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TERMS OF TRADE AND THE CURRENT ACCOUNT UNDER UNCERTAINTY*

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Abstract:

This paper analyses the effect of stochastic terms of trade shocks on the current account of a small and open economy. Under uncertainty the distinctions between expected and unexpected, transitory and permanent shocks of the previous perfect foresight literature break down. It is shown that under terms of trade presenting stochastic structural changes, the rational consumer makes use of an error learning model to form his expectations on future relative prices.

In this framework the consumer's optimal plan is derived for a two-good certainty equivalence utility function. Finally a simulation for the foreign debt profile in the aftermath of a terms of trade deterioration is performed.

1. Introduction

The recent literature on the effects of relative price shocks on optimal borrowing (or on the current account) distinguishes between permanent versus transitory and anticipated versus unanticipated changes in a framework of intertemporally optimizing agents. All papers emphasize the very different impacts of these shocks as a result of agents with rational expectations and perfect foresight.

The basic methodological distinction between the models refers to the number of periods and to the life-length of the corresponding agents. In the category of two-period models with one generation of agents, Sachs (1981a), Svensson (1982), Svensson and

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Razin (1983) and Marion and Svensson (1982) deal with the effects of oil-price increases and other exogenous shocks to the current account. Another class is represented by Persson and Svensson's (1983) model with two overlapping generations.

A third category is that of infinite-horizon models as the papers by Sachs (1981b) and Lipton and Sachs (1983). Within this class, the specific effect of anticipated (and certain) future relative price changes on optimal consumption and borrowing, via changes in expenditure-based real interest rates in two-goods models, has been studied only recently. Dornbusch (1983) and Martin and Selowsky (1984) analyse optimal consumption and debt profiles under anticipated relative-price changes in two-sector dependent-economy models. Obstfeld (1983) does the same for anticipated future terms-of-trade fluctuations for a fixed-endowment economy with consumption of exportable and importable goods.

The purpose of this paper is to study the effects of stochastic terms-of-trade on the current account. For rational agents confronting uncertainty, the permanent/transitory distinction made by the perfect-foresight literature breaks down. In addition, when current and future expected prices differ –and this is the normal case– the corresponding anticipated price change causes a revision of planned consumption and borrowing profiles.

The framework I use here is a standard intertemporally-optimizing infinitely-lived representative-consumer fixed-endowment economy, which therefore involves no investment or production decisions. The household consumes both an importable and an exportable good.

For intertemporal consupmtion optimization under uncertainty one confronts two modelling choices. Stochastic calculus offers a wide range for the choice of the utility function, but severely restricts the choice of the stochastic structure to two particular (and stable) time-series processes, as shown by Merton (1971).

As I am interested in considering time-series processes with stochastic and unpredictable structural changes, I will use a second alternative, which is certainty equivalence. This, at the cost of restricting the utility function to a quite particular one. In my two-good case, I derive a quadratic-hyperbolic utility function which satisfies certainty equivalence.

The paper is organized as follows. In section 2, I derive an optimal linear predictor for the terms of trade, which present stochastic structural changes in the underlying time-series processes. Following the signal-extraction technique based on the Kalman filter model, I derive an error-learning model for the optimal predictor, which takes a specific distributed-lag form.

In the next section the consumer problem for stochastic terms of trade is solved. The wealth and intra and intertemporal substitution effects stemming from price shocks are derived for consumption demands and for the current account.

Section 4 is devoted to simulate the time profile of net foreign debt affected by permanent price shocks. A discussion of the main conclusions closes the paper.

2. Optimal Linear Prediction

In this section I derive an optimal linear predictor for stochastic variables presenting stochastic shifts in their time-series structure. This predictor will be used in the following sections when analysing the effects of terms of trade shocks on the current account.

To derive linear predictors for these variables we confront two basic choices with regard to the underlying time-series processes. One alternative is to consider the long-

term process governing each of these variables, by explicitly modelling their structural changes or their long-term cycles. Otherwise we could view them as entirely stochastic sequences of short or medium-term structurally stable, stochastic processes.

The advantage of the former choice is to obtain predictors which are consistent with the long-term structure of the process –if such a structure is really stable. But if no such structure exists, *i.e.* when the structural changes between one short-term process and the following are entirely stochastic, the latter option is preferable. In addition it requires less data for its implementation.

Here I follow the second option. I will derive an optimal linear predictor for a stochastic variable under stochastic structural changes of the underlying short or medium-term processes. Optimality is defined in the sense of minimizing mean square prediction errors. I will show that predictions taking a specific form of distributed lags of past observations are consistent with rational forecast in the presence of stochastic structural changes.

To simplify the analysis, I assume that the structure of the short-term processes is white noise around a given mean (WNAM). A more general process (as any ARIMA) would significantly complicate the derivation, without altering the basic result I want to show, which is the emergence of distributed-lag forms for the optimal predictors. Naturally, the particular lag structure for the short-term ARIMA process would be different from that corresponding to the WNAM. Nevertheless I will use in subsequent sections a simplified version of the particular linear predictor derived for the WNAM process, which could also correspond to a predictor derived from an ARIMA process. The structural change takes the form of stochastic shifts of the mean at stochastic time intervals. There is uncertainty about the mean of the new process after a structural change has taken place, and about the breaking point at which it takes place.

Econometric theory has been developed to deal with estimation of models with structural parameter changes which are stochastic but have a systematic component[1]. Naturally there is no estimation technique dealing with purely stochastic structural changes. My aim is to derive a linear predictor for entirely stochastic structural changes applying the signal-extraction technique based on the Kalman-filter model[2].

2.1 *Preliminary Assumptions and Analysis*

The observer knows with certainty that until some period t–j in the past ($j \geqslant 0$ and j unknown) the stochastic variable p followed a white-noise process around a given mean:

(1) $p_s = \bar{p} + \eta_s \quad \text{for } s \leqslant t{-}j$

where η_s is white noise i.i.d. satisfying:

$$E[\eta_s] = 0, \quad E[(\eta_s)^2] = \sigma_\eta^2, \quad E[\eta_s\, n_{s-1}] = 0.$$

Mean $\bar{p}$ and error variance σ_η^2 are known by the observer. In addition she suspects that the process in (1) (which I will call the old process) may not more be valid for time periods t–j+1 to the present (t). The observer faces the possibility that at an unknown period t–j+1 there was a structural change in the time-series process defined as a mean shift. Therefore the stochastic character of this change is referred to both the magnitude ot the mean shift and to the period in which it happened.

Once the shift has taken place –a fact never known with certainty– it is assumed that p will be governed by the new process for a stochastic time length of unknown mean and variance. Hence I exclude the special case of a time-dependent cumulative distribution function for the probability of a structural change (like a Poisson). This implies that the observer will make use of present and past observations in a framework in which passing of time does not represent an additional piece of information. It is in this sense that the structural changes are entirely stochastic.

The process followed by p from t–j+1 to the present (which I will call the new process) is given by:

(2) $p_s = \bar{\bar{p}} + \eta_s \quad$ for $s > t-j$.

For identification purposes I assume that the white-noise error term in (2) has the same distribution as that in (1).

The mean $\bar{\bar{p}}$ in (2) is unknown by the observer. She does not know if the true mean $\bar{\bar{p}}$ is identical to the old mean $\bar{p}$ (*i.e.*, the old process still holds) or if there has been a structural change defined as a mean shift so that $\bar{\bar{p}} \neq \bar{p}$ (*i.e.*, the old process is not more valid).

The observer's task is to predict p_{t+1} at time t, given the described possibility of a mean shift. Using this information she projects p_{t+1} as a weighted average of the old process' known mean and of an estimated mean $\hat{\bar{\bar{p}}}_t$ corresponding to the new process:

(3) $E_t[p_{t+1}] = \theta_t \bar{p} + (1-\theta_t)\hat{\bar{\bar{p}}}_t \qquad$ where $0 \leqslant \theta_t \leqslant 1$.

