

ESCAPE CLAUSES AND TARGETING OF THE REAL EXCHANGE RATE: THE CASE OF NOMINAL EXCHANGE RATE PEGGING

CLAUSULAS DE ESCAPE Y OBJETIVOS SOBRE EL TIPO DE CAMBIO REAL: EL CASO BAJO TIPO DE CAMBIO FIJO

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Abstract

We consider an economy under a fixed exchange rate system, but with bounds (a minimum level or a band) on the real exchange rate. The international price of the tradable good is characterized by the continuous arrival of shocks that change its level. In a model with microfoundations, we investigate the effects of targeting the real exchange rate through nominal exchange rate changes that preclude the real exchange from trespassing the imposed bounds.

A stochastic general equilibrium model with two goods and fixed non-tradable goods price level is developed. We analyze the cases in which a lower bound or a band on the real exchange rate is introduced. The general conclusion is that when bounds are established, then welfare effects can be expected, which are generated at the expense of the levels of consumption that go in the opposite direction than what the policy intended. This short-run effect is present even in the case the targeting policy is never exercised. This result is similar to the one we find in the target zones literature, in the sense that just the existence of this tolerance band changes the behavior of the economy.

An interesting result is that, in the case in which home goods prices are fixed, the imposition of the band on the real exchange rate does not change its behavior within the band. However, this result is not true of other real variables in the economy. In other words, although the targeted variable within the band behaves identically to the case in which there are no bounds, the rest of the real variables in the economy behave differently, even if the targeted variable remains within the band and the escape clause is not triggered.

Keywords: *Real exchange rate targeting, escape clauses.*

JEL Classification: *E58, F31, F41.*

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Resumen

El presente trabajo considera una economía bajo tipo de cambio fijo que se caracteriza por la presencia de límites sobre el tipo de cambio real, ya sean éstos en forma de un mínimo nivel admisible o de una banda. Las variaciones en el tipo de cambio real son introducidas a través de perturbaciones sobre el precio de los bienes transables. En un modelo microfundado, se investigan los efectos de este “targeting” del tipo de cambio real, permitiendo microvariaciones en el tipo de cambio nominal solamente para evitar que el tipo de cambio real sobrepase los límites anunciados.

El estudio se realiza a través de un modelo de equilibrio general estocástico con dos bienes y precio fijo de los bienes no transables. Se analizan los casos de un límite inferior para el tipo de cambio real, así como también para el establecimiento de una banda (rango). La conclusión general es que cuando estos límites son impuestos pueden evidenciarse efectos en términos de bienestar, pero éstos son causados a expensas de niveles de consumo que van en dirección contraria a la que la política intenta establecer. Este efecto de corto plazo se evidencia incluso en el caso de que nunca se ejerza efectivamente la política “correctiva”. Este resultado es similar al que encontramos en la literatura de bandas cambiarias en el sentido que la imposición de estos límites al tipo de cambio real cambia el comportamiento de la economía, incluso si los límites no son alcanzados.

Un resultado interesante es que, en el caso de precios de no transables fijos, la imposición de un rango de tolerancia para el tipo de cambio real no altera el comportamiento de éste, pero sí el del resto de las variables reales de la economía. En otras palabras, si bien la variable objetivo se comporta dentro del rango objetivo idénticamente a como lo haría sin la cláusula de escape, el resto de las variables de la economía son las que se ven modificadas, incluso en el caso de que la cláusula de escape nunca sea activada.

Palabras Clave: *Objetivos sobre el tipo de cambio real, cláusulas de escape.*

Clasificación JEL: *E58, F31, F41.*

1. INTRODUCTION

In the past, an exogenous pre-announced path of the nominal exchange rate has been widely used by developing countries as a stabilization tool –the so-called “exchange rate anchor”. In some cases this pre-announced path involved a fixed devaluation rate; in others, a fixed level of the nominal exchange rate, including, at times, the creation of a currency board, with severe limitations on the central bank’s powers –the case of Argentina between 1991 and 2001 probably being the most important recent example.

Although in many of these cases stabilization was achieved, even with a dramatic fall in inflation rates, many if not most of these programs were eventually abandoned. The “post mortem” discussion suggests different possible reasons for this lack of sustainability. In the Argentine experiment of 1978-1981, with a pre-announced path of devaluation, the culprit seems to have been of a fiscal nature, with the fixed rate of devaluation implying a too low inflation rate and a level of the inflation tax insufficient for the financing of the primary deficit—the typical “unpleasant monetarist arithmetic” of Sargent and Wallace (1985). More frequently, rigidities in home goods prices and shocks in the price of internationally traded goods generated a “too low” level of the real exchange rate (i.e. the relative price of traded goods in terms of home goods), often associated with “too high” levels of unemployment, and more often than not with the perception of “lack of competitiveness” —a favorite argument, in some quarters, for a strong preference for “high” levels of the real exchange rate. In other words, many countries, and in particular developing countries, face the conundrum of either benefit from the price stability advantage of a strict nominal exchange rate and suffer real or perceived losses in economic activity generated by too low levels of the real exchange rate, or give up on the benefits of a strict rule and diminish output losses.^{1/2} As a result, either strict exchange rate regimes have been made implicitly conditional on “acceptable” low levels of the real exchange rate, or acceptable ranges of the real exchange rate have been pursued via the use of other monetary instruments, such as interest rates. Notice that a hidden “escape clause” for the case of an otherwise strict exchange rate (i.e., a “realignment” of the pre-announced path of the nominal exchange rate when too low a level of the real exchange rate is reached) would have an analogue in the same implicit clause by which strict monetary policy (i.e., the pre-announced path of the money supply) would be modified or abandoned under the same circumstances. In this last case, the nominal exchange rate is left to float, but monetary instruments are used to maintain minimum levels of the real exchange rate. The implicit or “hidden” escape clauses are justified given the need to preserve “flexibility”.³

The specification of escape clauses, in various otherwise completely exogenous policies, has received some attention in the literature. Flood and Isard (1989), for example, discuss escape clauses that allow the policymaker to act different when certain previously predefined circumstances are met.⁴

¹ Drazen and Masson (1994) point out that circumstances can seriously erode the credibility and ability of the policy maker to honor his commitment, especially when the policies carried out can be blamed for something as highly visible as a persistently high level of unemployment. Along the same lines, Blanchard (1985) stresses the historical evidence that even if a government wanted to keep its promise about the policy to be followed, it could still be removed from power by different means. See Miller and Weller (1989) and Masson (1995) for developed countries experiences.

² See Neut and Velasco (2003).

³ Gerlach (1995) uses option pricing theory for valuing that flexibility. Lockwood *et al.* (1998) show that, in that case, keeping the option of using discretion may be optimal when shocks are persistent.

⁴ Obstfeld (1997) distinguishes between discretionary and non-discretionary escape clauses. The former assumes that the policymaker faces a personal cost for making use of the escape clause —similar to Lohmann (1992)— and then, at discretion, the policymaker can choose what to do, but only at a “personal cost” dictated by the final decision. The latter is in line with the more purist interpretation of Flood and

This work attempts to analyze the case in which the central bank follows a strict exchange rate, i.e., a pre-announced path of the nominal exchange rate, but subject to an “explicit” conditionality establishing bounds on the levels of the real exchange rate, in the form of either a single minimum level or as a band, with both a minimum and a maximum “tolerable” level. If those levels are reached, then the central bank would proceed to devalue (or revalue) the currency (i.e., to increase or decrease the nominal exchange rate) to the extent that it is necessary to preclude the real exchange rate from trespassing those limits. In our work, we design an escape clause that takes the most restrictive form from the point of view of the policymaker. We define an escape clause that would allow the government to realign the nominal exchange rate only if some predetermined circumstance were observed. In our case the monitored variable is the level of the real exchange rate. More precisely, although the government is committed to buying and selling currency at a predetermined rate, if the real exchange rate variable reaches a pre-announced level, the nominal exchange rate will actually be adjusted to a level that prevents the real exchange rate from surpassing the admissible level (this mechanism is similar to that in the exchange rate bands literature, pioneered by Krugman (1991), also known as target zones).

One point must be clarified here before continuing. In the context of our work, the escape clause is a well defined contingent rule that the government makes explicit. This escape clause summarizes the government’s objective function. We assume throughout the work that the policy is credible and believed by the public, and fulfilled by the monetary authority. Thus, we ignore any kind of time inconsistency problems.

In order to implement the analysis, we use a rather simple model of a small country that is a world taker of both the world price of the traded commodity and the world real interest rate. We assume unrestricted, perfect capital mobility, and model the behavior of individuals who exhibit rational expectations and maximize the discounted present value of their lifetime utility. Although very simple, our model is motivated by microeconomic foundations. In order to motivate the analysis, we will discuss the case in which changes in the real exchange rate are brought about by continuous random changes in the foreign price (i.e., denominated in foreign exchange) of the traded commodity.

Despite the simplicity of the model, the treatment of continuous random shocks in international prices is not without difficulties, and in discussing the imposition of boundaries on the real exchange rate, we need to resort to the very simple, rather naïve assumption of the price of home (non-tradable) goods to be constant, with output being solely demand-determined—an assumption that can be defended on the basis of some short-run rigidities, but which is clearly unsatisfactory in the long run.

Very much in the same vein as in Krugman’s (1991), at issue here is exploring what effects arise from the mere imposition of well defined bounds on the real exchange

Isard’s original work. There are well-defined threshold values for a monitored variable which indicate to the policymaker what to do according to the observed level of that variable. However, even in the case in which the policymaker faces well-defined tolerance bands, he may have discretion about how to react, or can receive specific instructions.

rate even at times when those bounds are not reached and the escape clause is not triggered.

1.1. Related Literature

Two main branches of the literature are related to the present work. First, from the conceptual point of view, this is a model of a purchasing power parity rule or real exchange rate targeting. Second, the modeling technique is associated with the literature on portfolio choice under uncertainty allowing for welfare evaluation.

