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PROJECTING POLICY EFFECTS WITH STATISTICAL MODELS*

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Abstract:

This paper attempts to briefly discuss the current frontiers in quantitative modeling for forecasting and policy analysis. It does so by summarizing some recent developments in three areas: reduced form forecasting models; theoretical models including elements of stochastic optimization; and identification. In the process, the paper tries to provide some remarks on the direction we seem to be headed.

Introduction

This paper attempts to provide a picture of the current frontiers in quantitative modeling for forecasting and policy analysis. It does so by summarizing some recent developments in three areas: reduced form forecasting models; theoretical models including elements of stochastic optimization; and identification. I believe these developments are in the process of changing the ways economists think about methodology, but this paper does not pay much attention to my views in that area[1]. Instead I try to provide a concrete idea of what is going on in current research, with some remarks on the direction we seem to be headed.

I. Macroeconomic Times Series Probability Structures

In carrying on the series of forecasts initiated by Robert Litterman and described in his paper (1986), I have explored the probability structure of a nine variable system of

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U.S. postwar macroeconomic time series. The immediate stimulus for the exploration was the fact that, while the model Litterman used had done very well in forecasting unemployment and output, it had done poorly –persistently making errors of the same sign– in forecasting inflation. Other modelers, by informally adapting the structure of their models to what they saw happening in the data, had consistently done better at anticipating inflation. The challenge was to see if a model fit under the principles which had motivated Litterman's initial efforts could do better in tracking nominal variables.

The main principle to be adhered to was that the model should not invoke supposed *a priori* knowledge which actually had been learned from the data. This meant realistically modeling our prior uncertainty about the model structure and our beliefs about how fast the prediction equations change through time. Litterman was allowing for time variation in the model's linear structure and was using a prior distribution on the model's form to explicitly document uncertainty about specification. The approach he took is called the Bayesian Vector Autoregression (BVAR) approach.

Though in some respects the original BVAR modeling framework that Litterman describes in his paper and that he, Doan and I described in (1984) is more general than commonly used econometric models, it is still restrictive. It assumes normal distributions for disturbances to equations and to parameters. It assumes the covariance structure of these disturbances is fixed. It assumes linearity of the one-step-ahead forecasting equations at each date. It assumes a simple random-walk form for the time variation in coefficients. An ideal Bayesian approach would recognize explicitly that we are uncertain about all these aspects of the model. However any actual statistical modeling effort, Bayesian or not, must compromise between accurately representing uncertainty and keeping the model computationally manageable[2]. I set out to relax some of the simplifying restrictions in the original BVAR framework which might account for its poor performance in forecasting nominal variables. I hoped at the same time to preserve as much as possible of the framework's computational convenience.

This is not the place for a detailed technical description of the results, which are presented in a forthcoming discussion paper of mine (1988). The most important new features of the model are that it allows for nonnormality in disturbances (with modest cost in computational burden), and that it allows for variation over time in variances of disturbances. Though the original specification allowed for variation over time in the parameters of the linear forecasting equations, the data implied that the time variation was small. The new specification, when confronted with the data, implies considerably greater time variation in the forecasting equations. With assumed normal disturbances, large forecast errors imply large changes in coefficients if time variation is assumed. The assumed nonnormality of disturbances allows a damping of the effect of large forecast errors on the estimated coefficients.

The likelihood function of the model implies that all three of these features –nonnormality, drifting variances, and increased time variation in forecasting equations– are important to improved fit. As can be seen from Table 1, the model's simulated forecast performance over the historical data is roughly the same for real variables as that of the original Litterman specification, and the performance for the GNP deflator is much improved. The free parameters in the model have increased from about 6 to about 12; that is, from Litterman's. 67 parameters per equation the new model has moved to about 1.33 parameters per equation[3].

The model is strongly nonlinear. A good index of a model's nonlinearity is the degree of nonlinearity in its impulse responses. In any forecasting model, a change in the data will imply a change in the forecast. In a linear model the change in the forecast follows

TABLE 1

THEIL U STATISTICS, 1949:3–1987:2

Current 9 variable model result listed above result for 6 variable model with no time variation in each pair of rows.

Variable	Quarters ahead 1	2	4	8
Treasury	.9636	1.0379	.9641	.9950
Bill Rate	.9467	.9723	.9567	.8576
M1	.4661	.4232	.3767	.3761
	.4807	.4353	.3968	.4060
GNP	.3892	.3219	.2850	.2592
Deflactor	.4471	.4182	.4436	.4664
Real GNP	.7618	.6984	.6968	.6481
	.7523	.7034	.7022	.6857
Business	.8650	.8791	.9356	.9548
Fixed Investment	.9040	.9382	.9698	.9305
Unemployment	.7956	.8554	.9212	.9775
	.8163	.8680	.9568	1.0477
Trade-Weighted Value of Dollar	.9207	.9640	1.0274	1.1715
S&P 500 Stock Price Index	.8775	.9016	.9201	.9915
Commodity Price Index	.7471	.8036	.8727	.8758

Notes: The Theil U statistic is the ratio of the model's root mean square forecast error (RMSE) to that of a naive no-change forecast. Each forecast in the period is constructed from a model estimated using data up through the forecast date only, except that the 12 parameters of the prior are chosen using the full sample.

the same pattern, regardless of the size of the change in the data; only the scale of the change in the forecast varies with the scale of the change in the data.