Given that the stochastic variable has followed the new process in (2) from t–j+1 to the present, the optimal forecast would be $E_t[p_{t+1}] = \bar{\bar{p}}$, if the observer knew $\bar{\bar{p}}$. However, the agent's use of eq. (3) reflects her uncertainty on how to interpret the observations belonging to the recent past. She does not know if they still are correctly described by the old process or if they belong to the new process with a mean differing from $\bar{p}$.

This reflects the simple fact that uncertainty about a structural change is reduced to uncertainty on the new process' true mean $\bar{\bar{p}}$. If the observer knew $\bar{\bar{p}}$, her forecast would trivially collapse to that value. On the other side, if she knew with certainty that a structural change has taken place, she would only use $\hat{\bar{\bar{p}}}_t$ in forecasting p. In both cases the weight parameter θ_t is zero and our problem vanishes.

When obtaining θ_t below as a solution to the signal-extraction problem, it should be an increasing function of the uncertainty associated to a structural change. As the uncertainty vanishes, we want θ_t converging to zero.

Let's derive the optimal θ_t from minimizing a standard mean square prediction error:

(4) $E_t[(p_{t+1} - E_t[p_{t+1}])^2]$

Even at time t + 1 it will not be known if p_{t+1} belongs to the old process or not. But the realization can be written either as:

(5) $p_{t+1} = \bar{p} + \epsilon_{t+1}$

or as:

(6) $p_{t+1} = \bar{\bar{p}} + \eta_{t+1}$

which has the implication that ϵ_{t+1} and η_{t+1} are related by:

(7) $\epsilon_{t+1} = (\bar{\bar{p}} - \bar{p}) + \eta_{t+1}$

The "error" term ϵ_{t+1} only collapses to η_{t+1} if the old process is still valid. Its unconditional expectation (in the sense of not knowing if the old process still holds) is:

(8) $E_t[\epsilon_{t+1}] = \hat{\bar{\bar{p}}}_t - \bar{p}$

We can write p_{t+1} as:

(9) $p_{t+1} = \theta_t[\bar{p} + \epsilon_{t+1}] + (1-\theta_t)[\bar{\bar{p}} + \eta_{t+1}]$

Eq. (7) implies that (9) is an identity. But it is a useful form because its expected value at t is:

(10) $E_t[p_{t+1}] = \theta_t E_t[\bar{p} + \epsilon_{t+1} \mid \text{old process in true}] + (1-\theta_t)$
$E_t[\bar{\bar{p}} + \eta_{t+1} \mid \text{old process is not true}]$

which is equal to (3).

Therefore, after sustituting (3) and (9) into (4), the objective function is:

(11) $E_t[(\theta_t(\bar{p} + \epsilon_{t+1}) + (1-\theta_t)(\bar{\bar{p}} + \eta_{t+1}) - \theta_t\bar{p} - (1-\theta_t)\hat{\bar{\bar{p}}}_t)^2]$

The mean of the new process is measured (or estimated) with a white-noise measurement error ω_t, uncorrelated with η_t:

(12) $\hat{\bar{\bar{p}}}_t = \bar{\bar{p}} + \omega_t$

where:

$E[\omega_t] = 0,\ E[\omega_t\ \omega_{t-1}] = 0,\ E[\omega_t\eta_t] = 0.$

Substitute (12) into (11) to obtain:

(13) $E_t[(\theta_t\,\epsilon_{t+1} + (1-\theta_t)\,\eta_{t+1} - (1-\theta_t)\,\omega_t)^2]$

The objective function used thoughout this section is obtained by substituting for second moments after squaring and taking expected values in (13), as shown in appendix 1:

(14) $\theta_t^2(\hat{\bar{\bar{p}}}_t - \bar{p})^2 + (1 - 2\theta_t + 2\theta_t^2)\,E_t[\omega_t]^2 + \sigma_\eta^2$

Note that because:

(i) the stochastic process supposedly followed by p from t–j + 1 to the present is described by our new process, and hence the second moment $E_t[(\epsilon_{t+1})^2]$ is related to (and is always bigger than) the variance σ_η^2, but

(ii) the mean of the new process is unobservable and thus is estimated with a measurement error,

the objective function (23) differs from that used in the standard signal-extraction problem[3].

2.2 *Optimal Prediction Under Known Point of Structural Change*

Let's assume that if the structural change has taken place, which is an uncertain event for the observer, she knows it took place at a given period.

If she knows that the possible breaking point is in the present period, the estimated mean is p_t. Substituting the corresponding variance derived in appendix 1, yields the following objective function:

(15) $\theta_t^2(p_t - \bar{p})^2 + 2(1 - \theta_t + \theta_t^2)\,\sigma_\eta^2$

Minimize (15) with respect to θ_t to obtain:

$$(16)\quad \theta_t = \frac{\sigma_\eta^2}{(p_t - \bar{p})^2 + 2\sigma_\eta^2}$$

This expression is intuitively appealing, as I will discuss below.

Now assume that if the structural change has taken place, the observer knows with certainty it happened in period t−n+1. The estimated mean of the new process is $\frac{1}{n}\sum_{j=1}^{n} p_{t-j+1}$, with measurement-error variance also derived in appendix 1. Substitute these expressions into (14) obtaining:

$$(17)\quad \theta_t^2 \Big(\frac{1}{n}\sum_{j=1}^{n} p_{t-j+1} - \bar{p}\Big)^2 + (1 - 2\theta_t + 2\theta_t^2)\frac{1}{n}\sigma_\eta^2 + \sigma_\eta^2$$

Minimize (17) with respect to θ_t to obtain:

$$(18)\quad \theta_t = \frac{\frac{1}{n}\sigma_\eta^2}{\Big(\frac{1}{n}\sum_{j=1}^{n} p_{t-j+1} - \bar{p}\Big)^2 + \frac{2}{n}\sigma_\eta^2}$$

From eqs. (17) and (18) and from appendix 1 it follows that:

(i) If the new process mean were known, then $\omega_t = 0 = E_t\,[(\omega_t)^2]$, impliying that the optimal θ_t is zero. Hence, we only use the new process mean in predicting p_t.

(ii) The weight attached to the old process mean increases with the variance of the measurement error of the new process mean, and decreases with the squared difference between the estimate of the new process mean and the known old process mean.

(iii) When the sample size for estimating the new process mean increases, θ_t decreases toward zero:

$$\lim_{n\to\infty} \theta_t = 0$$

(iv) The highest possible value attainable by θ_t is 0.5, which is the case when the estimate of the new process mean is equal to $\bar{p}$. This result also has intuitive appeal: even when our last n observations' average coincides with $\bar{p}$, we face a 0.5 chance that they belong to a new process with a true mean $\bar{\bar{p}}$ which differs infinitesimally from $\bar{p}$.

2.3 *Optimal Linear Prediction Under Unknown Point of Structural Change*

A more general problem than that addressed by the preceding subsection is that refered to an unknown point of structural change, which we will address now. Therefore, in addition to deriving the optimal θ_t, we have to define how to obtain the estimate of the new process mean.

Under unknown breaking point, but assuming that the structural change took place in any of the past m periods, I will assume that $\bar{\bar{p}}$ is estimated as a weighted average of the m possible arithmetic averages:

$$(19)\quad \hat{\bar{\bar{p}}}_t = \mu_{1t}\, p_{1t} + \mu_{2t}\, p_{2t} + \ldots + \mu_{mt}\, p_{mt}$$

where:

$$p_{1t} \equiv p_t,\quad p_{2t} \equiv \frac{1}{2}(p_t + p_{t-1}), \ldots\, p_{mt} \equiv \frac{1}{m}(p_t + \ldots + p_{t-m+1}),$$

and $\sum_{j=1}^{m} \mu_{jt} = 1.$

This estimation alternative for $\bar{\bar{p}}$ may seem arbitrary. But we should note that the weight μ_{jt} assigned to each average (*i.e.*, to each possible breaking point) also will be derived from minimizing the square prediction error.

