggregate before tax earnings of the private sector amount to

$$\sum_H E_i = O(p^*).$$

If possible, an additional lump sum tax making up the difference

$$(5.1) \quad (1-\theta) E_i - Z_i$$

for the other income groups would be necessary if the social optimum is to be achieved.

Let the rate at which sector i was earning foreign exchange in the past be equivalent to $G_i \exp(nt)$ in the domestic currency, and assume it should sell that amount forever at an exchange rate related to the free market rate by a factor of h_i . The net present value of this income stream, given the international interest rate i , is

$$(5.2) \quad \int (1-h_i) G_i \exp[(n-i)t] = (1-h_i) G_i / (i-n),$$

which when compared with the expression in (5.1) yields the equation

$$(1-\theta) E_i - Z_i = (1-h_i) G_i / (i-n),$$

from which the needed exchange rate par-factor can be computed as

$$(5.3) \quad h_i = 1 - (i-n) [(1-\theta) E_i - Z_i] / G_i$$

Table of symbols

The asterisk (*) denotes concepts evaluated at international prices if they differ from the domestic prices, as in x^* . The operator D denotes differentiation with respect to time, so that for example $Dx \equiv dx / dt$. The circumflex ($\hat{}$) denotes the relative rate of change of a variable, so that for example $\hat{x} \equiv d \log x$.

The numbers in the first column of the following list indicate the section in which the concept is used for the first time.

Section

↓ Symbol	Description
1 a_{ij}	share of capital good j in the price of the composite investment good of sector i = capacity elasticity of capital good j in i = elasticity of the cost of the i -th sector capacity with respect to the price of the j -th capital good
2 αf_j	share of consumer expenditure on good j in total consumption expenditure
A b	reciprocal of elasticity of substitution between present and future consumption

4 B	net external indebtedness or borrowing from abroad
3 c_i	consumption rate of household of type i
1 $c(p)$	unit cost function
1 D_j	dismantling services used by sector j
1 D	subscript for dismantling services
1 d	rate of capital depreciation
6 DRC	domestic resource cost
1 E	nominal exchange rate
5 E_i	socially optimal earnings of group i
2 e	a vector with all its coordinates equal to unity
2 F_K^j	marginal product of capacity in sector j
2 F_L^j	marginal product of labor in sector j
1 $F^j(L_j, K_j)$	production function of the j—th sector
4 ϕ	rate of devaluation
5 G_i	rate at which sector i earns foreign exchange
4 $G_j(p)$	discounted output stream of sector j
3 $H \equiv \{T, N, M, L\}$	set of household types
5 h_i	exchange rate par-factor for group i
1 I_j	gross investment of sector j
1 i	international interest rate
1 K_j	capital used in sector j
2 k_j	capital/labor ratio in sector j
1 $\Lambda(L)$	labor transformation function
1 Λ_j	derivative of $\Lambda(L)$ w.r.t. labor in sector j = the marginal population requirement of j
1 Λ_j / Λ_j	marginal rate of labor mobility
1 L_j	labor employed in sector j
2 lam_j	elasticity of population w.r.t. employment in sector j
A l_j	ratio of labor employed in sector j to labor employed in T
6 LDC	less developed country
1 m_j	shadow value of capacity of sector j
1 M	subscript for importable goods
1 N	subscript for non-traditional exportable commodities
1 n	population growth rate
1 $O(p)$	net present value of the private sector at domestic prices
1 P	population

1	p_D	value of dismantling services to the producer of D
1	p_j	domestic output price of good j
1	q_j	cost of investment in a unit of capacity expansion of sector j
3	r	rate of time preference
2	ρ	degree of labor mobility
1	$S \equiv \{T, N, M, D\}$	set of productive sectors
2	S_{ij}	degree of labor mobility between sectors i and j
1	SSK	specific sector capital
2	s_j^K	the share of capacity in the output of sector j
2	s_j^L	the share of labor in the output of sector j
1	s_j	value of dismantling services to the user sector j
2	σ_j	elasticity of substitution in sector j
1	t_j	ad-valorem tariff rate
1	T	subscript for traditional exports
1	t	time
4	θ	value added tax rate
1	∞	planning horizon
3	$u(c)$	instantaneous utility function
4	$V_j(p)$	discounted investment stream of sector j
3	$v(p)$	consumer price index
1	w_j	wage rate of sector j
3	Y_i	present value of the i-th group income
4	$Y(p)$	net present value of aggregate income stream
2	x	share of non-traditional exports in the composition of the capacity of the same sector
1	y	income of the average worker family
4	$Z(p)$	net present value of aggregate consumption expenditures.