Real exchange rate targeting rules have been used by governments as a device to isolate the economy from domestic and external shocks that can weaken the competitiveness of the economy, or simply to obtain a higher real exchange rate, a key relative price of the economy⁵. Dornbusch (1982) used a Mundell-Fleming set-up with overlapping contracts to analyze the effects of introducing a real exchange rate targeting policy in the output-price stability trade-off and concluded that a policy that introduced indexation of the nominal exchange rate results in a greater price level instability with an uncertain result in terms of output stability.⁶

Calvo *et al.* (1995) used a continuous time representative individual model with a cash in advance constraint and flexible prices to show that achieving a higher real exchange rate by adjusting the rate of devaluation can only temporarily be attained. However, this is only possible at the cost of higher inflation, a higher real interest rate or a combination of both, according to the degree of capital mobility. They show that the long-run value of the real exchange rate does not depend on the rate of devaluation, leaving no room for a higher rate in the long run. However, changes in the rate of devaluation can certainly cause a short-run effect. In particular, if the rate of devaluation is perceived to be lower in the future, then the nominal interest rate will also be lower, reducing the cost of holding money. Therefore, as a consequence of the cash in advance constraint, consumption will also be cheaper in the future. Consumption of tradable goods will thus be postponed. Since the non-tradable goods sector needs to be in equilibrium, the relative price between tradable and non-tradable goods must be adjusted.

In recent years a different approach in the literature about adjustable peg systems has been developing. Those papers investigate the optimal devaluation policies when the government wants to restore the competitiveness of the economy. Among them the work by Flood and Marion (1997) is one of the first to analyze the optimal policy decision regarding the size and timing of the devaluation. They establish an economy in which there exist controls over capital mobility. The private agents' behavior is exogenous and the government minimizes a loss function that considers the cost of real exchange rate misalignments and a fixed cost to devaluing.

⁵ A complete historical perspective that covers most of the different regimes used in the 20th century can be found in Williamson (1981).

⁶ Adams and Gros (1986), Lizondo (1991), Montiel and Ostry (1991) and Lizondo (1993) conclusions are also on the same line.

In this context, they conclude that there is a positive relation between the drift component that governs the real exchange rate and the size of the devaluation to send the real exchange rate back to its target level. The idea is that the faster the real exchange rate deviates from its target level, the more frequently must the policymaker impose the economic and political costs of devaluing. Therefore, when he uses the option of devaluing, he tries to set the real exchange rate as far as possible from the lower admissible level to avoid paying that cost too often. The volatility of the real exchange rate also has a positive relationship with the size of the devaluation. Regarding the timing of the devaluation, the theoretical model does not allow them to sign its correlation with the drift and variance of the real exchange rate. Empirical evidence for Latin American countries over the period 1957-1990 suggested that the drift factor has had a negative relation with the timing of the devaluation while the variance has a positive correlation that increases with the gap between the target and actual real exchange rate that the policymaker is willing to tolerate during the peg.

Assumptions about capital controls and the exogenous behaviour of the private agents are relaxed in a work by Pastine (2000) under a (S,s) devaluation rule, finding that, depending on parameter values such as the degree of capital mobility, the size of the political and economic costs of devaluing and those costs caused by the misalignment of the exchange rate are low. The probability of devaluation is not monotonic in the target variable (the real exchange rate) and the monetary authority can thus deter speculation. One possible outcome is that the probability of devaluation decreases as its expected size gets bigger.

Our work departs from the literature in several ways. First, in general there is no trigger point on the variable that the government and the public use to evaluate the possible implementation of the policy rule. In most of these models, at each point in time, the government reacts in such a way that a predefined targeted level of real exchange rate is achieved.

Second, the government does not look for a real exchange above its equilibrium level. Moreover, changes in the nominal exchange rate will emerge as a policy response to keep the real exchange rate close to its equilibrium level. Consequently it does not apply the general approach followed in the real exchange rate targeting literature in which the policy consists of adjusting the rate of nominal devaluation with the aim of offsetting the gap between a high domestic inflation rate and the lower inflation rate in the rest of the world.

Third, as can be inferred from the introductory section, we do not explore the question of the optimal size and timing of a one-time change in the nominal exchange rate aimed at correcting a real exchange rate misalignment. In our work, changes in the nominal exchange rate are conducted only when the actual rate reaches the lowest level and the size of those devaluations will be just big enough to prevent the real exchange rate from perforating the band. The target zones literature mentions this as inframarginal interventions. The escape clause rule we use only allows the government to intervene when the limit rate is reached.

The second body of literature related to this work deals with the modeling technique. We use a general equilibrium framework with microfoundations in which the economy

is faced with a permanent series of shocks introduced through a Brownian motion process on the foreign price of tradable goods. This allows us to make explicit the relevant marginal conditions that individuals in the economy fulfill at each point in time while simultaneously understanding how the behavior of the individual affects the equilibrium of the whole economy.

The economy is characterized by the existence of individuals that have the opportunity of consuming tradable and/or non-tradable goods, and of allocating their wealth between two assets, a foreign bond and domestic money. From this perspective, the model can be solved as a portfolio choice problem like those in the celebrated papers by Merton (1969) and Merton (1973). This work follows the main thrust of Asea and Turnovsky (1998), Turnovsky and Grinols (1996) and Venegas-Martinez (2001) with the addition that it allows for the analysis of real exchange rate issues in an open monetary economy. One of the virtues of this approach is that it allows for policy evaluation through an explicit welfare unit of measure, which is the value function of the dynamic program.

1.2. Structure of This Work

The body of this work includes five sections. Section 2 develops the basic model. We assume an economy with fixed non-tradable goods prices and stochastic non serially-correlated disturbances to the world price of the tradable goods which may drive the real exchange rate away from its targeted level. Individuals have rational expectations and derive utility from the consumption of tradable and non-tradable goods as well as real cash balances. Then, an optimal decision rule is obtained.

Section 3 analyzes the effects of introducing the escape clause that takes the form of a lower bound that the government makes explicit and is committed to obey. Our results indicate that the existence of a lower bound can have two opposing effects on welfare. In the long run the promise to intervene by adjusting the nominal exchange rate when the real exchange rate touches the lowest admissible level ensures a floor on non-tradable goods production. This increases welfare in the long run. The smaller the admissible gap between the initial real exchange rate and its bound, the higher the expected welfare gain in the long run. However, in the short run the probability of intervention in the nominal exchange rate reduces the levels of consumption of both goods and the desired demand for real cash balances.

In Section 4 we analyze how the results of Section 3 change when a two-sided band (i.e., a higher as well as a lower bound) is introduced, maintaining the non-tradable goods price rigidity.

Section 5 explores the consequences of relaxing the assumption of rigid non-tradable goods prices, by considering sluggish adjustment instead. When reversion to an equilibrium level is allowed for the non-tradable goods price, the welfare effects of the escape clause are reduced.

In Section 6, we summarize our conclusions and comment on possible extensions of the model and future research.

2. A STOCHASTIC MODEL FOR A SMALL OPEN ECONOMY

We consider a small open economy with a large number of identical individuals who live forever. Individuals derive utility from the consumption of non-tradable goods, tradable goods and the services provided by the stock of real money they hold. The economy can lend and borrow freely from the world capital market at a fixed world interest rate. The government prints money and implements lump-sum transfers.

2.1. Individuals

Individuals derive utility from the consumption of tradable and non-tradable goods. We denote those levels of consumption by c_T and c_H respectively. They also derive utility from the services provided by their money holdings. We define the real money stock in terms of the tradable goods, that is

$$m = \frac{M}{EP} \quad [1]$$

where M is the nominal stock of money, E is the nominal exchange rate and P is the international price of the tradable good. The nominal exchange rate is defined as the price of the foreign currency in terms of the domestic one. If frictionless arbitrage is possible, with no transportation costs and/or customs duties, then the law of one price applies to the tradable good, and its price in the domestic currency (T) is given by the product of the nominal exchange rate, E , and its price in the rest of the world P , i.e. $T=EP$.

An individual's total wealth v , defined in terms of the tradable good, is the sum of real money balances m and the stock of foreign bonds b that yield a real interest i ,

$$v = \frac{M}{EP} + b \quad [2]$$

International bonds are denominated in terms of the tradable good. Each individual also receives flows X_T and X_H of tradable and non-tradable goods respectively. We assume X_T to be constant. They also receive or pay lump-sum transfers $d\tau$.

We introduce uncertainty in the model through the international price of the tradable good. Specifically, we assume that P is exogenously given and evolves according to

$$dP = \sigma P dz \quad [3]$$

In other words, the international price of the tradable good receives continuous independent random shocks dz that are normally distributed with zero mean and variance dt . No secular inflation in the price of tradable goods over the instant dt is assumed. The diffusion parameter σ is finite and non-negative, i.e. $0 \leq \sigma < \infty$.

As the international bonds are denominated in terms of the tradable goods, then the return, R_b , is given by the real interest rate i .

$$R_b = i dt \quad [4]$$

The real return on money holdings, keeping the nominal exchange rate fixed, is given by $d(1/EP)/(1/EP)$, the proportional change on its price in terms of the tradable good. Applying Ito's Lemma, this results in the expression

$$R_M = \sigma^2 dt - \sigma dz \quad [5]$$

This expression indicates that the real return on money holdings, in a period of length dt , has a deterministic component that is related to the diffusion parameter σ , plus a stochastic component given by the process dz governing the international price of the tradable good, which has an expected value of zero.