In order to illustrate how this problem is solved we will concentrate on the special case of a possible breaking point at any of the last two periods (m = 2). Thus we have two choices for the estimate of the new process mean. Define μ_t as the weight attached to the first choice. According to (19), the estimate of $\bar{\bar{p}}$ is now:

$$(19')\quad \hat{\bar{\bar{p}}}_t = \mu_t\, p_{1t} + (1 - \mu_t)\, p_{2t}$$

Substitute (19') and the measurement-error variance (eq. (A13) in appendix 1) into (14) to obtain the following objective function:

$$(20)\quad \theta_t^2\,[\mu_t\, p_{1t} + (1-\mu_t)\, p_{2t} - \bar{p}]^2 + (1 - 2\theta_t + 2\theta_t^2)\, 2\mu_t\,(1-\mu_t)^2$$
$$[(p_{1t})^2 - (p_{2t})^2] + (1 - 2\theta_t + 2\theta_t^2)\big(\tfrac{1}{2} + \mu_t - \tfrac{1}{2}\mu_t^2\big)\sigma_\eta^2 + \sigma_\eta^2$$

Minimize this expression with respect to θ_t to obtain a first equation in θ_t and μ_t:

$$(21)\quad \theta_t = \frac{2\mu_t\,(1-\mu_t)^2\,[(p_{1t})^2 - (p_{2t})] + (\frac{1}{2} + \mu_t - \frac{1}{2}\mu_t^2)\,\sigma_\eta^2}{[\mu_t p_{1t} + (1-\mu_t)p_{2t} - \bar{p}]^2 + 4\mu_t(1-\mu_t)^2\,[(p_{1t})^2 - (p_{2t})^2] + 2(\frac{1}{2} + \mu_t - \frac{1}{2}\mu_t^2)\,\sigma_\eta^2}$$

Now minimize (20) with respect to μ_t to obtain the following equation in θ_t and μ_t:

$$(22)\quad \frac{\theta_t^2}{\theta_t^2 + (1-\theta_t)^2} = -\frac{(1 - 4\mu_t + 3\mu_t^2)\,[(p_{1t})^2 - (p_{2t})^2] + (1-\mu_t)\,\sigma_\eta^2/2}{[\mu_t p_{1t} + (1-\mu_t)\, p_{2t} - \bar{p}]\,[p_{1t} - p_{2t}]}$$

There is no analytical solution for θ_t and μ_t from the system of equations (21) and (22). In fact, when substituting (21) into (22), one obtains a polynomial of sixth degree for μ_t, with no general solution.

Nevertheless we can draw some general conclusions from these results:

(i) Both equations are well behaved in the sense of rendering solutions for θ_t and μ_t which lie in the 0−1 range, for any parameter values.

(ii) There is no restriction placed on the solution of μ_t resulting from our optimization process. This implies that the weights attached to p_{1t} and p_{2t} in estimating $\bar{\bar{p}}$ may have any value between 0 and 1, subject to that they add 1.

(iii) If $p_{1t} = p_{2t}$, then $\mu_t = 1$ (from eq. (22)) and $\theta_t = \sigma_\eta^2 / [p_{1t} - \bar{p})^2 + \sigma_\eta^2]$ (from eq. (21)), which is our result obtained in subsection 2.2.

2.4 *A Simplified Linear Predictor*

We have derived a particular structure of expectations formed as distributed lags which are consistent with rational foresight in the presence of stochastic structural changes of the underlying time-series processes, assuming a simple time-series structure for short or medium-term processes. Uncertainty about the mean of the new process, combined with certainty about the breaking point at t−n+1 (if there was a structural change), gives rise to an optimal forecast which attaches fixed weights to past observations. From eq. (3) and subsection 2.2 we have:

$$(23)\quad E_t[p_{t+1}] = \theta_t\,\bar{p} + \left(\frac{1-\theta_t}{n}\right)\sum_{j=1}^{n} p_{t-j+1}$$

If in addition the observer faces uncertainty about the period in which the new process started, the optimal forecast will attach declining weights to past observations. From (3) and subsection 2.3 we have:

$$(24)\quad E_t[p_{t+1}] = \theta_t\,\bar{p} + (1-\theta_t)\sum_{j=1}^{m} \mu_{jt}\,p_{jt} =$$

$$\theta_t\bar{p} + (1-\theta_t)\left[\frac{\mu_{1t} + \ldots + (1-\mu_{1t} \ldots - \mu_{mt})}{m}\right] p_t +$$

$$+ (1-\theta_t)\left[\frac{1-\mu_{1t} - \ldots - \mu_{mt}}{m}\right] p_{t-m+1} =$$

$$= \theta_t\,\bar{p} + (1-\theta_t)\sum_{j=1}^{m} \xi_j\,p_{t-j+1}$$

where the ξ_j correspond to the linear combinations of the μ_j obtained in the previous expressions, satisfying $\xi_1 > \xi_2 > \ldots . \xi_m$.

In subsection 2.3 we have shown that in the case of general uncertainty the optimal coefficients $\mu_{1t}, \ldots \mu_{mt}$ are not related in a particular way to each other. Therefore, although we obtain optimal predictors with declining parameters ξ_j ($j = 1, \ldots m$) for lagged observations, we do not obtain truely adaptive expectations or any other particular linearly or non-linearly declining pattern for the general case.

Now let's focus on a specific form for eq. (24), simplifying it in a way such that it will be useful in section 4.

Let's introduce the following simplifying assumptions:

(i) $m-1$, the number of lagged variables, is fixed over time,

(ii) actual realizations coincide with the means of the underlying time-series processes (a fact not known ex ante by the agent),

(iii) θ_t converges linearly to zero $m-1$ periods after a structural change has taken place (instead of declining exponentially over the entire future time horizon), and

(iv) the ξ_j (the weights for present and lagged observations) decline linearly from $j = 1$ to $j = m$, and are constant over time.

With these assumptions the linear predictor takes the following form:

$$(25)\quad E_t[p_{t+1}] = \theta'_t\,\bar{p} + (1-\theta'_t)\,\frac{2}{m(m+1)}\sum_{j=1}^{m}(m+1-j)\,p_{t-j+1}$$

where:

$$(26)\quad \theta'_t = \begin{cases} \dfrac{1}{2} & \text{for } t < t_0 \\[2ex] \dfrac{m-(t+1-t_0)}{2m} & \text{for } t_0 \leqslant t < t_{0+m-1} \\[2ex] 0 & \text{for } t \geqslant t_{0+m-1} \end{cases}$$

and where t_0 is the period from which on a structural change might have taken place.

Finally note that an implication of having white-noise deviations around means, and given that passing of time is not an additional variable in the agent's information

set, optimal predictions of future prices are constant over the time horizon, at a given period t:

$$(27)\quad E_t[p_{t+1}] = E_t[p_{t+k}] \qquad k = 1, 2, \ldots\ldots$$

This equivalence states that the observer will project p_{t+k} as a constant into the future, although she knows that the present short or medium-term WNAM process (which is of finite but entirely stochastic length) will be replaced with probability one by another WNAM process as k converges to infinity. This seems to be appropriate in the absence of any information on the "big" long-term process which encompasses all shorter-term "old" and "new" processes.