Charts 1-3 display the impulse responses of the fitted model in three ways. In all three the model is responding to a matrix of first-period impulses derived from the sample covariance matrix of one-step ahead forecast errors. Chart 1 simply uses the end-of-sample coefficient estimates as if they were fixed and displays responses to orthogonalized innovations as has been conventionally done in VAR analysis. This chart can be thought of as a scaled-up display of the effects on the model's projections of infinitesimally small current disturbances. Very small disturbances will have very small effects on coefficient estimates and hence will have predicted future effects close to those implied by current estimated coefficients. Charts 2 and 3 are calculated by taking the first period disturbances to be .5 and 1.5 times, respectively, the usual orthogonalized shock matrices. These first period disturbances are fed in to the Kalman filter, coefficient estimates are updated, and forecasts from the second period on are generated holding the updated coefficients constant at their new values. If the model were approximately linear, Charts 1-3 would be approximately the same. As can be easily seen, they are quite different in certain respects. For example, the fixed-coefficient interest rate shock generates a persistent negative response in GNP, but the response is modest. The .5 standard error shock strengthens this response, making it larger and more persistent. (The scale of the output response line increases by a factor of 5 or so between Charts 1 and 2, so the response looks smaller, even though it is quite a bit larger). The 1.5 standard error shock eliminates the negative response of output to interest rates.

It does appear feasible, then, by allowing for time-variation in a model's equations and for nonnormality in disturbances, to obtain explicit probability models that seem capable of doing as well at forecasting as the usual procedure of estimating a restricted model whose structure is adapted informally to the data over time. The resulting explicit models are likely to be strongly nonlinear and nonnormal. They have the advantages as