Therefore, the stochastic return on total individual's wealth is the weighted average of the returns on each of the two assets

$$R = v_M R_M + v_b R_b = [v_M \sigma^2 + (1 - v_M) i] dt - v_M \sigma dz \quad [6]$$

where the weights v_M and v_b are the proportions of total real wealth that is held in the form of money and bonds respectively,

$$v_M = \frac{1}{v} \frac{M}{EP} \quad \text{and} \quad v_b = \frac{b}{v} \quad [7]$$

The sum of these portfolio shares must total one

$$v_M + v_b = 1 \quad [8]$$

Under these conditions an individual's total wealth, defined in terms of tradable goods, evolves in accordance with

$$dv = v R + \left[X_T - c_T + (X_H - c_H) \frac{1}{\varepsilon} \right] dt + d\tau \quad [9]$$

where $\varepsilon = EP/H$ is the relative price of the non-tradable good in term of the tradable good, i.e. the real exchange rate and H is the nominal price of the non-tradable goods. We assume H is fixed⁷. After some substitutions, the budget constraint can be written as

⁷ If the price of the non-tradable goods were fully flexible, the clearing market condition in that sector would result in an adjustment of that price that guarantees that the long-run level of production would be always attained. In the other extreme, a fixed price level allows for departures from the long-run level of production on the non-tradable goods sector, keeping the economy out of its equilibrium forever. As it is generally accepted, some markets present a certain level of price rigidity (among the possible explanations are the overlapping contract arguments and the existence of monopolistic competition, for

$$dv = v f dt - v v_M \sigma dz + d\tau \quad [10]$$

$$\text{where } f = \left\{ v_M \sigma^2 + (1 - v_M) i + \frac{1}{v} \left[X_T - C_T + (X_H - C_H) \frac{1}{\varepsilon} \right] \right\}$$

In a stochastic context as the one described in this work, with non-tradable goods price fixed, a fixed exchange rate system and the international price of the tradable goods following the process given by equation [3], the real exchange rate follows the process

$$d\varepsilon = \sigma \varepsilon dz \quad [11]$$

with expected variation $Exp(d\varepsilon)=0$ and variance $Var(d\varepsilon)=\varepsilon^2 \sigma^2 dt$.

2.2. The Government

For this work we define government as the conglomerate of the fiscal and monetary authorities. The government also has a stock of foreign assets, b_G , that renders the real interest rate i , and prints fiat money, M . As we do not intend to study the effect of the government's fiscal position, the government sector is kept at the simplest level. We assume that government's consumptions of both tradable and non/tradable goods are equal to zero. The government also implements lump-sum transfers, which are denominated in terms of the tradable good. There are no distortionary taxes implemented and the monetary regime is characterized by a hard peg, i.e. the monetary authority commits to buying and selling money at the predetermined nominal exchange rate E .

The government's real net wealth v_G is the difference between the stock of foreign assets it holds and the real money stock held by the public, expressed in terms of the tradable good.

$$v_G = b_G - \frac{M}{EP} \quad [12]$$

Its stochastic wealth accumulation equation, the budget constraint, is given by

$$dv_G = b_G R_G - \frac{M}{EP} R_M - d\tau = \left(b_G i + \frac{M}{EP} \sigma^2 \right) dt + \frac{M}{EP} \sigma dz - d\tau \quad [13]$$

The government's per capita transfer policy is defined by

example). A logic assumption is that although the price of the non-tradable goods can depart from its long-run level, there exists some degree of price flexibility that allows this price to converge to its long-run value over time. We adopt the fixed price assumption for tractability purposes and, consequently, the results we find have to be interpreted as limited to the very short run. We make some conjectures about the effects of introducing non-tradable goods price sluggishness in Section 5.

$$d\tau = \left(\frac{M}{EP} \sigma^2 + ib_G \right) dt + \frac{M}{EP} \sigma dz \tag{14}$$

In other words, the government returns to the public, in a form unrelated to their money holdings, the revenues from money creation plus the interest it earns on its holdings of international bonds.

2.3. The Individuals' Problem

Individuals' preferences are defined over tradable and non-tradable goods consumption as well as the real money stock they hold. These preferences are summarized by a strictly increasing, concave and continuously differentiable utility function $U(c_H, c_T, m)$ that satisfies the usual Inada conditions, $\lim_{j \rightarrow 0} \frac{\partial}{\partial j} U(c_H, c_T, m) = \infty$ and $\lim_{j \rightarrow \infty} \frac{\partial}{\partial j} U(c_H, c_T, m) = 0$ for $j = c_H, c_T, m$. We assume both goods are perishable. The individuals' problem can be defined by choosing the optimal consumption c_H and c_T and the portfolio allocation policy (ie. v_M and v_B) to maximize the von Neumann-Morgenstern utility functional at $t=0$

$$\Omega[v(0), \varepsilon(0), 0] = \underset{c_H, c_T, v_M, v_b}{Max} \text{Exp}_0 \left[\int_0^\infty U(c_H, c_T, v_M v) e^{-\rho t} dt \right] w \tag{15}$$

where ρ is the rate of time preference that is assumed to be constant, Exp_0 is the expectation operator conditioned on all information available at $t=0$, and subject to the stochastic behavior of wealth accumulation [10], along with the stochastic process governing the real exchange rate [11] and the wealth constraint [8]. The initial stock of wealth $v(0) = v_0$ and real exchange rate level $\varepsilon(0) = \varepsilon_0$ are given. The solution method we follow is that of dynamic programming.

Given the exponential time discounting, the value function can be assumed to have a time separable form

$$\Omega(v, \varepsilon, t) = e^{-\rho t} J(v, \varepsilon) \tag{16}$$

In this case, applying the differential operator L^δ , one can write

$$L[e^{-\rho t} J(v, \varepsilon)] = e^{-\rho t} \left(-\rho J + g v J_v + \frac{1}{2} v^2 \sigma^2 J_{vv} + \frac{1}{2} \varepsilon^2 \sigma^2 J_{\varepsilon\varepsilon} - \frac{1}{2} v \varepsilon \sigma^2 J_{v\varepsilon} \right) \tag{17}$$

where the subscripts v and ε refer to partial derivatives.⁹

⁸ See Turnovsky (1997 Chap. 9).

⁹ For notational convenience, we suppress the arguments of the value function J .

Therefore, the individuals' objective is to select the rates of consumption for each good and their portfolio shares to maximize the Lagrangian expression

$$U(c_H, c_T, v v_M) e^{-\rho t} + L[e^{-\rho t} J(v, \varepsilon)] + \Lambda(1 - v_b - v_M) \quad [18]$$

The corresponding optimality conditions with respect to c_H , c_T , v_B , v_M and Λ are given by

$$U_H - \frac{J_v}{\varepsilon} = 0 \quad [19]$$

$$U_T - J_v = 0 \quad [20]$$

$$-\Lambda = 0 \quad [21]$$

$$U_m v \varepsilon - v (i - \sigma^2) J_v - \Lambda = 0 \quad [22]$$

$$1 - v_b - v_M = 0 \quad [23]$$

After some manipulations, we find the marginal conditions that must be fulfilled at all times

$$\frac{U_T}{\varepsilon} = U_H \quad [24]$$

$$U_T = U_m \frac{1}{(i - \sigma^2)} \quad [25]$$

The two equations described above have clear and meaningful interpretations. Equation [24] expresses the usual condition for an optimal choice. In this case, it means that the marginal utility of the consumption of tradable goods divided by its relative price have to equal the marginal utility of the consumption of the non-tradable good. In other words, for any given level of the consumption of the tradable goods, we can derive the demand of the non-tradable good as a function of the relative price between these two goods. Equation [25] has a similar interpretation, in the sense that a demand function for real money holdings that depends on the cost of holding that stock of money for a period of length dt can be obtained. Equations [19] and [20] indicate that the marginal utility of consuming one extra unit of the good must equal the marginal valuation that one could have derived from saving those resources and consuming them in the future, i.e. the marginal valuation of the state variable v .

For this work, we assume a constant relative risk aversion (CRRA) and concave individual utility function of the form

$$U(c_H, c_T, m) = \frac{1}{\gamma} (c_H^\alpha c_T^\beta m^{1-\alpha-\beta})^\gamma \quad [26]$$

where $0 < \gamma < 1$ and $(1-\gamma)$ is the Arrow-Pratt measure of relative risk aversion. The coefficients α , β and $(1-\alpha-\beta)$ measure the relative weights of each of the arguments in the utility function. This CRRA utility function with money as an argument fulfills the necessary regularity conditions required to be functionally equivalent to the cash-in-advance-constraints or the transactions-cost approaches to modeling a monetary economy (see Turnosvky and Grinols (1996) and Feenstra (1986)). We also choose this particular form for the utility function in the interest of keeping the problem manageable and deriving closed-form solution in those cases where that is possible.

Simple manipulation of equations [19]-[22] and [26] results in the marginal conditions that must be fulfilled at all point in time. Those conditions are

$$c_H = \frac{\alpha}{\beta} \varepsilon c_T \quad [27]$$

$$m = \frac{(1-\alpha-\beta)}{\beta(i-\sigma^2)} c_T \quad [28]$$

That is, for a given level of consumption of the tradable good, the consumption of non-tradable goods is a function of its relative price, the real exchange rate. Expression [28] is the demand for real money stock as a function of the opportunity cost of holding it.

2.4. The Resource Constraint for the Aggregate Economy

Consolidation of the private sector and the government results in the balance of payment identity

$$dV = [iV + (X_T - c_T)] dt \quad [29]$$

where the V is the sum of private sector's (v) and government's (v_G) real wealth (or equivalently, the sum of the foreign bond hold by each sector). Notice that the expression above is the usual balance of payment equation we find in a model with no uncertainty and it indicates that for the external sector to be in equilibrium, the interest earned by total non-monetary domestic wealth should be enough to finance the balance of trade flow. Note that even in the case of this stochastic environment, equation [29] is non-stochastic. This is a consequence of assuming that the international bonds are expressed in terms of the tradable goods and, therefore, changes in the nominal price of that good in the international market do not affect the aggregate economy's level of wealth.