3. Stochastic Terms of Trade and the Current Account

In this section I will analyze the effects of stochastic terms-of-trade shocks on the current account (or on net external borrowing) of a small and open debtor economy. With no investment and with fixed and certain endowments, the representative consumer's infinite-horizon optimizing plan will give rise to an optimal path of the country's debt. The optimal consumption and debt plan is revised each period with new information on the stochastic terms-of-trade process. With regard to information processing, the consumer uses the framework of time-series sequences presenting stochastic structural changes, developed in section 2. Applying the corresponding error-learning model, she predicts future terms of trade according to the particular lag structure obtained above.

The perfect-foresight case for anticipated and unanticipated, transitory and permanent price changes has been extensively treated in the recent literature.

The specific effect of anticipated and certain future relative price changes on consumption and borrowing paths (through changes in the consumption-based real interest rates) has been analyzed recently. Dornbusch (1983) and Martin and Selowsky (1984) study this effect in traded / non-traded goods models with substitution in consumption and in production. Obstfeld (1983) analyses the same effect for anticipated future terms-of-trade fluctuations for a fixed-endowment economy with consumption of importable and exportable goods.

Under uncertainty of future relative prices the traditional distinction between transitory and permanent, expected and unexpected shocks, made by the perfect-foresight literature, breaks down. As a result of uncertainty, in each period there is a "shock" defined as the error of the prediction conditional on the information of the previous period. In the framework of stochastic structural changes governing the time series, the consumer will assess if this error is white noise of an old, known process or if the new observation belongs to a structurally different, new process. Although there are similarities with the transitory/permanent distinction, the nature of the shock will be known more precisely only after more information is accumulated, *i.e.*, ex-post.

The expected-unexpected distinction disappears in this model. Any deviation of relative prices from an old and known process mean gives rise to a difference between the current period's relative price and the price expected today to prevail in the future. Therefore, with probability 1 there is a future price change expected today. So, each period the consumer predicts a terms-of-trade change, which obviously might or might not be realized. The expected price change affects the expenditure-based real interest rate, and therefore causes a revision of (ex-ante) optimal consumption and borrowing paths, in a similar way as analysed by the abovementioned authors.

When introducing uncertainty into intertemporally-optimizing consumptio problems, there are two different alternatives to derive an analytical solution.

One alternative is to assume the stochastic variable follows either a Wiener process (or Brownian motion) or a Poisson process. As Merton shows in his 1971 article, only these processes satisfy Itô's Lemma, a fundamental tool to solve a dynamic stochastic programming problem of this type. Merton solves the consumption-portfolio problem for the case of aggregate consumption and two assets, with uncertainty related to capital and/or wage income, assuming the consumer's utility function belongs to the big hyperbolic absolute risk-aversion (HARA) family.

An alternative to this approach consists in allowing the time-series process to take any form –even to present stochastic structural changes– at the cost of restricting the utility functions to a smaller family: the class of functions satisfying certainty equivalence (CEQ)[4].

As I am interested in analyzing the effects of freely changing terms of trade and interest rates (both presenting structural changes) on consumption and on the current account, I will pursue this second alternative.

The CEQ solution to the intertemporal consumption problem normally requires the following conditions to hold[5]:

(i) the utility function has a zero third partial derivative with respect to each of its arguments,

(ii) there are no liquidity constraints, and consumption may vary between $-\infty$ and $+\infty$, and

(iii) all loans must be repaid with certainty.

Under these conditions the consumption path is identical to what it would be under no uncertainty.

From a theoretical (but not necessarily empirical) point of view, conditions (ii) and (iii) do not seem to be very restrictive for my general problem. Anyway I will limit the analysis to non-negative consumption levels.

Condition (i) might be more costly in the sense of restricting the utility function to a quadratic form in the one-good case, as in Zeldes (1984), or to a quadratic-hyperbolic function in the two-goods case, as I present below. Although risk-aversion is assured by having negative second derivatives, the homogeneity property is lost under certainty equivalence.

In the more general case of non-CEQ, HARA utility functions, positive third derivatives (or decreasing absolute risk aversion), imply that added uncertainty about future income or prices decreases optimal consumption at all levels of wealth, and increases the sensitivity of consumption to current income. By contrast, under CEQ savings are independent of the variance of the stochastic variable, *i.e.*, there is no additional "precautionary demand for saving", as originally called by Leland (1968).

3.1 *The Consumer Problem*

A small and open debtor economy is composed by identical infinitely-lived households. The representative consumer's concave period utility function $U(x_s, m_s)$ is defined for her consumption of an exportable good (x_s) and an importable good (m_s).

The path of the price of exports in terms of imports, p_s, is exogenous to this economy. With perfect integration to world capital markets, so is the interest rate r. The interest rate is restricted to be equal to ρ, the household's constant rate of time preference. Following Obsteld (1983, 1984), I assume that r is fixed in terms of the importable

good, and reflects the return to the only available asset, which is an internationally-traded bond. This assumption seems to be relevant for many developing economies, whose debts are mostly US dollar-denominated, while a substantial fraction of their imports is also priced in US$. Anyway, below I also discuss the implications of having the interest rate fixed in terms of the exportable good.

The number of bonds hold at the beginning of the planning period is b_t. In this endowment economy the representative household holds a claim to an exogenous and constant flow of y units per period of the exportable good. There is no domestic production of the importable good. Both goods are non-storable.

In order to determine her optimal consumption and debt paths, the infinitelly-lived consumer maximizes discounted lifetime utility:

$$(28)\quad V = E_t\left[\sum_{s=t}^{\infty} U(x_s, m_s)\left(\frac{1}{1+\rho}\right)^{s+1}\right]$$

subject to the ex-post budget constraint:

$$(29)\quad \sum_{s=t}^{\infty}[p_s x_s + m_s]\left(\frac{1}{1+r}\right)^{s+1} = b_t + y\sum_{s=t}^{\infty} p_s\left(\frac{1}{1+r}\right)^{s+1}$$

and to the transversality condition:

$$(30)\quad \lim_{s\to\infty} b_s\left(\frac{1}{1+r}\right)^{s} = 0$$

This specification assumes the consumer taking her decisions at the beginning of each planning period t, when current terms of trade are already known, but makes all transactions at the end of period[6].

Ex post the budget constraint (29) holds exactly. Any difference between expected terms of trade and realized values will be reflected in wealth changes, and hence will take the household to revise her optimal plans ex post. But in order to solve her ex-ante problem, the consumer uses the ex-ante version of (29), which is[7]:

$$(31)\quad (p_t x_t + m_t)\frac{1}{1+r} + (x_t^e p_t^e + m_t^e)\frac{1}{r(1+r)} =$$

$$= b_t + y\,p_t\frac{1}{1+r} + y\,p_t^e\frac{1}{r(1+r)} \equiv W_t$$

where prices p_t^e and consumption levels x_t^e and m_t^e, expected at t to prevail in the future, are constant over the future horizon:

$$p_t^e = E_t[p_s],\quad x_t^e = E_t[x_s],\quad m_t^e = E_t[m_s],\ \forall s > t$$

The flat expected future price profile, at a given planning period t, is a consequence of the time-series structure assumed to prevail, and therefore is a restatement of eq. (27) in section 2. Naturally, constant expected future prices imply constant expected future consumption levels.

Wealth is noted by W_t, the sum of bond holdings and the present value of constant income streams in eq. (31).

Next let's introduce the following quadratic-hyperbolic non-homogeneous period utility function:

(32) $U_s = \alpha x_s + \beta m_s + \gamma x_s m_s - \frac{\delta}{2}(x_s + m_s)^2$

This particular form has to satisfy the following requirements:

(i) Certainty Equivalence

This utility function belongs to the class of quadratic functions, so that:

$U_i' > 0,\ U_i'' < 0,\ U_i''' = 0,$ for $i = x, m$.