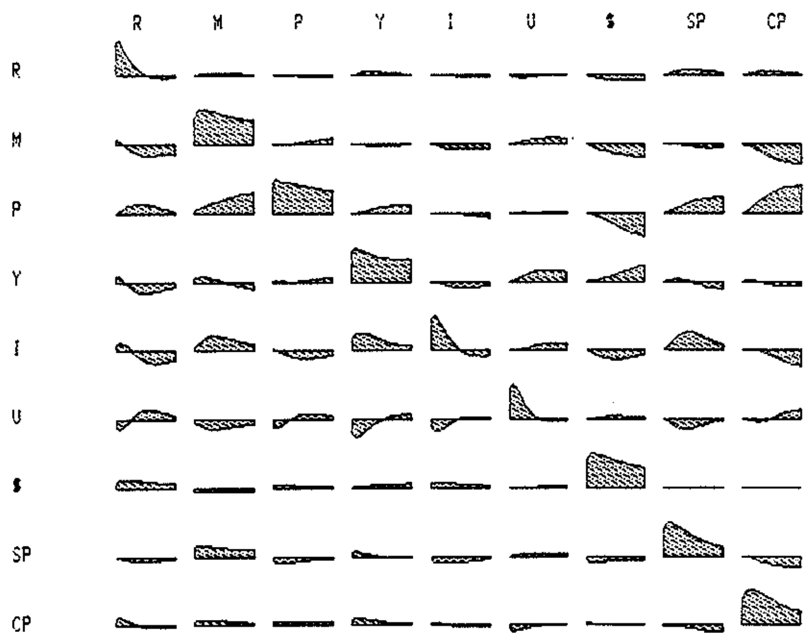


CHART 1

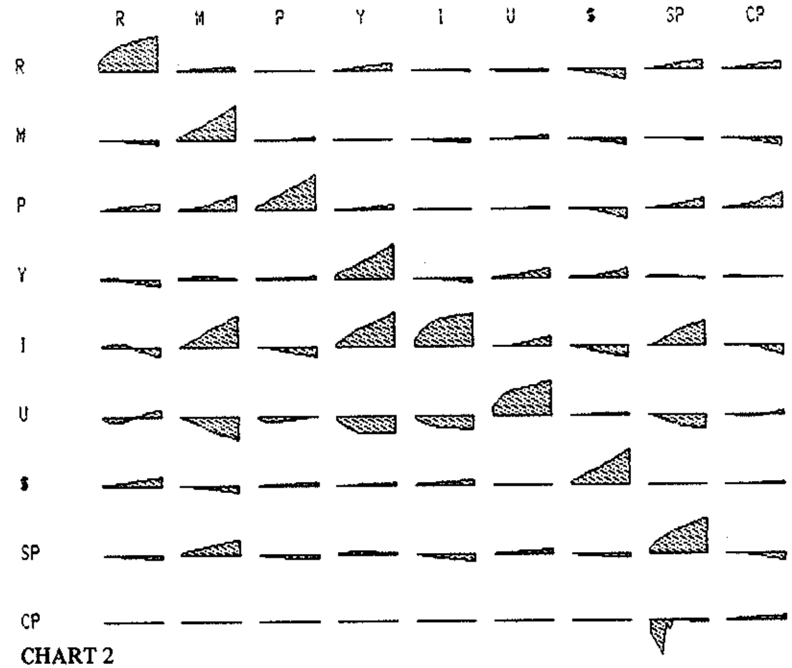


CHART 2

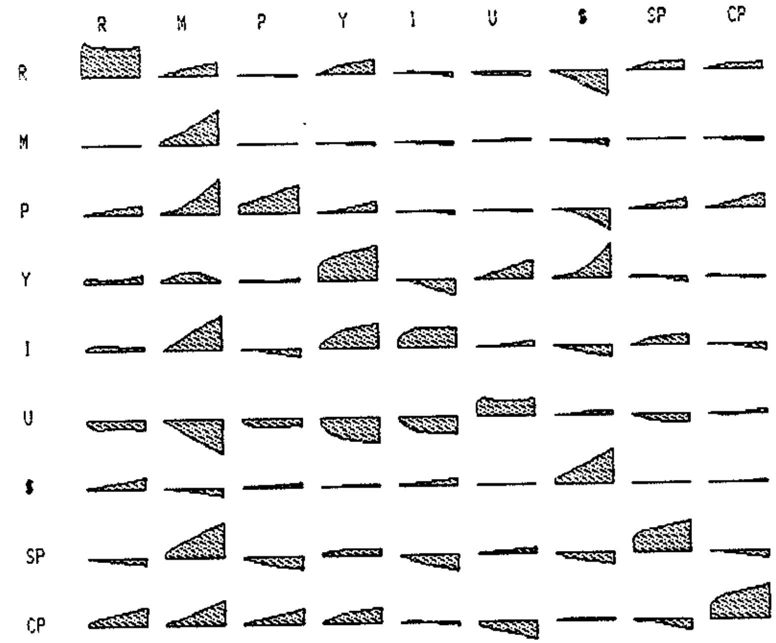


CHART 3

forecasting models that they treat "specification uncertainty" explicitly and can therefore give a more realistic picture of likely forecast error and that their forecasts can be used without expert adjustment for reasonableness. They have the advantage for economic science that they have a stronger claim to be a foundation for structural interpretation than models in which much of the economic environment's real randomness is assumed away.

II. Behavioral Modeling

It has become the standard in macroeconomic theory to use aggregated general equilibrium models in which agents face uncertainty and at least some of them optimize[4]. Solving such models is computationally difficult, and as a result they have not often been used directly for empirical modeling. This gap between macroeconomic theory and macroeconometrics represents a challenge, however, and it is spawning a great deal of promising research activity[5].

The standard approach to stochastic optimization is dynamic programming, with the problem's state space discretized to allow casting the constraints into Markov form. This approach grows more demanding computationally as the dimension of the problem increases, and the rate of increase is particularly severe here. If we can successfully handle a one-sector growth model with a single capital stock as state variable by considering an approximate problem with, say 50 distinct values for the capital stock, what happens when we consider a model with two forms of capital? To retain the same degree of refinement in the discrete approximation in both dimensions of the state variable requires $50^2 = 2{,}500$ distinct points in state space. For three kinds of capital the number is 125,000, etc. Nonetheless the method has been applied successfully. Larry Christiano[6] has shown it to be quite feasible computationally for small macroeconomic problems and Kenneth Wolpin and John Rust[7] among others have used it in applied work. George Tauche[8] has developed methods for greatly speeding computations in certain problems of this type by careful choice of the discretization.

The main prospect for avoiding the exponential growth in computational burden with dimensionality in these problems appears to be in approximating the solution to the original problem by a method which is not so sensitive to dimensionality. Kydland and Prescott (1982) suggested a linear-quadratic approximation to the original problem and work by Christiano suggests that for many macroeconomic problems the approximation error may be small. I have suggested modifying the problem so that the decision rule is assumed or derived from the linear-quadratic approximation, and the distribution of stochastic elements in the environment is then derived. Rust has suggested assuming a form for the value function in dynamic programming, then deriving the implied form of the objective function. Pam Labadie and Albert Marcet[9] have suggested methods based on the idea of assuming an approximate form the expectations functions appearing in the Euler equations. Ray Fair, Joseph Gagnon, and John Taylor[10] have worked out an approximate solution method which is feasible in large models with rational expectations elements. There are a number of other people I know to be working in this area. I expect that within 2-3 years there will be packaged programs available allowing simulation of stochastic dynamic economic equilibrium models of up to 15 variables, in which some agents optimize, on microcomputers.

III. Identification

I take the econometric identification problem to be that of finding a behavioral interpretation of a probability model that allows it to be useful in guiding decisions[11]. Economists are taught the importance of identification in this sense as part of their standard graduate training and are generally aware of the pitfalls of treating it informally. This contrasts with the situation among statisticians, engineers, and natural scientists, who tend to treat identification in this sense informally.

The increased sophistication of reduced form time series econometric models like that described in section one and of fully interpreted behavioral models like those discussed in section II has not, unfortunately led to convergence in form of the two types of model. Theoretical models do tend to be nonlinear, but in narrow, tightly parameterized ways. They may allow nonstationarity, but not the free-form drift of forecasting equations of section I. They seldom focus attention on the distribution of disturbance terms. Hard as they are to solve, they tend to generate implied behavior for the data which is considerably less complex than what is implied by the best current reduced form models.

This situation leaves two main possibilities for obtaining quantitative guidance for decision making in the immediate future: one can use a reduced form model with an informal, approximate, or incomplete identification; or one can use a fully interpreted behavioral model, connected to the data as well as possible but acknowledged not to forecast as well as a good reduced form model. Academic economists tend to be excessively suspicious of the former option, while the latter raises some conceptual difficulties.

While some economists tend to proceed as if theoretical models can yield policy conclusions without any examination of how well they correspond to the data, few would seriously defend this idea. But if fully interpreted behavioral models must for the time being remain too simple to compete with reduced form models in forecasting, the data reject them all. If we decide we are willing to use a false model because we are more confident of its interpretation, how do we choose among candidate models?