It should also be noticed that once you know the level of wealth in this economy, there is one and only one level of consumption of tradable goods c_T that is consistent with equilibrium in the balance of payment. Therefore, the marginal condition [24] (or [27] in the particular case of the utility function we have chosen) and the market clearing condition for the non-tradable goods sector leads to the conclusion that

there exists one and only one equilibrium level of the real exchange rate ε . That level is denoted as ε^F , and the correspondent production level of non-tradable goods is denoted as X_H^F .

In a context characterized by sticky prices, misalignments of the real exchange rate are possible. In this work, those out-of-the-equilibrium values for the real exchange rate are generated by a fixed price of the non-tradable goods¹⁰ and a continuously moving price for the tradable goods. Hence, when the latter is lower (higher), the real exchange rate becomes higher (lower). In the case the non-tradable goods sector is demand-determined, the level of production of those goods is lower (higher) than its long-run equilibrium state.

2.5. The Solution of the Model

The value function of the problem, additionally, must satisfy the Hamiltonian-Jacobi-Bellman equation [30]

$$\text{Max} \{U(c_H, c_T, v, v_M) e^{-\rho t} + J[e^{-\rho t} H(v, \varepsilon)]\} = 0 \quad [30]$$

where we replace the optimized values for c_H , c_T and m in the utility function. We reach for a solution for $J(v, \varepsilon)$ that solves the resulting differential equation.

We define the total amount of resources (in terms of the tradable good) that the individual needs to allocate to get utility $U(\cdot)$ as

$$G = \frac{c_H}{\varepsilon} + c_T + m(i - \sigma^2) \quad [31]$$

and the ratios of each of the components of the utility function with respect to the function G as

$$g_H = \frac{c_H}{\varepsilon G} \quad g_T = \frac{c_T}{G} \quad g_m = \frac{m(i - \sigma^2)}{G} \quad [32]$$

where $g_H + g_T + g_m = 1$.

After several manipulations using marginal conditions [27] and [28] it can be shown that the consumption of both types of goods and real money holdings are proportional to G

$$c_H = \alpha \varepsilon G \quad c_T = \beta G \quad m = \frac{(1 - \alpha - \beta)}{(i - \sigma^2)} G \quad [33]$$

¹⁰ In fact, only the lack of fully flexible prices is necessary.

Substituting [33] in the utility function [26] we obtain an expression for the utility function that depends on parameters and G only,

$$U(c_H, c_T, m) = \frac{1}{\gamma} \left(G \varepsilon^\alpha \alpha^\alpha \beta^\beta \left(\frac{1-\alpha-\beta}{i-\sigma^2} \right)^{1-\alpha-\beta} \right)^\gamma \quad [34]$$

As noted before, the solution to the problem consists of finding a function $J(v, \varepsilon)$ that satisfies the marginal conditions [19]-[22] and the Bellman equation [30]. A candidate solution to that problem has the form

$$J(v, \varepsilon) = A v^\gamma \varepsilon^\phi \quad [35]$$

where the constants A and ϕ have yet to be determined.

Using the condition [20], the CRRA utility function [26], and the candidate function [35] we obtain an expression for the utility function that can be rewritten as

$$U(c_H, c_T, m) = \frac{1}{\gamma} \left(A \gamma \alpha^{-\alpha} \beta^{-\beta} \left(\frac{1-\alpha-\beta}{i-\sigma^2} \right)^{-(1-\alpha-\beta)} \right)^{\frac{\gamma}{\gamma-1}} v^\gamma \varepsilon^{\frac{\gamma(1-\beta)}{\gamma-1}} \quad [36]$$

Finally, using [36], [35] and its derivatives into the Hamiltonian-Jabobi-Bellman equation we conclude that a possible solution for the candidate function [35] exists only if $\phi = \gamma\alpha$. After several manipulations that involve equations [20], [26], [33], we obtain an expression for G of the form

$$G = \left(A \gamma \alpha^{-\alpha} \beta^{-\beta} \left(\frac{1-\alpha-\beta}{i-\sigma^2} \right)^{-(1-\alpha-\beta)} \right)^{\frac{\gamma}{\gamma-1}} v \quad [37]$$

and the equation

$$\frac{G}{v} - \rho + f\gamma + \frac{1}{2} \sigma^2 \gamma(\gamma-1) + \frac{1}{2} \sigma^2 \phi(\phi-1) - \frac{1}{2} \sigma^2 \gamma\phi = 0 \quad [38]$$

It should be noted that imposing the markets clearing condition in the non-tradable goods sector, the transfer policy and [33], the expression for f in [10] reduces to

$$f = \sigma^2 + i + \frac{1}{v} X_T + \beta \frac{G}{v} \quad [39]$$

The optimal decision rule for G can be found by using equation [39] in [38]

$$G = v \frac{1}{1 + \beta} \frac{1}{\gamma} \left[\rho - \left(i + \frac{X_T}{v} \right) \gamma - \frac{1}{2} \sigma^2 \gamma (\gamma - 1) - \frac{1}{2} \sigma^2 \phi (\phi - 1) + \frac{1}{2} \sigma^2 \gamma \phi \right] \quad [40]$$

The optimal levels of consumption and the optimal portfolio can be found using [7], [8] and [33]. Hence, given that $\phi = \gamma\alpha < 1$, the level of G in the face of uncertainty (which in our setup implies $\sigma^2 > 0$) can be shown to be higher than when the source of uncertainty is not present (setting $\sigma^2 = 0$).

The constant A can be found by substituting the result from [40] back into equation [38]

$$A = \left(\frac{G}{v} \right)^{\gamma-1} \frac{1}{\gamma} \left(\alpha^\alpha \beta^\beta \left(\frac{1 - \alpha - \beta}{i} \right)^{(1 - \alpha - \beta)} \right)^\gamma \quad [41]$$

Therefore, the function

$$J(v, \varepsilon) = A v^\gamma \varepsilon^{\alpha\gamma} \quad [42]$$

is the solution for the differential equation on $J(v, \varepsilon)$ that satisfies the marginal conditions [19]-[22]. The optimal choices for c_H , c_T , v_B , v_M can be derived using [7] and [33]. Notice that G is a function of a set of parameters and the state variable v . The real exchange rate ε does not enter in that function. Therefore, the level of consumption of the tradable goods and real money holdings are completely determined without reference to the real exchange rate.

The equilibrium solution must satisfy the feasibility condition. This is simply the transversality condition

$$\lim_{t \rightarrow \infty} \text{Exp}[v J_v(v, \varepsilon) e^{-\rho t}] = \lim_{t \rightarrow \infty} \text{Exp}[J(v, \varepsilon) e^{-\rho t}] = 0 \quad [43]$$

By Ito's Lemma and using [10] and [11] we find

$$\frac{dJ(v, \varepsilon)}{J(v, \varepsilon)} = \left[f\gamma + \frac{1}{2} \sigma^2 \gamma (\gamma - 1) + \frac{1}{2} \sigma^2 \phi (\phi - 1) - \frac{1}{2} \sigma^2 \gamma \phi \right] dt + (\phi - \gamma) \sigma dz \quad [44]$$

which means that

$$J(v, \varepsilon, t) = J(v, \varepsilon, 0) e^{\left[f\gamma + \frac{1}{2} \sigma^2 \gamma (\gamma - 1) + \frac{1}{2} \sigma^2 \phi (\phi - 1) - \frac{1}{2} \sigma^2 \gamma \phi \right] t + (\phi - \gamma) \sigma z_t} \quad [45]$$

Replacing this expression in the transversality condition and applying the expectation operator we can easily show the expression

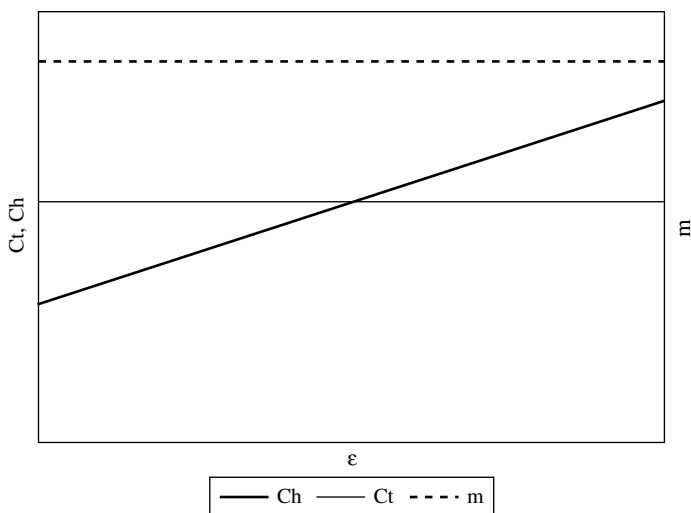
$$\rho - f\gamma - \frac{1}{2}\sigma^2\gamma(\gamma - 1) - \frac{1}{2}\sigma^2\phi(\phi - 1) + \frac{1}{2}\sigma^2\gamma\phi > 0 \tag{46}$$

By equation [38], this ensures that the levels of G and the choice variables are all positive.

As a summary and illustration of the results, Figure 1 shows the demand of money and the consumptions of tradable and non-tradable goods. Clearly, as the function G does not depend on the real exchange rate, money holdings (the m line, which is measured in the secondary axis of the figure) and the consumption of tradable goods (the C_t line) have flat paths, i.e. they are independent of the level of real exchange rate observed in the economy. To understand the intuition behind these results, one should note that when the exchange rate regime is a fully credible hard peg, there is no risk of changes in the nominal exchange rate, and consequently the opportunity cost of holding money stays constant. As the level of consumption of tradable goods is determined by the external sector, where no changes have occurred, the demand for money must be constant. The Ch line shows how the consumption of non-tradable goods reacts to changes to its relative price with respect to tradable goods, i.e. the

FIGURE 1

CONSUMPTION OF TRADABLE, NON TRADABLE GOODS AND REAL MONEY HOLDINGS



real exchange rate, a result that comes from the marginal condition that relates the consumptions of tradable and non-tradable goods.