(ii) Non-negativity of consumption levels

For non-negative x and m we have to impose a certain relationship between parameters of the utility function and the annuity value of wealth (or permanent income). For the particular case of current and expected future prices equal to one, this condition is:

(33) $r W_t \geq |A - B|$

were A and B are the bliss points (the levels of consumption the household would choose in the absence of any wealth constraint) defined by:

$$A \equiv \frac{\alpha\delta + \beta(\gamma - \delta)}{\gamma(2\delta - \gamma)} \qquad B \equiv \frac{\beta\delta + \alpha(\gamma - \delta)}{\gamma(2\delta - \gamma)}$$

Condition (33) imposes a constraint on how much relative preferences can differ *vis-a-vis* permanent income to avoid having either x_t or m_t negative.

(iii) Positive marginal utilities

To insure a negative marginal rate of substitution we have to consume at the left side of the bliss points. That implies that permanent income can not exceed the expenditure required to consume at the bliss points. For the particular case of current and expected future prices equal to one, this conditions is:

(34) $r W_t \leq A + B$

(iv) Normal intertemporal substitution and normal shadow value of wealth

In order to obtain normal intertemporal substitution in the sense of having relative present to future consumption levels a negative function of current expenditure-based real interest rates we have to impose:

(35) $2\delta > \gamma > \delta$

This condition also allows the shadow value of wealth to be a negative function of wealth and it is a necessary (but not sufficient) condition for (34) to hold.

Note that a simpler form for the period utility function than eq. (32) can not satisfy simultaneously conditions (iii) and (iv). If in (32) we have $\alpha = \beta = 0$, condition (iii) would imply $2\delta > \gamma$, which is the opposite of (35) for condition (iv) to hold.

(v) Normal intratemporal substitution

To obtain a positive (intratemporal) elasticity of substitution between goods, we require $\gamma > \delta$ to hold (which is part of (35)), and therefore the third term in (32) can not be dropped.

3.2 *Optimal Consumption and Expenditure*

The necessary first-order conditions of the consumer problem are:

(36) $\alpha + (\gamma - \delta) E_t[m_s] - \delta E_t[x_s] = E_t[\lambda_s p_s]$

(37) $\beta + (\gamma - \delta) E_t[x_s] - \delta E_t[m_s] = E_t[\lambda_s]$

(38) $E_t[\lambda_{s+1}] - E_t[\lambda_s] = (\rho - r) E_t[\lambda_s]$

From assuming $\rho = r$, the expected shadow value of wealth is constant over the planning horizon, at a given period t:

(38') $E_t[\lambda_s] = \lambda_t \qquad \forall\ s \geq t$

From equations (36)-(38') and from the intertemporal budget constraint (31) obtain equilibrium current consumption levels for each good:

$$(39)\ x_t = r W_t (1+r)\left[\frac{\gamma - \delta + \delta p_t}{D_t r + D_t^e}\right] + A\left\{1 - [p_t r + p_t^e]\left[\frac{\gamma - \delta + \delta p_t}{D_t r + D_t^e}\right]\right\} - B(1+r)\left[\frac{\gamma - \delta + \delta p_t}{D_t r + D_t^e}\right]$$

$$(40)\ m_t = r W_t (1+r)\left[\frac{\delta + (\gamma - \delta) p_t}{D_t r + D_t^e}\right] + B\left\{1 - (1+r)\left[\frac{\delta + (\gamma - \delta) p_t^e}{D_t r + D_t^e}\right]\right. - A[p_t r + p_t^e]\left[\frac{\delta + (\gamma - \delta) p_t}{D_t r + D_t^e}\right]$$

where:

$$D_t \equiv \delta + 2(\gamma - \delta) p_t + \delta (p_t)^2$$

$$D_t^e \equiv \delta + 2(\gamma - \delta) p_t^e + \delta (p_t^e)^2$$

Analogous expressions can be obtained for expected future consumption levels x_t^e and m_t^e.

Now evaluate (39)-(40) at $p_t = p_t^e = 1$, to obtain:

(39') $x_t \big|_{p_t = p_t^e = 1} = x_t^e \big|_{p_t = p_t^e = 1} = 0.5\ [r W_t + (A - B)]$

(40') $m_t \big|_{p_t = p_t^e = 1} = m_t^e \big|_{p_t = p_t^e = 1} = 0.5\ [r W_t + (B - A)]$

At constant unit current and expected future prices, current and planned future consumption levels coincide. Not surprisingly, these levels differ from 50% of permanent income by a proportion of the consumer's relative preference for each good.

An interesting result, consistent with the earlier literature, is having the difference between current and expected future consumption levels a negative function of the difference between current and expected future terms of trade:

(41) $(x_t - x_t^e) = (p_t - p_t^e)\,\delta \left\{ r W_t - A\left(\frac{r}{1+r} p_t + \frac{1}{1+r} p_t^e\right) - B \right\}$

$$(42)\quad (m_t - m_t^e) = (p_t - p_t^e)(\gamma - \delta)\ \left\{ r\,W_t - A\left(\frac{r}{1+r}\,p_t + \frac{1}{1+r}\,p_t^e\right) - B \right\}$$

This reflects the effect of the consumption-based real interest rate on intertemporal consumption decisions in a framework of uncertain future prices.

As a result of having the importable good as our numeraire, an increase in p_t denotes a rise in the relative price of the exportable. An increse in p_t *vis-a-vis* p_t^e implies an expected terms-of-trade decline in the following period. This has an immediate implication for the real cost of foreign borrowing. A unit of foreign borrowing (equivalent to a unit of the importable good) today "has relatively little purchasing power in terms of the consumption basket today but costs a lot in terms of the consumption basket upon repayment of the loan next period"[8]. Hence, the consumer substitutes intertemporally in response to changes in the real rate of interest, which is a weighted average of the real own-rate for exportable consumption (which is higher because of the expected price drop) and of the real own-rate for importable consumption (which is permanently equal to the constant nominal interest rate r).

An important result to note is that relative present to expected future consumption levels of *both* goods are negative functions of the difference $(p_t - p_t^e)$. This is more clearly seen when instead of an increase in p_t, the consumption-based real interest increases because of a fall in p_t^e. In this case there is no current price change, and therefore relative present consumption levels remain unaltered. (This assumes A = B, otherwise the wealth effect caused by a lower p_t^e affects x_t/m_t; see eq. (46) below). But the increase in the real interest rate generates an incentive to substitute future for current consumption and therefore, given unaltered x_t/m_t, consumption of both the exportable and the importable good will decline.

This result also stems from assuming normal intertemporal and intratemporal substitution, and from consuming at the left side of the bliss points (conditions (iv), (v) and (iii)), which are quite general conditions. It is also a result from having the importable as the numeraire. If alternatively we had the exportable good as numeraire, a rise in the terms of trade is a reduction in the current relative price of importables, implying that foreign borrowing is cheaper in the present period. This decline in the consumption-based real interest causes an increase in consumption of both goods; the opposite of the result presented in (41)-(42). Similary, all results we will discuss below depend crucially on choosing the importable good as numeraire.

The role of real interest rates relevant for intertemporal consumption decision has been previously analysed by Martin and Selowsky (1984) and by Dornbusch (1983) in a framework of traded/non-traded goods and by Obstfeld (1983, 1984) in a exportable/importable goods model. These authors model the perfect-foresight case, while the result in (41)-(42) is a natural extension for price uncertainty.

Now let's focus on the effects of current and expected future price increases on absolute current consumption levels.