The procedure which ought in my view to be standard would compare behavioral models to good-fitting reduced form models, measuring how unlikely is the stochastic variation generated by the behavioral model under the distribution implied by the more accurate reduced form model. One way to do this would be to simulate the behavioral model using the current situation as initial conditions, generating a Monte Carlo sample of time paths for the economy from the current date t to $t+k$. The reduced form forecasting model can then treat these Monte Carlo paths as data, transforming each path into a sequence of implied forecast errors for the reduced form model. The measure of inaccuracy of the behavioral model is then minus the average log likelihood of the Monte Carlo forecast errors using the reduced form model's likelihood function. This approach generates an absolute measure of distance between the behavioral model and the reduced form and at the same time allows some diagnosis of the reason for lack of fit. The Monte Carlo forecast errors may be especially large for some subset of the variables, indicating the behavioral model handles them incorrectly, or the Monte Carlo errors might show cross-variable correlation indicating the relations among certain variables are mishandled, for example[12].

It may not be long before the solution methods discussed in section II are powerful enough to allow treatment of models of substantial size with explicit modeling of measurement and specification error. For the time being, though, it is likely that practical methods of macroeconomic policy evaluation will continue to rely on incomplete, approximate or informal identification of reduced form models.

We do not need complete identification to evaluate policy. We do need an accurate model of how changes in variables controlled by policy affect the rest of the economy, but this does not require that we also understand all of the mechanisms by which they have their effect. Thus a model can be useful for policy evaluation even if it does not explicitly identify expectation-formation mechanisms. Econometricians often informally assume that the observed variation in "policy variables" is statistically exogenous, so that models can be simulated under given assumptions about the time paths of these variables without explicit attention to possible implications of those time paths for the model's stochastic disturbances. This practice can be misleading indeed it can be argued that monetarist macroeconomics is essentially a mistake arising from fallacious identification along these lines. Nonetheless it is not necessarily worse than mathematically more explicit identification schemes which are limited by computational tractability.

Work by Blanchard and Watson (1984), Bernanke (1986), and me (1986) has shown how identification based only on restrictions on the contemporaneous interactions among variables can allow convenient behavioral interpretation of reduced form VAR models. These methods do not generalize immediately to models with time-varying parameters, but they probably can be generalized. This sort of identification foregoes some types of identifying restrictions for the sake of computational tractability.

Economists are probably too suspicious of approximate, informal, or incomplete identification. Exact, formal, complete identification based on mistaken assumptions can lead to error fully as bad as inexact identification. Indeed honest reduced form modeling of uncertainty can partially compensate for inexact identifying assumptions, as is illustrated in the next section.

IV. The Robustness of Careful Empiricism

Here we re-examine the Kydland-Prescott (KP) example of the perils of ignoring rational expectations. In that example policy makers use a false model and lead the economy to a suboptimal, high inflation equilibrium in a vain attempt to reduce unemployment by exploiting the Phillips curve. Of course policy makers who use a false model will nearly always generate suboptimal policy, whether or not their model explicitly invokes rational expectations. Nonetheless, the Kydland-Prescott example is misleading in that its results depend on unrealistic naivete on the part of policy-makers. Policy makers who use a flexible empirical model which makes reasonable allowance for uncertainty can generate nearly optimal policy without allowing explicitly for rational expectations in their model.

In the simple Kydland-Prescott economy, the government controls the inflation rate directly, except for a random component. Deliberately created variation in inflation has no effect at all on unemployment, while random changes do affect unemployment. If the government does not begin at the Kydland-Prescott suboptimal equilibrium, it must get there by deliberately changing the inflation rate. These deliberate changes in the inflation rate have no effect on the unemployment rate, and anyone plotting a scatter diagram of inflation and unemployment during the period of active policy intervention would observe the truth –a weak relation between inflation and unemployment. Whether or not the government understands or believes in rational expectations, observation of the fact of this weak relation will lead it to nearly optimal policy –an inflation rate near

zero. In the KP framework it is still true that the near optimal policy cannot persist –while it is maintained, it generates spurious statistical evidence of an exploitable Phillips curve.

Though it does not undermine the KP conclusion about the nature of the unique equilibrium, the self-correcting tendency for policy intervention to generate statistical data reflecting the true vertical Phillips curve means that convergence to the Kydland-Prescott suboptimal equilibrium from a initial position with lower inflation is extremely slow. Chart 4 shows the time path of an economy for which the KP equilibrium is 6 per cent inflation and 6 per cent unemployment, assuming the initial inflation rate is one per cent. The upper part of the graph shows unemployment oscillating randomly about six per cent, the natural rate in the model. The lower part shows the rising path of the inflation rate. The parameters have been chosen to be more or less realistic for annual data (see the Appendix). The vertical bars on the graph represent centuries. While the economy moves in just a few decades from one per cent to three per cent inflation, the remaining nine centuries of simulated data show a further move up only to a mean inflation of less than four per cent.

But there is a further respect in which the KP model makes its data analysis unreasonably naive. In the eventual equilibrium, policy makers retain confidence in an absolutely fixed Phillips Curve, despite the evidence of drift in the Phillips curve during the approach to equilibrium. Because they are willing to believe the Phillips curve is constant, when their estimates tell them that no policy-generated change in inflation is appropriate, they do not intervene. The equilibrium can persist only because their belief that an intervention to change the inflation rate would have a precisely known effect leads them *not* to intervene. If they did intervene, they would observe the ineffectiveness of intervention and the equilibrium would be undermined.