3. INTRODUCING A LOWER BOUND ON THE REAL EXCHANGE RATE

Under a fixed exchange rate system, misalignment of the real exchange rate can be generated by changes either in the world price of the tradable good or in the price of the non-tradable good. In either case some lack of flexibility in the non-tradable goods price needs to be assumed¹¹. If this element is not present, then, just after receiving the shock, the non-tradable goods price will instantaneously adjust so that consumption at each point in time of the non-tradable goods will equal its full employment level of production, and the economy will be always at its long-run equilibrium.

In the previous section we assumed that the world price of the tradable goods P is continuously exposed to shocks. The extreme assumption, that the non-tradable goods price was fixed was also made for tractability purposes. Therefore, the real exchange rate ε evolved according to [11].

Consequently, if we define the target level of the real exchange rate ε^* and the nominal exchange rate is fixed, then there exists one and only one value for P that is compatible with that level of the real exchange rate.

In this Section, we analyze the case in which there exists a lower bound on the real exchange rate.¹² The general idea is that the government, in a context characterized by fixed non-tradable goods price, decides to set a minimum level of non-tradable goods production with the aim of avoiding lower levels of production in that sector. This lower bound or limit can be interpreted as a representation of the society distaste for unemployment. In other words, a restriction to the stochastic process that governs the motion of the real exchange rate is introduced. An escape clause with the following specification is introduced: the government uses the nominal exchange rate to realign the real exchange rate if and only if it reaches a predefined level that is considered too low. More specifically, the exchange rate policy implies a commitment to keep the nominal rate fixed as long as the real exchange rate is above the lowest admissible value $\underline{\varepsilon}$, but as soon as this boundary is reached, the government is commanded to adjust

¹¹ One rationale for this, and one not all too uncommon in the economic literature, is the existence of overlapping contracts of different “ages” at a given point in time. There are two consequences of the existence of these contracts. One is that if there is an unexpected change in the economy, the contract upon renewal introduces the new equilibrium price. But those that are still in effect need to wait until the expiration date to adjust their price. As a result, the average price in the economy shows a delayed adjustment toward the new equilibrium price. The other effect is that, in anticipation of a future change, the contracts that are going to be in effect after the event takes place are going to adjust their prices in advance.

¹² The case of a band targeting on the real exchange rate is analyzed in next section. We first introduced a lower bound rule for two reasons. First, because it allows to understand the effects that arise for the existence of a bound in a simpler setup. Second, because, generally, the concerns about the real exchange rate level grow when that variable is below its long-run level.

the nominal exchange rate so that the real exchange rate is not below the established lowest admissible value.

In general, the complete solution to any differential equation includes the particular and complementary solutions. The first gives the equilibrium value of the general equation, the second provides the dynamic behavior of the equation relative to equilibrium, indicating how the system evolves towards or away from the equilibrium. This complementary solution results from solving the homogeneous part of the same equation, which is given, in this case, by

$$-\rho J(v, \varepsilon) + f\gamma J_v(v, \varepsilon) + \frac{1}{2}\sigma^2 \varepsilon^2 J_{\varepsilon\varepsilon}(v, \varepsilon) = 0 \quad [47]$$

As we did before, we need to guess the solution. A candidate solution function for this problem has the form

$$J(v, \varepsilon) = K v^\gamma \varepsilon^\omega \quad [48]$$

Using the correspondent partial derivatives, and solving for ω we obtain the complete solution for our differential equation given by

$$J(v, \varepsilon) = Av^\gamma \varepsilon^{\gamma\alpha} + K_1 v^\gamma \varepsilon^{\omega_1} + K_2 v^\gamma \varepsilon^{\omega_2} \quad [49]$$

where

$$\omega_{1,2} = \frac{1}{2} \left[(1 + \gamma) \pm \sqrt{q} \right] \text{ and } q = (1 + \gamma)^2 + 4\gamma(1 - \gamma) + \frac{8(\rho - f\gamma)}{\sigma^2} \quad [50]$$

are the roots that solve the complementary solution. The two of them are real and of different signs provided that $\rho - f\gamma > 0$. The constants K_1 and K_2 remain to be determined. For convention, denote the negative root as ω_j .

Let us recall the Hamiltonian-Jacobi-Bellman equation we solved has the form

$$U(c_H, c_T, m) e^{-\rho t} - \rho e^{-\rho t} J(v, \varepsilon) + e^{-\rho t} f v J_v(v, \varepsilon) + \frac{1}{2} \sigma^2 \varepsilon^2 e^{-\rho t} J_{\varepsilon\varepsilon}(v, \varepsilon) = 0 \quad [51]$$

This expression can be rewritten as

$$\rho J(v, \varepsilon) dt = U(c_H, c_T, m) dt + \text{Exp}[dJ(v, \varepsilon)] \quad [52]$$

As pointed out by Dixit (1993 pp. 14-15), this is an arbitrage equation in which $J(v, \varepsilon)$ is the value of the entitlement to a flow of utility. It can then be interpreted as a capital asset. If the asset is held over a period $(t, t+dt)$, then the normal return is

given by the left-hand side of the equation [52]. On the other side of the equation, we have that the asset yields a dividend of $U(\cdot)dt$ and an expected capital gain of size $Exp[J(v, \varepsilon)]$. If no arbitrage possibilities are present, then this last equation holds. This also means that, in the absence of barriers (restrictions on the stochastic process), as the one described in Section 2, the particular solution [42] is the complete solution to the problem and the constants K_1 and K_2 equal zero.

When barriers are present, then the complementary solution should capture their effect, and the constants K_1 and K_2 are determined according to the kind of barrier imposed in the problem.¹³ These are the cases analyzed in this and the next sections.

3.1. Changes to the Basic Model

As before, the general rule the government follows is an exchange rate rule. However, at a predefined and publicly known level of the real exchange rate (associated with a too low level of non-tradable goods production) it is mandated to exercise the necessary adjustment to the nominal exchange rate to avoid allowing the real one to go beyond its lowest admissible level. Under this rule, the nominal exchange rate follows a process that can be represented by

$$dE = E dN_L \quad [53]$$

where the regulator dN_L is a non-negative non-decreasing process that is only positive when the real exchange rate hits the predefined lower level. It is equal to zero elsewhere and involves infinitesimal accommodations of the nominal rate to keep the real exchange rate just at that lowest admissible level. N_L stands for the cumulative interventions on the nominal exchange rate.

Because of those interventions of the monetary authority changing the nominal exchange rate at the edge of this one-sided band, small accommodations of the individuals' real money stocks occur. Then, the instantaneous real return to money holdings is given by a modified version of expression [5]

$$R_M = \sigma^2 dt - \sigma dz - dN_L \quad [54]$$

For simplicity, we assume the government's revenues from the realignment of the nominal exchange rate are returned to the public as part of the lump-sum transfers.

Finally, the real exchange rate behavior needs to be redefined too. The proportional changes originally represented by equation [11] should include possible corrections to the nominal exchange. The real exchange rate follows the process

$$d\varepsilon = \sigma \varepsilon dz + \varepsilon dN_L \quad [55]$$

¹³ See Dixit (1993).

The newly defined problem to be solved has a similar setup as the one analyzed in the previous section, with the only differences given by equation [53], and equations [54] and [55] in replacement of [5] and [11]. As the procedure to solve the system follows the same steps that were followed in the previous Section, we just present the main differences.

3.2. The Escape Clause and the Smooth-Pasting Condition

For the particular problem at hand, the escape clause constitutes a lower bound on ε . In other words, ε is allowed to follow the process described by [55] as long as $\varepsilon > \underline{\varepsilon}$, but if $\varepsilon = \underline{\varepsilon}$, then the next increment cannot be negative. This is referred to as a reflecting barrier at $\underline{\varepsilon}$.

Suppose for a moment that the real exchange rate ε follows a path that goes further from the targeted level $\underline{\varepsilon}$ as time passes. In this context, it is unlikely that the real exchange rate will reach the lower admissible level $\underline{\varepsilon}$ in a reasonable time span. In that case, the particular solution should be a more relevant part of the complete solution of [49]. Recall we assumed $\omega_1 < 0$ and $\omega_2 > 0$. So, the last term on the right hand side of [49] goes to infinity as the real exchange rate increases. This result contradicts the idea that the particular solution should be a good approximation to the problems as the real exchange rate increases, unless $K_2 = 0$. On the other hand, the term in [49] that contains the constant K_1 gets smaller as ε reaches higher levels. That is, the effect of this part of the complementary solution vanishes as the monitored variable gets further from its lower admissible level $\underline{\varepsilon}$, and gets more influential when ε approaches $\underline{\varepsilon}$. So it seems logical to allow that term to be present in the complete solution of the differential equation.

So far, we have established that $K_2 = 0$ and $K_1 \neq 0$. The precise value of K_1 will be determined by endpoint conditions that come from the existence of this reflecting lower barrier. In particular, that condition is given by the smooth-pasting condition.¹⁴

$$J_\varepsilon(v, \underline{\varepsilon}) = A \phi v^\gamma \underline{\varepsilon}^{\phi-1} + K_1 \omega_1 v^\gamma \underline{\varepsilon}^{\omega_1-1} = 0 \quad [56]$$

Solving for K_1 we find

$$K_1 = -A \frac{\phi}{\omega_1} \underline{\varepsilon}^{\phi-\omega_1} \quad [57]$$

and the complete solution of the differential equation is¹⁵

¹⁴ For an intuitive explanation of the smooth-pasting condition see Krugman (1991) and Dixit (1993 pp. 26-27). Malliaris and Brock (1982 pp. 200) provide a formal derivation. Applications of this condition can be found in Constantinides (1986), Dixit (1989), Merton (1973) and Dumas (1992) among others and is frequently found in the literature on option pricing and investment under uncertainty.