The impact of a current-price increase is obtained by differentiating (39) and (40) with respect to p_t. Evaluate these expression at $p_t = p_t^e = 1$[9], to obtain:

$$(43)\quad \left.\frac{\partial x_t}{\partial p_t}\right|_{p_t = p_t^e = 1} = \frac{1}{2\gamma(1+r)}\ \Big\{ [r\,W_t - (A+B)]\ [(1+r)\delta - r\gamma] + (y - A)\,r\gamma \Big\} \gtrless 0$$

$$(44)\quad \left.\frac{\partial m_t}{\partial p_t}\right|_{p_t = p_t^e = 1} = \frac{1}{2\gamma(1+r)}\ \Big\{ [r\,W_t - (A+B)]\ [-(1+r)\delta + \gamma] + (y - A)\,r\gamma \Big\} \gtrless 0$$

The ambiguous effect of p_t on current consumption levels reflects the simultaneous operation of three effects with opposing signs: a positive wealth effect, a negative intertemporal substitution effect and an intratemporal substitution effect. The wealth effect stems from the one-period transfer from the rest of the world which comes from higher terms of trade. But higher prices today imply an expected price decline tomorrow, and therefore the consumption-based real interest rate declines, affecting negatively both consumption levels. In addition, higher p_t induces intratemporal substitution today, increasing m_t at the expense of x_t. The latter effect explains why an increase in p_t affects m_t less negatively, or more positively, compared to x_t.

With regard to relative current consumption levels x_t/m_t, a current price rise will unambiguously reduce relative exportable/importable consumption levels, as a result of condition (v) assuming intratemporal substitutability among goods.

To analyze the impact of p_t^e differentiate (39) and (40) with respect to expected future prices, and evaluate again these expressions at $p_t = p_t^e = 1$:

$$(45)\quad \left.\frac{\partial x_t}{\partial p_t^e}\right|_{p_t = p_t^e = 1} = \left.\frac{\partial m_t}{\partial p_t^e}\right|_{p_t = p_t^e = 1} = 0.5\,(-r b_t + B)\,\frac{1}{1+r} \gtrless 0$$

For any debtor economy ($b_t < 0$), and for most creditor economies, a rise in expected future prices increases consumption of both goods in response to positive wealth and positive intertemporal substitution effects. The partial effect is the same for exportable and importable consumption because of the absence of any intratemporal substitution effect: current terms of trade remain unchanged.

The effect of p_t^e on relative current consumption levels will depend on the relative preference-intensities for each good in the following way:

$$(46)\quad \left.\frac{\partial (x_t/m_t)}{\partial p_t^e}\right|_{p_t = p_t^e = 1} = \frac{2}{(m_t)^2}\,[-r b_t + B]\,(B - A)\,\frac{1}{1+r} \gtrless 0$$

Current relative consumption levels x_t/m_t will increase whenever the relative preference for good m is stronger ($B - A > 0$), because in that case, for $p_t = p_t^e = 1$, x_t must be lower than m_t and from (45) we know the partial effect on absolute consumption levels is the same for each good.

Now let's turn to aggregate consumption expenditure Z_t, which is given by the following expression:

$$(47)\quad Z_t = r W_t \left[\frac{D_t(1+r)}{D_t r + D_t^e}\right] + A \left\{ p_t - \left[\frac{D_t}{D_t r + D_t^e}\right] [p_t r + p_t^e] \right\} + B \left\{ 1 - \left[\frac{D_t(1+r)}{D_t r + D_t^e}\right] \right\}$$

When current and expected future prices coincide, the expenditure profile is obviously flat and equals permanent income:

$$(47')\quad Z_t \Big|_{p_t = p_t^e} = Z_t^e \Big|_{p_t = p_t^e} = r\,W_t$$

While current consumption levels of each good are lower than expected future levels when p_t exceeds p_t^e (eqs. 41-42), this is not the case of aggregate consumption expenditure:

$$(48)\quad Z_t - Z_t^e = (p_t - p_t^e)\left[\frac{1+r}{D_t\,r + D_t^e}\right]\Big\{\,[2(\gamma-\delta) + \delta(p_t + p_t^e)]$$

$$[r\,W_t - (A+B)] + \delta\,[1 + p_t + p_t^e - p_t p_t^e]\;A\Big\}$$

The two terms in the big right-hand side parenthesis have opposite sign simply because when current prices are high, current quantities are low, and therefore current total expenditure is not necessarily lower than expected future expenditure.

The effects of current and expected future prices on total consumption expenditure are consistent with the corresponding effects on absolute consumption levels (eqs. 43-45). Here higher p_t and p_t^e have an ambiguous and a positive effect on consumption expenditure, respectively:

$$(49)\quad \frac{\partial Z_t}{\partial p_t}\Big|_{p_t = p_t^e = 1} = y + [rb_t - B]\,\frac{1}{1+r} \gtrless 0$$

$$(50)\quad \frac{\partial Z_t}{\partial p_t^e}\Big|_{p_t = p_t^e = 1} = [-rb_t + B]\,\frac{1}{1+r} > 0$$

3.3 *The Current Account*

Following the tradition of the "real" intertemporal-optimization literature, in this model real variables alone determine aggregate expenditure and net foreign debt. Hence changes in net asset holdings merely reflect differences between income and absorption. There are no independent forces determining separately the capital account. The current account surplus (CAS_t) equals domestic savings defined by the excess of income over consumption expenditure, and is made possible by an identical reduction of net foreign debt:

$$(51)\quad CAS_t \equiv rb_t + y\,p_t - Z_t \equiv b_{t+1} - b_t$$

Substitute for Z_t to obtain:

$$(52)\quad CAS_t = [rb_t - B]\left[\frac{D_t^e - D_t}{D_t\,r + D_t^e}\right] + (y - A)\left[\frac{p_t\,D_t^e - p_t^e\,D_t}{D_t\,r + D_t^e}\right]$$

Any positive current price shock will induce an expected price decrease in the following period, because expected future prices are only revised by a fraction of the current price increase, according to the error-learning model which culminates in eq. (24).

Abstracting from the effect of current prices on p_t^e (see footnote (9)), a current price rise will induce a current account surplus, which is (evaluated at $p_t = p_t^e = 1$):

$$(53)\quad \frac{\partial CAS_t}{\partial p_t}\Big|_{p_t = p_t^e = 1} = [-rb_t + B]\,\frac{1}{1+r} > 0$$

Although the effect of higher current terms of trade on aggregate consumption expenditure is indeterminate, anyway it is smaller than the effect on current income.

Therefore we have four effects generating a current-account surplus: a positive income effect which dominates over the remaining three effects which act upon consumption: a positive wealth effect, a negative intertemporal substitution effect and an ambiguous intratemporal substitution effect.

The reason behind the dominating income effect is that the consumer does not believe current high prices will be maintained in the future. Through intertemporal smoothing she increases current consumption by less than her current income gain.

When the household expects higher future prices, her consumption smoothing induces a current account deficit equal in size to the previous surplus (evaluated at $p_t = p_t^e = 1$):

$$(54)\quad \frac{\partial CAS_t}{\partial p_t^e}\Big|_{p_t = p_t^e = 1} = [rb_t - B]\,\frac{1}{1+r} < 0$$

In the absence of any current income effect, the amount of dissaving obviously equals the increase in consumption expenditure (eq. 50), which is not the case when comparing the effects of current price increases on the current acount and on consumption expenditure (eqs. 53 and 49).

4. A Simulation for the Current Account Under Permanent Terms-of-Trade Shocks

In the preceding section I analyzed the effects of changes in the terms of trade on current consumption and expenditure levels and on the current account.

In this section I simulate the effects of permanent terms of trade or price shocks on the one-period and on the cumulative current account (or on net foreign borrowing). The reason for distinguishing between the one-period and cumulative effects is because expected future prices and interest rates adjust gradually to actual shocks during a period of length m, as a result of using optimal predictors represented by eqs. (25) and (26). The changes I will model now are (ex post) permanent shocks. From our preceding discussion we know the consumer can not distinguish ex ante between permanent and transitory shocks.

The equations used in the simulations are (25) and (26) for optimal price predictors, and an equation for foreign debt accumulation consistent with (52).