There are many reasons why in reality policy-makers are unlikely to behave this way. The simplest is that real economies do not retain constant structure, so that real quantitative models are constantly adapting in form to recent history. If policy-makers believe that the Phillips Curve is shifting in form, they will constantly change their estimates of it, regardless of how long a sample of data is available. The changing estimates will generate changes in policy, and the changes in policy will make the empirical Phillips Curve more vertical. It is interesting, therefore, to modify the KP framework by having policy makers use a statistical model which allows explicitly for random time-variation in the parameters of the Phillips curve. Instead of least-squares regression, they use the Kalman filter to update their estimates of the Phillips curve.

I have no analytical results about the nature of equilibrium in such a modified KP model, but it is easy to simulate it. Chart 5 shows the path of an economy exactly like that of Chart 4 except with an allowance for random change in the parameters of the Phillips curve (which begin at 2 and –.1) with a standard deviation of .1 per year. Here there is no tendency to approach the KP equilibrium. The model is constructed so that the optimal policy would yield inflation fluctuating around 0 with a standard deviation of .4. This simulation shows a mean of around two per cent with a standard deviation of over one per cent. It is clearly not the optimum, but it is much closer to the optimum than to the KP equilibrium of six per cent mean inflation.

This modified KP model seems to have two possible steady-state equilibria, one around the original KP equilibrium another like the Chart 5 plot, with inflation oscillating around a mean near zero. Char 6 shows a simulation, differing from that of Chart 5 only in the relative amounts of time variation in constant term and slope coefficient assumed in the Phillips curve, in which the inflation rate does rise to the KP equilibrium level. It is thought-provoking that this result flows from policy-makers' assuming that the "natural rate" of unemployment implied by the constant term in the Phillips curve is highly variable through time.

Chart 7 shows that if the economy begins with policy-makers assuming a Phillips curve like that of the KP equilibrium, it can break out of that situation. Note, however, that breaking out of the KP equilibrium in this particular version of the model (with somewhat more randomness in the natural rate and less in the inflation rate than in Chart 5) takes hundreds of years. Not every simulation run breaks out even over the thousand year span of the simulation. By increasing the amount of time variation in the policy makers' model or by decreasing the size of random shocks to inflation relative to random shocks in the Phillips curve, the KP equilibrium can be made less persistent and the low-inflation mode of the model can be made to have a lower average inflation rate. On the other hand, with small enough time variation or large enough random variation in inflation, the model can be made to behave just like the original KP model, indeed with somewhat quicker convergence to a KP-like equilibrium.

This modified KP model gives the policy-makers an only slightly less naive statistical model, and this gives them a reasonable chance of generating policy close to the optimum. The biggest element of naivete in the model is left unchanged, however. Policy-makers presumably know at least approximately what they have done to the price level, and can therefore see the difference between that and the actual price level. Suppose they modify the Phillips curve regression so that it includes policy-generated inflation and uncontrolled random disturbances to inflation as two distinct variables. (In a more realistic model, this might amount to distinguishing between changes in monetary policy variables like reserves and controlled interest rates and related uncontrolled variables like the price level.) Then they are using a statistical model which is actually structural: the true regression function has a coefficient of zero on policy-induced inflation, and this is the actual effect of policy-induced inflation on unemployment, regardless of how the policy is generated.

With this structural Phillips curve model, policy makers of course are likely to quickly learn the true Phillips curve parameters and settle into the optimal policy equilibrium. It is interesting to note, however, that if they make no allowance for time variation, they can still settle into a suboptimal equilibrium. If estimates lead them to confidence in a nonzero coefficient on government-induced inflation, they may set the policy variable at a nearly fixed suboptimal value. Steadily increasing collinearity between the policy variable and the constant term then can prevent them from learning the true value of the coefficients[13].

Note that to achieve this perfect result, policy-makers do not have to understand that because of rational expectations the effect of policy-induced inflation on unemployment is zero. They do not even have to model expectation formation. They need only have a correct understanding of what variable in the economy they control and be able to estimate regressions of the outcome variables they are interested in (price and unemployment) on the variable they control.

While this simple model cannot be taken seriously as applying to U.S. historical data, it does I think contain lessons on pitfalls in interpretation of those data. Economist who have used large econometric models in forecasting claim that implied Phillips curves in those models quickly became steep after the 60's, so that there is no basis for the notion that the inflation of the 70's arose from misguided pursuit of low unemployment along a flat econometric Phillips curve. The simple model in this section shows this could be true; the econometric models could have quickly adapted to give the right answers on policy choices without having incorporated rational expectations into their structures.

And indeed this seems reasonable. The actual history of inflation and employment in the U.S. in the 1970's and 80's (see Chart 8) seems unlikely to have allowed long persistence of any econometric illusion of a stable inflation-unemployment tradeoff.

V. Remarks on Computation

Most of the developments surveyed in this paper are related to the increased availability of computational power, and the 9 variable model described in section I, for example, is ordinarily estimated and simulated on a Cray 2 supercomputer. Nonetheless these developments should not be regarded as esoteric methods useful only to those few economists with easy access to a supercomputer. The 9 variable model does not at all tax the capacity of a supercomputer. Its likelihood function can be evaluated in about 20 seconds on the Cray 2. A six variable version of the same model can be evaluated on an AT in about 35 minutes. The 9 variable model itself can probably be handled on an 80386 PC with one of the new operating systems that can address over a megabyte of memory.