¹⁵ The underlining of J means this is the solution form for the case of a lower band in the real exchange rate.

$$\underline{J}(v, \varepsilon) = A v^\gamma \varepsilon^\phi - A \frac{\phi}{\omega_1} \underline{\varepsilon}^{\phi-\omega_1} v^\gamma \varepsilon^{\omega_1} \quad [58]$$

The presence of the lower bound for the real exchange rate in equation [58] implies that it is not inconsequential for the economy. In the next section, we analyze those consequences.

3.3. Long and Short-Run Effects of the Lower Bound Policy

Policy evaluation needs the development of a welfare criterion. In our case, short and long-run effects of the policy under consideration can be selected. In particular, the instantaneous utility function and its lifetime version evaluated at the optimal path are the natural candidates for this purpose.

For the latter, expressions [58] and [42] are the relevant ones for the cases of an economy with and without a real exchange rate lower bound, respectively.

Notice that equation [58] can be written as

$$\underline{J}(v, \varepsilon) = \underline{A} v^\gamma \varepsilon^\phi \quad [59]$$

where

$$\underline{A} = A \left[1 - \frac{\phi}{\omega_1} \left(\frac{\underline{\varepsilon}}{\varepsilon} \right)^{\phi-\omega_1} \right] \quad [60]$$

Clearly, any difference in the long run on welfare expected at time zero with and without the escape clause is explained by the term between square brackets in [60]. That term is definitively positive provided that $\omega_1 < 0$. That is, the existence of a known lower limit on the real exchange rate, in this context with fixed prices of the non-tradable goods, is a welfare improving measure.

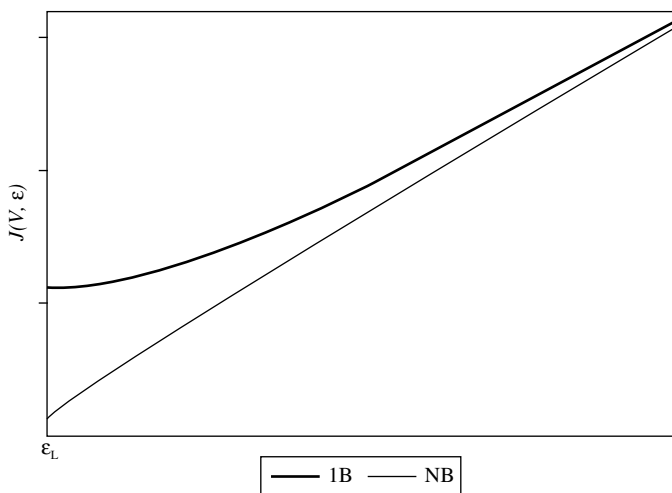
Figure 2 compares the levels of welfare attained under a permanent fixed exchange rate (line *NB*) and a nominal exchange rate defined by the lower bound on the real exchange rate (line *IB*).¹⁶ As can be observed, the level of welfare attained under the existence of the lower bound on the real exchange rate is higher than an exchange rate policy with no target on the real exchange rate for any level of the real exchange rate. Notice that the difference is smaller as the real exchange rate reaches levels further from its lowest admissible level.¹⁷

¹⁶ The relevant parameter values to generate the figures are: $\alpha=\beta=1/3$, $\gamma=0.5$, $\rho=i=0.01$, $\sigma=0.01$, $b=0.9$ and $v(0)=10,000$. These parameters are kept constant for the rest figures included in this work, unless a change is specifically mentioned.

¹⁷ We run a simulation that involved generating 10,000 different random paths for the real exchange rate and computing the expected welfare level under a hard peg and three different levels of the lower bound on the real exchange rate. Both standard errors and covariances of the series were estimated by

FIGURE 2

EXPECTED WELFARE UNDER NO TARGETING AND A LOWER BOUND



The short-run effects of the policy can be evaluated by using the marginal condition [20] once again. After several manipulations, it can be determined that

$$\underline{G} = G \left[1 - \frac{\phi}{\omega_1} \left(\frac{\underline{\varepsilon}}{\varepsilon} \right)^{\phi - \omega_1} \right]^{-1/(1-\gamma)} \tag{61}$$

which is lower than G , provided that $0 < \gamma < 1$ and the term between square brackets is bigger than one. The level of consumption of tradable goods, and also of non-tradable goods and money holdings, are lower than when no lower bound exists.

To understand the result discussed in the previous paragraph it is necessary to recognize the effects of the lower bound at the individual level. In particular, as the real exchange rate approaches its lowest admissible level, the probability of a realignment of the nominal exchange rate grows. For example, suppose that the observed real exchange rate is at the level $\underline{\varepsilon}$. If no lower bound exists, then the probability of facing a devaluation, i.e. a change in the level of the nominal exchange rate, equals zero

bootstrapping. At a 99% confidence level we rejected the null hypotheses that the expected welfare attained under the hard peg and each of the different lower bounds was equal. Moreover, the expected welfare gains were bigger as the level of the lower bound was increased.

(provided that the shock to the real exchange rate has an expected value equal to zero) and the expected value for the real exchange rate is $\underline{\varepsilon}$. Figure 3 shows a probability distribution for the possible value of the real exchange rate. Now suppose that there is a lower bound policy under effect and that the observed real exchange rate in the economy equals that lower bound, which we call $\underline{\varepsilon}$. In this case, the probability of facing a devaluation equals 50% and it is given by area B. Under the lower bound policy, individuals know that the monetary authority will never let the real exchange rate to go to the left of $\underline{\varepsilon}$ in Figure 3 and that it will react by a devaluation of the necessary size to keep in the real exchange rate at $\underline{\varepsilon}$. The probability distribution of the real exchange rate is truncated, such that the new probability distribution is the sum of areas A and C, and the expected value of the real exchange rate is now higher than $\underline{\varepsilon}$. In the event that the next shock triggers a devaluation, the instantaneous return on money holdings is reduced. This effect is associated in the above expression by the term $(\underline{\varepsilon}/\varepsilon)$ that measures the distance between the actual real exchange rate and the edge of the band. In other words, now the variables that the individuals control (especially the demand for real cash balances) are sensitive to the real exchange rate and they act accordingly.

Figures 4, 5 and 6 show the optimal levels of real money holdings, the consumption of tradable and non-tradable goods under the two alternatives schemes for conducting the nominal exchange rate. They clearly show that for any level of the real exchange rate, the existence of the lower limit in the real exchange rate makes individuals consume less of both types of goods and hold lower stocks of real cash balances.