For the simulations I will use the following values for the exogenous variables and parameters:

$\overline{p} = 1.1$, $\overline{\overline{p}} = 0.90$ (a negative terms-of-trade shock),
$y = 3.0$, $r = 0.10$, $b_{t_0} = -1.0$,
$\gamma = .75$, $\delta = .50$, $\alpha = .80$, $\beta = .90$ (therefore $A = 3.33$, $B = 3.47$).

These values satisfy conditions (i)-(v) imposed in section 3 on the utility function and on the consumer problem in general. Pre and post-shock prices have been chosen to lie in the neighborhood of 1.0, in order to make the results comparable to the analytical results discussed above, which were evaluated at unit current and expected prices in most cases. The initial debt/GDP ratio is 33%, a figure which represents a typical developing debtor economy.

Arbitrarily, I assume a higher relative preference for importable goods.

In addition we want to compare the effect of the terms of trade shock under different lag structures of the optimal price predictor. In one case a lag structure of 3 periods (m = 3) is assumed, in the second it is doubled to 6 periods (m = 6).

When analysing the results it is important to keep in mind that the arbitrariness of the chosen values is an obstacle to evaluating them independently from each other. In fact, the purpose of the simulation is to portray the relative impact of the shock under varying lag structures on the debt profile. In table 1 the results for the time profiles of expected future prices and of foreign debt under the two cases are presented.

TABLE 1

SIMULATED TIME PROFILES OF EXPECTED FUTURE TERMS OF TRADE AND OF THE NET FOREIGN DEBT AFTER A PERMANENT TERMS OF TRADE DETERIORATION

Period	Lag Length of Terms of Trade Predictors			
	m = 3		m = 6	
	p_t^e	b_t	p_t^e	b_t
t_{-1}	1.1	−1.0	1.1	−1.0
t_0	1.033	−1.0	1.067	−1.0
t_1	0.961	−1.413	1.030	−1.510
t_2	0.900	−1.614	0.993	−1.918
t_3			0.957	−2.217
t_4			0.925	−2.408
t_5			0.900	−2.493

In period t_0 a negative terms of trade shock arises as permanent p decreases by 18%. This causes a slow revision of expected future prices, because the rational agent only learns over time that the shock is "permanent" in the sense of inaugurating a new short or medium-term time-series process of entirely stochastic length. Under a lag structure of 3 (6) periods, only in period t_2 (t_5) the consumer expects a future value equal to the new current level, $\bar{\bar{p}} = 0.90$.

Table 1 also portrays the time profile of foreign debt after the shock. A decrease in the country's terms of trade causes an income reduction which is stronger than the effect on expenditure (the latter has ambiguous sign), implying an unambiguous current account deterioration.

It is interesting to note that when the number of lags used in forming expected future prices is doubled, the cumulative current account deficit[10] is multiplied by 2.5, increasing from .614 (20% of GDP) to 1.493 (50% of GDP). The latter is a high proportion of output. If parameter values were realistically chosen (or estimated), and periods were

interpreted as years, a lag structure of six years is probably at the upper limit for a "reasonable" predictor for the terms of trade.

In conclusion, these simulations show that under stochastic structural changes of the time-series processes governing the terms of trade, current account imbalances caused by ex-post permanent shocks are very sensitive to changes in the terms of trade and to the length of the lag structure of the underlying predictors. A permanent terms of trade deterioration will cause an increase in foreign debt during the periods rational agents require to learn about the structure of the new process.

5. Conclusions

In this paper I have analysed the effects of uncertain terms-of-trade shocks on the current account of a representative-consumer fixed-endowment debtor economy. To model shocks of variables which present stochastic structural changes in their underlying time-series processes, I used a certainty-equivalence framework to solve for optimal consumption and borrowing paths.

In section 2, I derived an error-learning model for the signal-extraction problem corresponding to the optimal prediction of a time-series process which presents stochastic parameter shifts. Confronting uncertainty related to both the magnitude and the period in which a structural change occurs, the agent's behavior will give rise to a particular distributed-lag form for the optimal linear predictor.

In the following section the consumer problem for stochastic terms of trade is solved. The wealth, intra and intertemporal substitution effects on consumption are analyzed.

The traditional distinction between expected and unexpected, transitory and permanent shocks of the perfect-foresight literature is not more valid under uncertainty. Any deviation of actual terms of trade from expected values constitutes a shock which induces a revision of optimal consumption and borrowing paths. Hence a current (expected future) price increase has a negative (positive) intertemporal substitution effect and a positive wealth effect on consumption expenditure.

In the case of higher current terms of trade there is an additional positive current-income effect. Therefore a current (expected future) permanent terms-of-trade rise unambiguously causes a current account surplus (deficit) during the adjustment period of expected future to higher current prices.

Finally a simulation for (ex post) permanent terms-of-trade shocks was performed. The effects on the time profile of the net foreign debt were obtained for given values of parameters and exogenous variables, and for two different lag lengths of the optimal predictor. It was shown that the path of net foreign debt is very sensitive to price shocks and to the lag length of the underlying predictor.

Appendix 1: Derivations for Optimal Linear Prediction

In deriving (14) note that it will not help to substitute (7) and (12) into eq. (13) at this stage. If ϵ_{t+1} is substituted from these equations we immediately obtain $\theta = 1$ from minimizing (13). On the other side, if η_{t+1} is substituted into (13) we obtain $\theta = 0$. Both extremes imply higher values of the objective function than the optimal solution.

To derive the latter substitute for the second moments after squaring and taking expected values in (13):

$$(A1)\quad \theta^2 E_t[(\epsilon_{t+1})^2] + (1-\theta)^2 \sigma_\eta^2 + 2\theta(1-\theta) E_t[\epsilon_{t+1}\eta_{t+1}] +$$
$$+ (1-\theta)^2 E_t[(\omega_t)^2]$$

The second moment of ϵ_{t+1} is obtained from (7) and (12):

$$(A2)\quad E_t[(\epsilon_{t+1})^2] = (\hat{\bar{\bar{p}}}_t - \bar{p})^2 + E_t[(\omega_t)^2] + \sigma_\eta^2$$

From (7) derive:

$$(A3)\quad E_t[\epsilon_{t+1}\eta_{t+1}] = \sigma_\eta^2$$

Now substitute (A2) and (A3) into (A1) to obtain the objective function in (14).

Let's turn now to the variances of the measurement errors.

When knowing that the breaking point is in the present period (if it has taken place), the estimated mean is p_t with a measurement error defined by:

$$(A4)\quad \omega_t^1 = p_t - \bar{\bar{p}}$$

Equation (A4) is the equivalent of eq. (6) for period t. Therefore the variance of the measurement error is:

$$(A5)\quad E_t[(\omega_t^1)^2] = E_t[(\eta_t)^2] = \sigma_\eta^2$$

When knowing that the structural change took place at t–n+1 (if it took place), the measurement error is:

$$(A6)\quad \omega_t^n = \frac{1}{n}\sum_{j=1}^{n} p_{t-j+1} - \bar{\bar{p}} = \frac{1}{n}\sum_{j=1}^{n} \eta_{t-j+1}$$

with variance:

$$(A7)\quad E_t[(\omega_t^n)^2] = \frac{1}{n}\sigma_\eta^2$$

As expected, this variance is a decreasing function of n, the number of periods used in estimating $\bar{\bar{p}}$.

Under unknown breaking point, but assuming that the structural change took place in one of the past m periods, $\bar{\bar{p}}$ is estimated as a weighted average of the m possible arithmetic averages, as specified in eq. (19).