The distance between large and small computers tends to be exaggerated. A Cray 2 has 256 million 8-byte words of memory, compared to about one million words on the most advanced PC's. Yet most economic models have storage requirements that grow at least cubically, sometimes quartically with model size. The 9 variable model's specification grows cubically with the number of variables, so if a 9 variable model hits memory limits on high-powered PC's, a model with about 60 variables hits limits on the Cray 2. Indeed if the model specification is generalized to take account of cross-equation correlation of disturbances, the storage requirements become quartic in model size, so that a six-variable model on a PC corresponds to 24 variable model on a Cray 2. While these are substantial differences in model size, they are not so great that methods developed for use with supercomputer sized models are irrelevant to work on a PC.

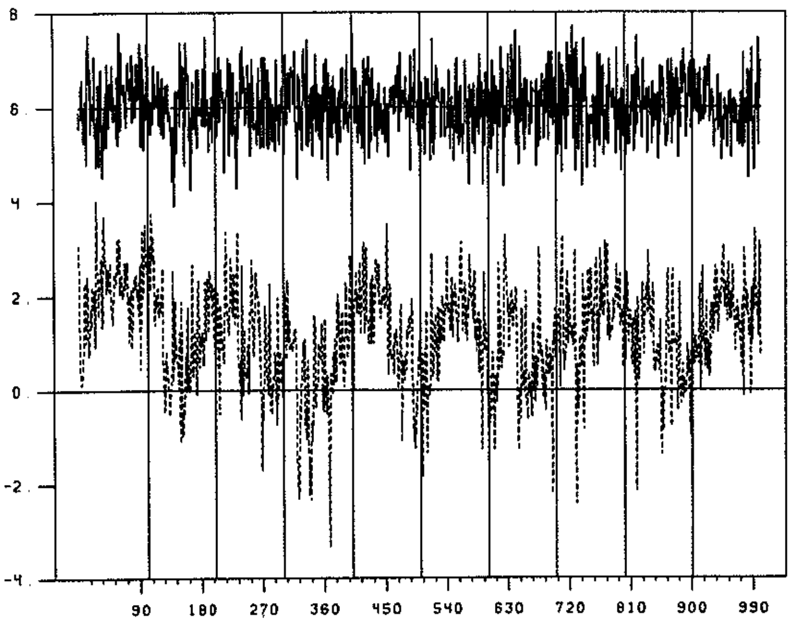
The same considerations apply to methods for model solution like those in section II. Dynamic programming with discretized state space, growing exponentially in storage requirements with model size, can easily grow from PC to supercomputer size with the addition of just a few new state variables.

The biggest and fastest computers will make a crucial difference to some problems, but for most problems the difference will be a modest increase in potential scale, not a qualitative difference. The improvements in computer technology imply that quantitative economic modeling is likely to change even in locations where only personal computers are available.

VI. Conclusion

Already forecasts and policy analyses are being carried out with probability models that take account of specification uncertainty. It is likely that such models will become more common and relieve econometricians of some of the burden of monitoring their models for breakdown of equations and unreasonable forecasts. It is not yet standard for macroeconomic theorists to fully explore the behavior of their models with computer simulations, but developments in this area are proceeding so rapidly that this could change within just a few years[14].