FIGURE 3

RANDOM SHOCKS ON THE REAL EXCHANGE RATE
AND THE PROBABILITY OF A DEVALUATION

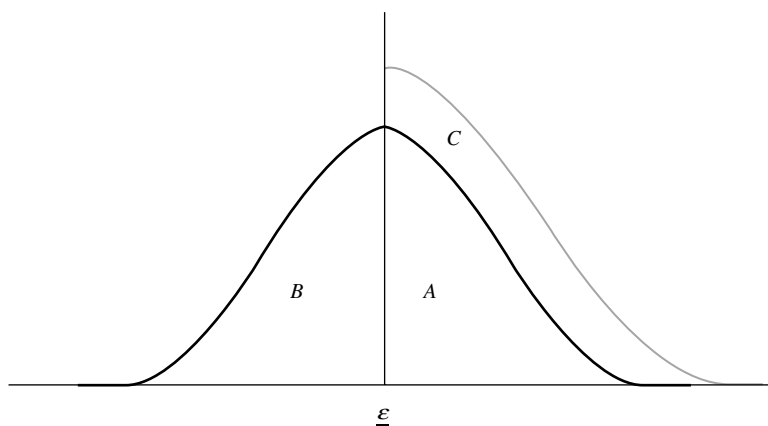


FIGURE 4
REAL MONEY HOLDINGS UNDER NO TARGETING
AND A LOWER BOUND

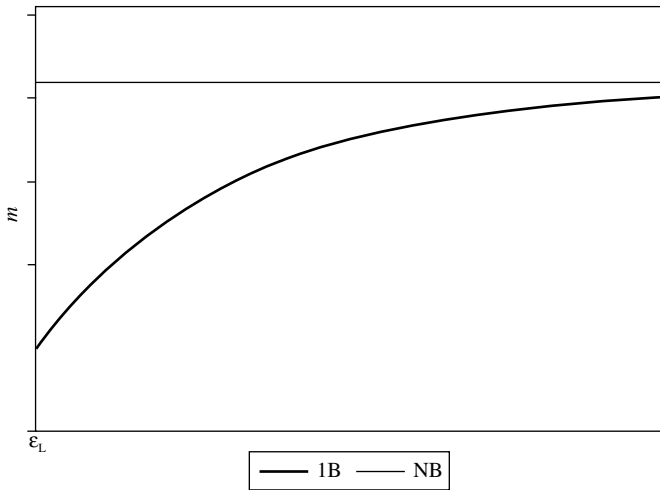


FIGURE 5
CONSUMPTION OF TRADABLE GOODS UNDER NO
TARGETING AND A LOWER BOUND

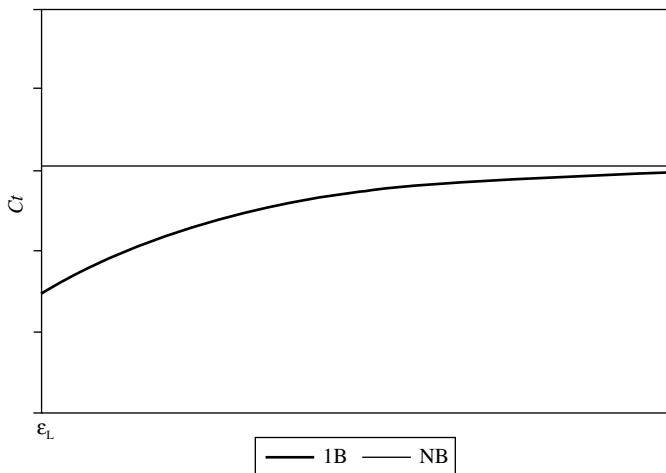
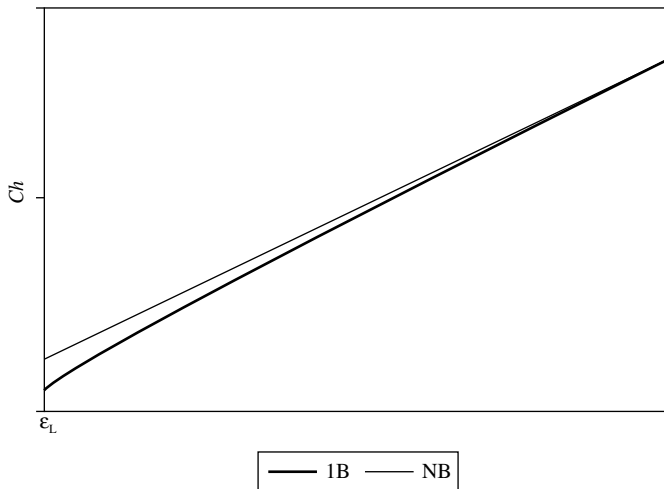


FIGURE 6

CONSUMPTION OF NON-TRADABLE GOODS UNDER NO TARGETING AND A LOWER BOUND



A lower level of consumption allows for a greater level of wealth that is made in form of foreign bonds and will ultimately provide the economy with a higher level of welfare. However, this welfare improving policy has contemporaneous effects. With the aim of avoiding possible “too low” levels of production in the non-tradable good sector, caused by an “inconvenient” level of the real exchange rate, the policy generates the necessary contradictory incentives in the private sector. The contingent devaluation policy reduces the expected gains from holding money and brings a contraction in the level of activity compared to the case where no lower bound exists. This effect is present even in the case when that lowest tolerance limit is not ever reached, as it is observed in the previous figures.

4. THE CASE OF A REAL EXCHANGE RATE BAND

In the previous section we assumed that the government was only concerned about a drop in the real exchange rate that can worsen the level of production in the non-tradable goods sector. This assumption allowed for a great simplification to understand first the consequences of introducing uncertainty over the path of the world price of the tradable goods, secondly, it allowed the examination of the effects of a policy that at first did not tolerate, and then imposed, limits on the size of the misalignment in the real exchange rate with respect to its targeted level.

In this section, we analyze a different scenario, by introducing a band on the real exchange rate, holding the assumption of a fixed price level in the non-tradable goods market. This upper limit to the real exchange rate represents the aversion of the government to an inflationary process derived from external shocks and its commitment to keep the real exchange rate as close as possible to its long-run equilibrium level.

4.1. A Band on the Real Exchange Rate

As in Section 3, the government is committed to a fixed exchange rate, but the government decides to keep the option of an automatic realignment of the nominal exchange rate if the real exchange rate reaches either a lower or an upper admissible level. If the real exchange rate touches the lower bound, then the government proceeds to devalue in a magnitude such that the real exchange rate stays at that lower limit. On the other hand, the government automatically revalues when the upper limit is reached.

As a consequence of this policy the nominal exchange rate follows a process given by

$$dE = E \left(dN_L - dN_H \right) \quad [62]$$

where the regulator dN_L is the non-negative non-decreasing process introduced in equation [53] and dN_H is the non-negative non-increasing regulator that is only positive when the real exchange rate hits its predefined admissible upper level. Both regulators are equal to zero when the real exchange rate fluctuates within the band and involves infinitesimal accommodations of the nominal rate to keep the real exchange rate just at the edges of the band. N_L stands for cumulative devaluations and N_H stands for cumulative revaluations.

The general solution to the problem follows the same steps described in the previous two sections with the difference that now the return on money holdings and the process for the real exchange rate are given by

$$R_M = \sigma^2 dt - \sigma dz - dN_L + dN_H \quad [63]$$

and

$$d\varepsilon = \sigma \varepsilon dz + \varepsilon \left(dN_L - dN_H \right) \text{ respectively} \quad [64]$$

As before, we assume the government's revenues from the realignment of the nominal exchange rate are returned to the public as part of the lump-sum transfers.

Now the two constants on the right hand side of [49], K_1 and K_2 will be determined by endpoint conditions that come from the existence of the reflecting barriers. In particular, a system of two equations (the two smooth-pasting conditions) renders

$$K_1 = A \frac{\phi}{\omega_1} \frac{\underline{\varepsilon}^\phi \bar{\varepsilon}^{-\omega_2} - \bar{\varepsilon}^\phi \underline{\varepsilon}^{\omega_2}}{\bar{\varepsilon}^{\omega_1} \underline{\varepsilon}^{-\omega_2} - \underline{\varepsilon}^{-\omega_2} \bar{\varepsilon}^{\omega_1}} \text{ and } K_2 = A \frac{\phi}{\omega_2} \frac{\bar{\varepsilon}^\phi \underline{\varepsilon}^{\omega_1} - \underline{\varepsilon}^\phi \bar{\varepsilon}^{-\omega_1}}{\bar{\varepsilon}^{\omega_1} \underline{\varepsilon}^{-\omega_2} - \underline{\varepsilon}^{-\omega_2} \bar{\varepsilon}^{\omega_1}} \quad [65]$$

where $\underline{\varepsilon}$ and $\bar{\varepsilon}$ are the lower and upper limit for the real exchange rate. The complete solution of the differential equation takes the form

$$\bar{J}(v, \varepsilon) = A v^\gamma \varepsilon^\phi \left[1 + \frac{\phi}{\omega_1} \varepsilon^{\omega_1 - \phi} \frac{\left(\underline{\varepsilon}^\phi \bar{\varepsilon}^{\omega_2} - \bar{\varepsilon}^\phi \underline{\varepsilon}^{\omega_2} \right)}{\left(\bar{\varepsilon}^{\omega_1} \underline{\varepsilon}^{\omega_2} - \bar{\varepsilon}^{\omega_2} \underline{\varepsilon}^{\omega_1} \right)} + \frac{\phi}{\omega_2} \varepsilon^{\omega_2 - \phi} \frac{\left(\bar{\varepsilon}^\phi \underline{\varepsilon}^{\omega_1} - \underline{\varepsilon}^\phi \bar{\varepsilon}^{\omega_1} \right)}{\left(\bar{\varepsilon}^{\omega_1} \underline{\varepsilon}^{\omega_2} - \bar{\varepsilon}^{\omega_2} \underline{\varepsilon}^{\omega_1} \right)} \right] \quad [66]$$

In the next section we analyze the effects of the existence of the band on the real exchange rate.

4.2. Effects of the Real Exchange Rate Band

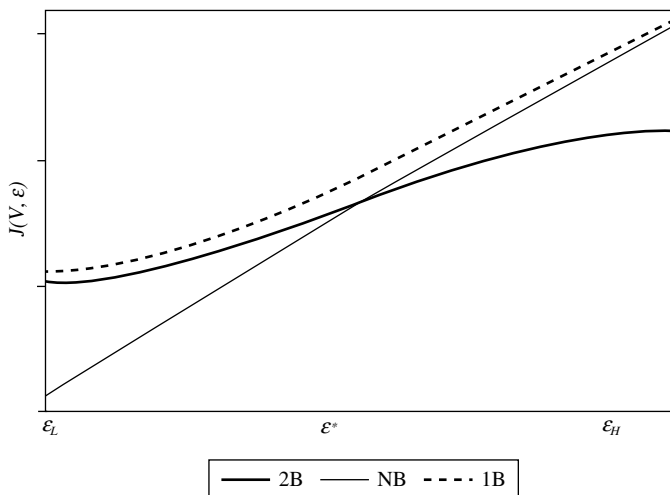
Once again, a welfare criterion needs to be used to evaluate the effects of the band on the real exchange rate. Given the complexity of equation [66], we proceed by simulating the value function $J(v, \varepsilon)$ under three different scenarios: no targeting at all, a lower bound (ε_L) and a symmetric band ($\varepsilon_L - \varepsilon_H$) on the real exchange rate. The function is represented in Figure 7. The thin NB-line shows the level of welfare attained under the no targeting scheme. The dotted 1B-line clearly shows the gains in terms of welfare (the vertical distance between the 1B and the NB lines) of introducing a lower limit to the real exchange rate, as explained in Section 3.

The S-shaped thick 2B-line represents the welfare attained at different levels of the real exchange rate within the band. As it should be expected, there is a cap on the welfare function, a straight result from the smooth pasting condition. The more valuable information is the reduction on the welfare gains the economy reaches when the real exchange rate approaches the lower band. That effect arises because even if the real exchange rate approaches that lowest admissible level and the probability of a revaluation gets smaller because the real exchange rate is getting further from its upper limit, that probability is always different from zero. On the other hand, when the real exchange rate is above the desired level, the existence of an upper limit eliminates the realization of states in which the production of non-tradable goods is expanded and consequently a permanent fixed exchange rate would have been a better alternative.

One important point to be remarked is the fact the even if the real exchange rate is at its long-run equilibrium level, there are still welfare gains generated by the establishment of the band. A sensitivity analysis practiced on the welfare function shows that the size of these welfare gains are correlated with the parameter of risk aversion γ . The more risk-averse the individual is, the higher the welfare gains are. Individuals have a relatively higher valuation for limiting the bad outcomes generated by a “too low” real exchange rate.