The measurement error corresponding to this estimate of $\bar{\bar{p}}$ is defined by:

$$(A8)\quad \omega_t^{*m} = \sum_{j=1}^{m} \mu_{jt}p_{jt} - \bar{\bar{p}} = \sum_{j=1}^{m} \mu_{jt}\,\omega_t^j$$

For the special case of a possible breaking point at any of the last two periods (m = 2), the estimate of $\bar{\bar{p}}$ is eq. (19') with measurement error:

$$(A8')\quad \omega_t^{*2} = \hat{\bar{\bar{p}}}_t - \bar{\bar{p}} = \mu_t\,\omega_t^1 + (1-\mu_t)\,\omega_t^2$$

and with variance:

$$(A9)\quad E_t[(\omega_t^{*2})^2] = \mu_t^2 E_t[(\omega_t^1)^2] + (1-\mu_t)^2 E_t[(\omega_t^2)^2] +$$
$$+ 2\mu_t(1-\mu_t) E_t[\omega_t^1\,\omega_t^2]$$

where:

$$(A10)\quad E_t[\omega_t^1\,\omega_t^2] = p_{1t}\,p_{2t} - (p_{1t} + p_{2t})\,\hat{\bar{\bar{p}}}_t + E_t[(\bar{\bar{p}})^2]$$

For obtaining $E_t[(\bar{\bar{p}})^2]$ I will substitute $\bar{\bar{p}}$ by $(p_{1t} - \omega_t^1)$. (Alternatively, I could have used $(p_{2t} - \omega_t^2)$, or any linear combination of these two expressions). Therefore:

$$(A11)\quad E_t[(\bar{\bar{p}})^2] = (p_{1t})^2 + \sigma_\eta^2$$

Now replace (19') and (A11) into (A10) to obtain:

$$(A12)\quad E_t[\omega_t^1\,\omega_t^2] = (1-\mu_t)[(p_{1t})^2 - (p_{2t})^2] + \sigma_\eta^2$$

Finally substitute (A7) (for n = 1, 2) and (A12) into (A9) to obtain the measurement-error variance for the case of a breaking point in any of the last two periods:

$$(A13)\quad E_t[(\omega_t^{*2})^2] = \mu_t^2\sigma_\eta^2 + [(1-\mu_t)^2\sigma_\eta^2/2] +$$
$$+ 2\mu_t(1-\mu_t)^2[(p_{1t})^2 - (p_{2t})^2] + 2\mu_t(1-\mu_t)\sigma_\eta^2 =$$
$$= 2\mu_t(1-\mu_t)^2[(p_{1t})^2 - (p_{2t})^2] + [\tfrac{1}{2} + \mu_t - \tfrac{1}{2}\mu_t^2]\,\sigma_\eta^2$$

Appendix 2: Shadow Value of Wealth and Equilibrium Consumption Levels

From equations (36)-(38') obtain present and expected future consumption levels:

$$(A14)\quad E_t[x_s] = A + \left[\frac{(\gamma-\delta) + \delta E_t[p_s]}{\gamma(\gamma-2\delta)}\right]\lambda_t$$

$$(A15)\quad E_t[m_s] = B + \left[\frac{(\gamma-\delta) + \delta E_t[p_s]}{\gamma(\gamma-2\delta)}\right]\lambda_t$$

where:

$E_t[i_s] = i_t$, for s = t

$E_t[i_s] = i_t^e$, for s > t, and for i = x, m.

Note that as a result of condition (35), current and expected future consumption levels are negative functions of λ_t, the shadow value of wealth. The same condition assures that λ_t (which can be obtained from combining (A14)-(A15) and the budget constraint) is a negative function of wealth:

$$(A16)\quad \lambda_t = r\,W_t(1+r)\left[\frac{\gamma(\gamma-2\delta)}{D_t r + D_t^e}\right] - A[p_t r + p_t^e]\left[\frac{\gamma(\gamma-2\delta)}{D_t r + D_t^e}\right] -$$
$$- B(1+r)\left[\frac{\gamma(\gamma-2\delta)}{D_t r + D_t^e}\right]$$

where:

$D_t \equiv \delta + 2(\gamma-\delta)p_t + \delta(p_t)^2$

$D_t^e \equiv \delta + 2(\gamma-\delta)p_t^e + \delta(p_t^e)^2$.

Also note that condition (33) guarantees having λ_t non-negative in the neighborhood of $p_t = p_t^e = 1$.

The shadow value of wealth is a function of output, the interest rate, and current and expected future terms of trade. Any change in these variables will affect λ_t and therefore will have an impact on optimal planned consumption and debt paths.

To obtain equilibrium current consumption levels (eqs. (39) and (40)) substitute (A16) back into (A14) and (A15).

NOTES

1 This is the case of the varying parameter models dealing with certain parameter changes with a stochastic location in time. It represents a special case of switching regression. (See Judge *et. al* (1980), chap. 10).

2 A basic discussion of the signal-extraction problem is in Sargent (1979). Applications of this technique are the rational-expectations models under imperfect information by Altonji and Ashenfelter (1980) and Gertler (1982).

3 A simple treatment of the standard signal-extraction problem is in Sargent (1979), p. 209.

4 An alternative approach to analytical solutions is to restrict neither the utility function nor the stochastic time series to any particular form, obtaining specific numerical solutions for given parameter values applying dynamic stochasting programming. An interesting example is Zeldes (1984), chapter I, where optimal consumption plans are derived for different deviations from certainty equivalence.

5 Zeldes (1984), chap. I, also discusses the theoretical and empirical implications of the CEQ solution and of different deviations from CEQ, for the one-good optimal consumption problem.

6 The reason for having all transactions made at the end of period is to simplify present-value calculations below.

7 The expectational ex-ante formulation of the budget constraint (31) is:

$$\sum_{s=t}^{\infty} [E_t[p_s x_s] + E_t[m_s]] \left(\frac{1}{1+r}\right)^{s+1} = b_t + y \sum_{s=t}^{\infty} E_t[p_s] \left(\frac{1}{1+r}\right)^{s+1}$$

The excepted value $E_t[p_s x_s]$ is by definition the sum of the product of expected values and the covariance of p and x. The consumer problem consists in determining the optimal consumption path, given the household's wealth and an expected exogenous terms-of-trade path. In solving this problem she sets the covariance equal to zero in order to avoid asset accumulation or decumulation which would violate the transversality condition (30). For instance, when prices are constant and so are expected prices ($p_t = E_t [p_s] = \bar{\bar{p}}\ \forall s, t$, if a negative covariance term is considered in the expectational budget constraint above, the corresponding stationary overconsumption and debt accumulation violates (30). Therefore the household chooses x and m in a way equivalent to assume independence of x and p. So the budget constraint is:

$$\sum_{s=t}^{\infty} \left\{ E_t[p_s] E_t[x_s] + E_t[m_s] \right\} \left(\frac{1}{s+1}\right)^{s+1} = b_t + y \sum_{s=1}^{\infty} E_t[p_s] \left(\frac{1}{1+r}\right)^{s+1}$$

which is reflected by (31).

8 Dornbusch, R. (1983), p. 145.

9 In order to analyse the effects of exogenous variables on control variables I always use partial derivatives, which are simpler than sometimes quite complicate expressions for elasticities –a consequence of our particular utility function. The reason for evaluating most expressions at unit or identical current and expected prices is similar: the corresponding expressions are significantly simpler. The signs of the partial derivatives correspond to the case of a debtor economy ($b_t < 0$). Finally, throughtout the paper the partial derivatives with respect to p_t do not consider the effect of current variables on expected future values, again in order to keep derivations as simple as posible. Therefore they are benchmark cases which abstract from the partial adaption of expectations to changes in current values.

10 The cumulative current account deficit or surplus is defined as the difference between the foreign debt stock reached after expectations have adapted completely to the terms-of-trade shock (b_{t_2} or b_{t_5}), and the initial stock (b_{t_0}).

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