As both these developments proceed, we will have to be rethinking the meaning and methodology of identification. It should be interesting.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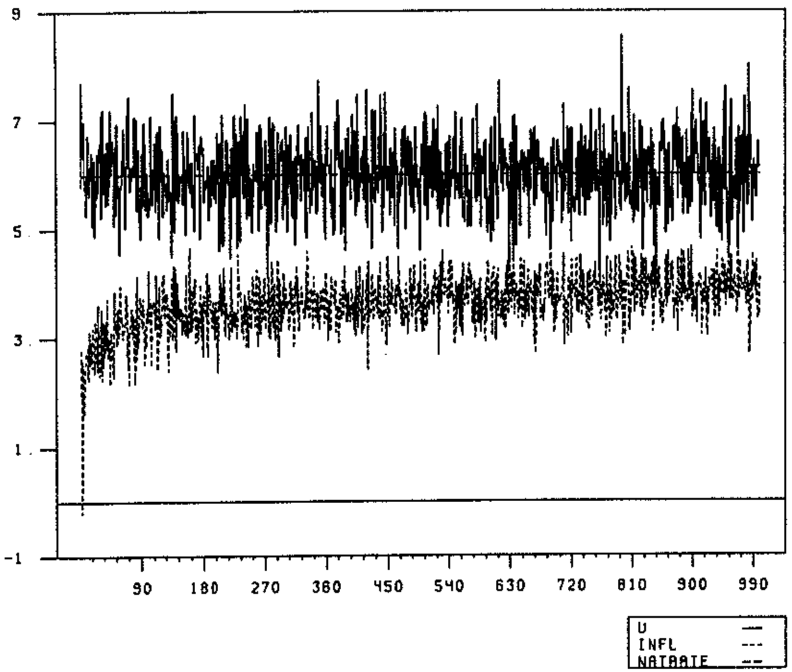
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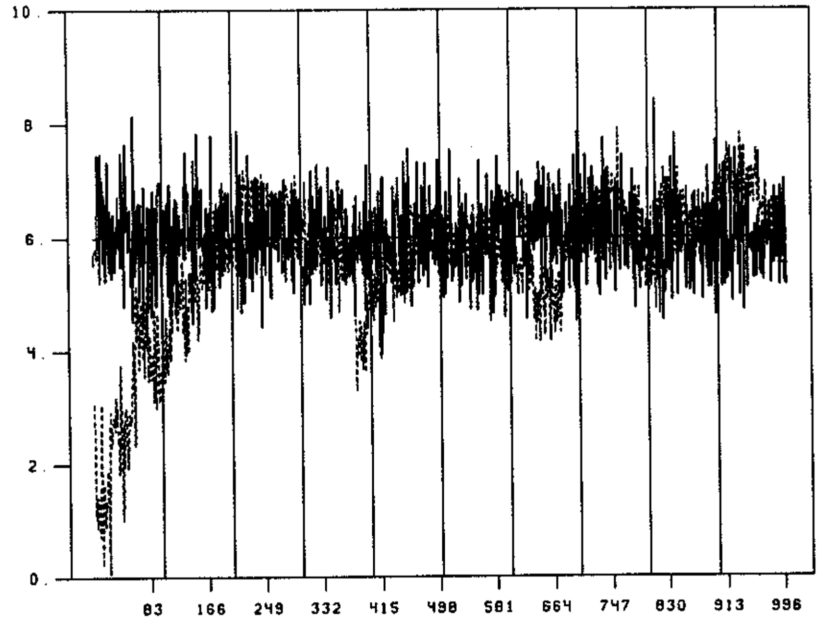
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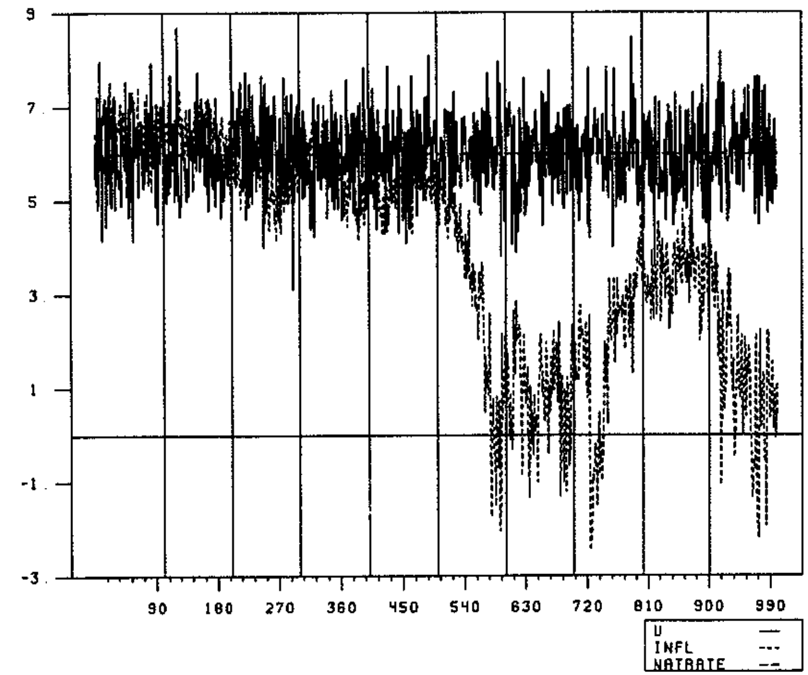
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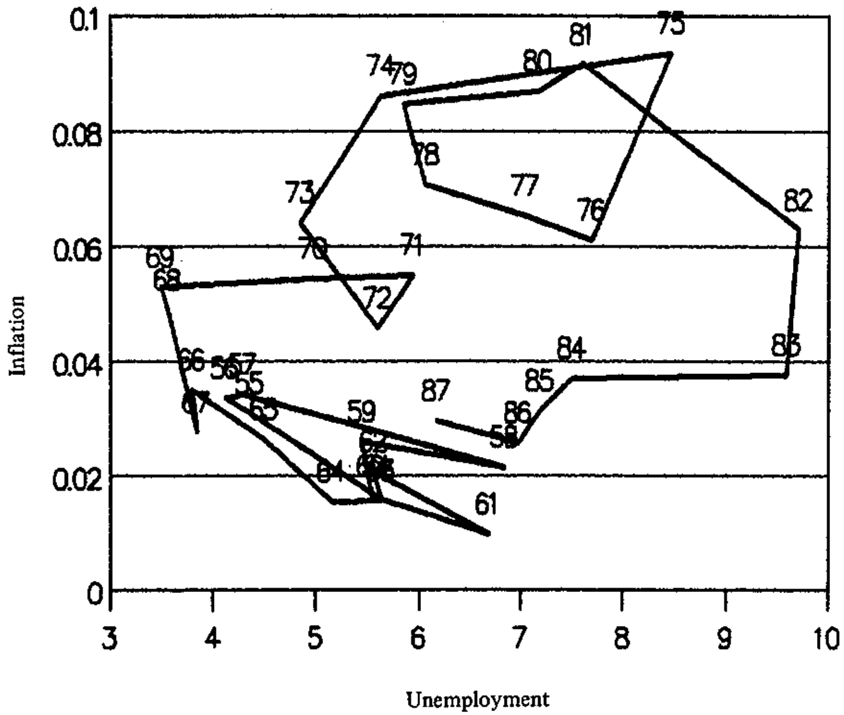

Notes

1 See (1986), (1982) for my views on those issues.

2 Because the Bayesian approach points directly to an optimum use of available data for any well-defined problem of inference, while classical approaches easily allow discussion of the properties of simple but inefficient estimators, the Bayesian approach tends to be regarded as difficult to apply. It is possible, though, to consider approximate computations or inference based on easily computed summary statistics, within a Bayesian framework. The fundamental distinction is that a Bayesian approach encourages formal treatment of what econometricians call "specification uncertainty" –uncertainty about the form of the model.

3 It is important to understand the distinction between parameters and coefficients in this kind of a model. The 11 free parameters are the only aspect of the model that is adjusted to conform to the data in the process of "fitting" it. The forecasting equations at each date nonetheless contain 414 coefficients, and these are modeled as different at each date. The coefficients are treated as random variables like the equation disturbance terms. The parameters are the parameters of the distribution of these random variables. The 12 free parameter ought, from the perspective of a Bayesian purist, also to be treated as random variables and given a distribution. The practice of choosing a single setting of these parameters which has high likelihood can be justified only as a convenient approximation to postulating a prior over the parameters and integrating them out of the posterior p.d.f. This is the Bayesian version of the problem os "overfitting". If there are too many free parameters, the peak of the likelihood with respect to the free parameters will be unrepresentative of the "average" behavior of the model over likely values of the free parameters. It seems probable that 12 parameters treated informally this way is not too many in a model with about a thousand data points, but clearly moving from 6 to 12 free parameters increases the need to check for overfitting.

4 By "general equilibrium" here I mean only that a complete environment of technological possibilities and assumptions about human behavior is specified and the implications for resource allocation derived, not necessarily that rationality or competitive markets are assumed.

5 Because most of the work cited in this section is still unpublished and still unfinished, I will often cite names and institutional affiliations rather than particular papers.

6 Minneapolis Federal Reserve Bank.

7 University of Minnesota and University of Wisconsin, respectively.

8 Duke University.

9 Columbia University and Carnegie-Mellon University, respectively.

10 Yale University, Board of Governors of the Federal Reserve, and Stanford University, respectively.

11 "Identification" as used in statistics, and sometimes in econometrics, concerns the mathematical question of whether all the parameters of the model can be determined from a large enough sample of data. Econometricians have commonly worked with a "reduced form" model constructed to be statistically convenient (and in particular to be statistically identified) and a more easily interpreted model called a "structural model". The question of identification is then the treated as the question of the nature of the mapping linking structural and reduced form models. This mapping is the economic interpretation of the reduced form probability model.

12 The procedure described here, linked to simulation starting from a particular date's initial conditions, is essential if either the reduced form or the structural model is nonstationary. If not, simulations could be based on arbitrary initial conditions and carried out long enough so that both models' behavior has settled into steady state patterns.

13 Again, I have no analytical result. In simulations sometimes such an equilibrium sometimes persists through a thousand "years", but sometimes it persists for a few hundred and then breaks up to yield to the optimal equilibrium with inflation oscillating about zero. Whether it eventually breaks up with probability one I don't know.

14 Econometricians have not commonly been trained in the mathematical of the new reduced form forecasting models, which may limit their use even after they become feasible. Macroeconomic theorists have now commonly been trained in the mathematics of stochastic equilibrium models, so if simulating them becomes easy, it is likely to spread quickly.

Appendix

The Extended Kydland-Prescott Model

The model used to generate the simulations of Charts 4-7 assumes that unemployment U(t) satisfies

$$U(t) = 6.0 + \epsilon(t) \tag{1}$$

where ϵ is i.i.d. normal with variance σ_ϵ^2. It supposes that policy-makers believe that U is related to inflation π according to

$$U(t) = \alpha_t - \beta_t \pi(t) + \nu(t)\ , \tag{2}$$

where they assume $\nu(t)$, α_t, and β_t to be normally distributed stochastic processes satisfying

$$\alpha_t = \alpha_{t-1} + \eta_1(t) \tag{3}$$

$$\beta_t = \beta_{t-1} + \eta_2(t),$$

with η_1, η_2, and ν jointly normal, mutually uncorrelated, and i.i.d. over time. The variances of η_1 and η_2 are labeled σ_1^2 and σ_2^2. The government begins with a prior mean $\bar{\alpha}$ for α and $\bar{\beta}$ for β and a diagonal prior covariance matrix for α and β which gives α variance of 1 and β a variance of .1. The policy authority is assumed to update its estimates of α and β at each date, then use them as certainty equivalents to minimize the expected value of $\pi(t)^2 + u(t)^2$. This means they set the controllable portion of inflation at t, $\gamma(t-1)$, equal to the current estimate of $\beta\alpha/(1+\beta^2)$. There is an uncontrollable component $\phi(t)$ to inflation as well, so that actual inflation satisfies

$$\pi(t) = \gamma(t-1) + \phi(t)\ , \tag{4}$$

with ϕ i.i.d. $N(O, \sigma_\phi^2)$.

The settings of these parameters for the simulations shown in the charts are as follows:

Chart	σ_ϵ	σ_ν	σ_1	σ_2	σ_ϕ	$\bar{\alpha}$	$\bar{\beta}$
4	.5	.1	O	O	.4	2	1
5	.5	.1	.1	.1	.4	2	1
6	.5	.1	.1	.2	.4	2	1
7	.7	.1	.1	.1	.3	12	1

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