FIGURE 7

EXPECTED WELFARE UNDER NO TARGETING, A LOWER BOUND AND A BAND



Summarizing, the band on the real exchange rate lessens the welfare improvement we observe when just a lower bound is imposed to the real exchange rate.

For the analysis of the short-run effects, the optimal level for the arguments of the utility function can be used. Figures 8, 9 and 10 present those values for real money holdings and the consumption of the tradable and non-tradable goods, respectively. These variables are related by the marginal conditions presented in equations [25], [26] and its proportionalities with respect to G , expressed by [31]. Once G is obtained, the three of them can be determined. The figures show that level of the real exchange rate closer to the lower (upper) limit of the band generates lower (higher) levels of utility than if no band existed and the nominal exchange regime was that of a hard peg.

An important consideration to be made is that the existence of the band on the real exchange rate does not influence the real exchange rate itself within the edges of the band, in contrast to the classical result expected in the target zone literature. With a nominal exchange rate that is kept fixed within the band, the assumption of fixed price level of the non-tradable goods, the only source of changes in the real exchange rate is provided by changes in the international price of the tradable-goods, which is independent of the nominal exchange rate regime. However, under the existence of the band, the real exchange rate acts as a device that transmits information about the probability of future changes in the nominal exchange rate, forcing individuals to correct their consumption of both goods and the level of real cash balances they want to hold.

FIGURE 8

REAL CASH BALANCES UNDER NO TARGETING,
A LOWER BOUND AND A BAND

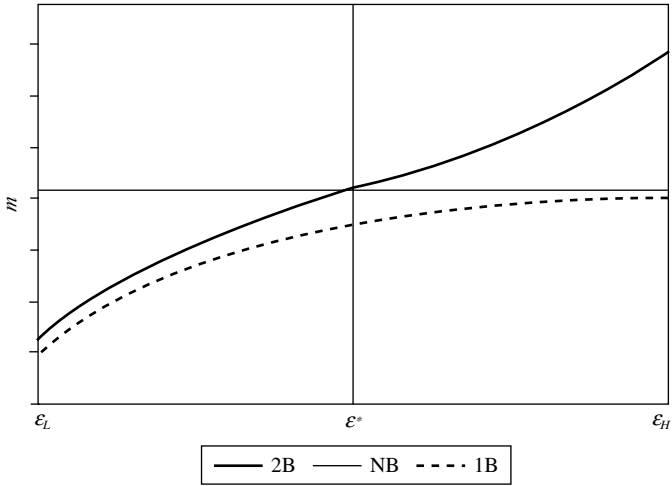


FIGURE 9

CONSUMPTION OF TRADABLE GOODS UNDER NO TARGETING,
A LOWER BOUND AND A BAND

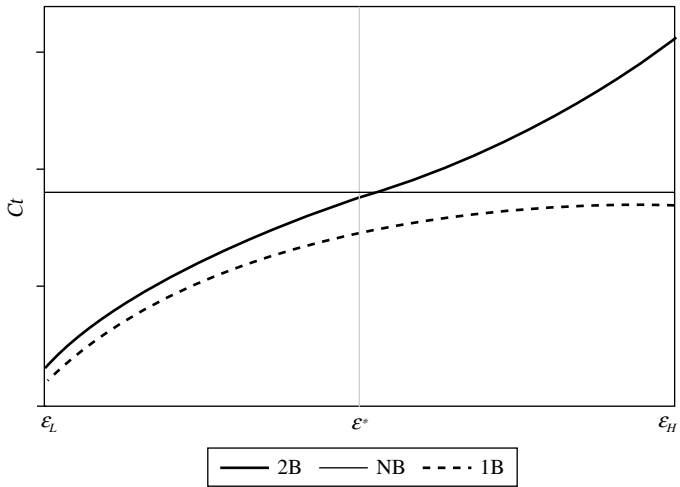
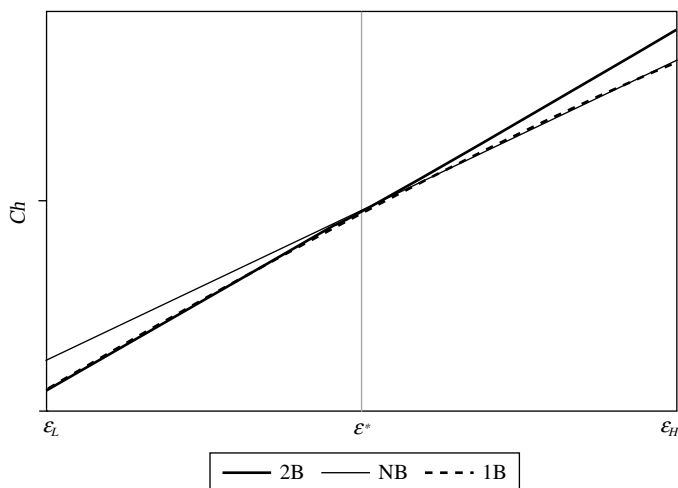


FIGURE 10

CONSUMPTION OF NON-TRADABLE GOODS UNDER
NO TARGETING, A LOWER BOUND AND A BAND



As a way to better understand the effect of the band we simulated four different possible paths for the real exchange rate. Figure 11 details the behavior of the real exchange rate, the consumption of tradable goods, the level of wealth and the consumption of non-tradable goods under two different decreasing paths of the real exchange rate. On the left, we assume that the real exchange rate approaches the lower bound of the band asymptotically. On the right, we let the real exchange rate follow a random path, but with a decreasing trend. We observe that a low real exchange rate induces individuals to consume less of both types of goods compared to the case of the pure hard peg regime (the *NB* line). That reduction on the levels of consumption is smaller in the case of the band (the *2B* line) compared to the case of having just a lower bound on the real exchange rate (the *1B* line). Reduced levels of consumption allows for the accumulation of wealth in both cases. An interesting observation is the fact that eventually the higher level of wealth may produce a level of consumption of both types of goods higher than in the case of a pure hard peg regime.

In a similar way, Figure 12 details the behavior of the real exchange rate, the consumption of both types of goods and the level of wealth but now, under increasing paths of the real exchange rate: one that approaches the upper bound of the band asymptotically (on the left) and another that follows a random path, but with an increasing trend. In the case of the existence of a band on the real exchange rate, the conclusions are exactly in the opposite direction compared to what it is observed in Figure 11. However, when only a lower bound is established, the paths for the consumption of tradable goods and wealth remain the same. These results

FIGURE 11

THE ECONOMY UNDER A DOWNWARD PATH OF THE RER

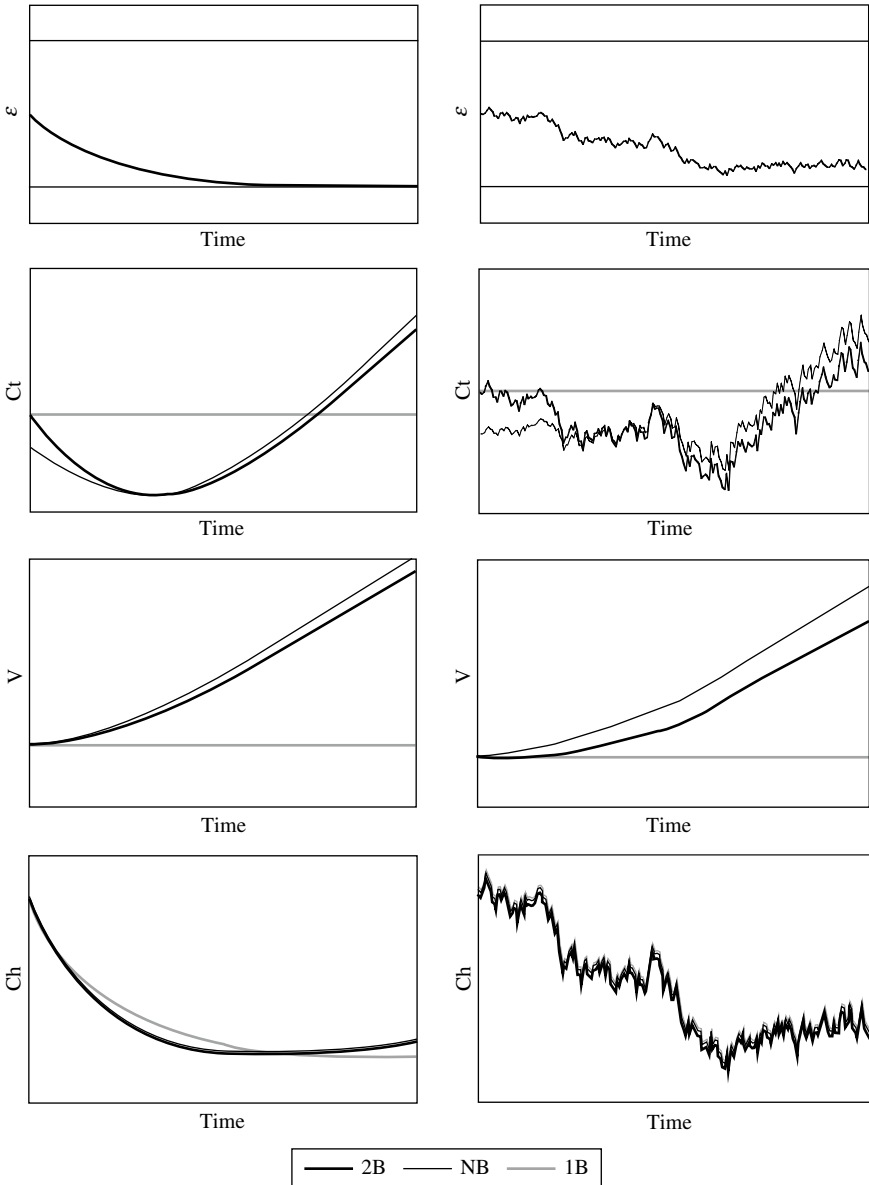
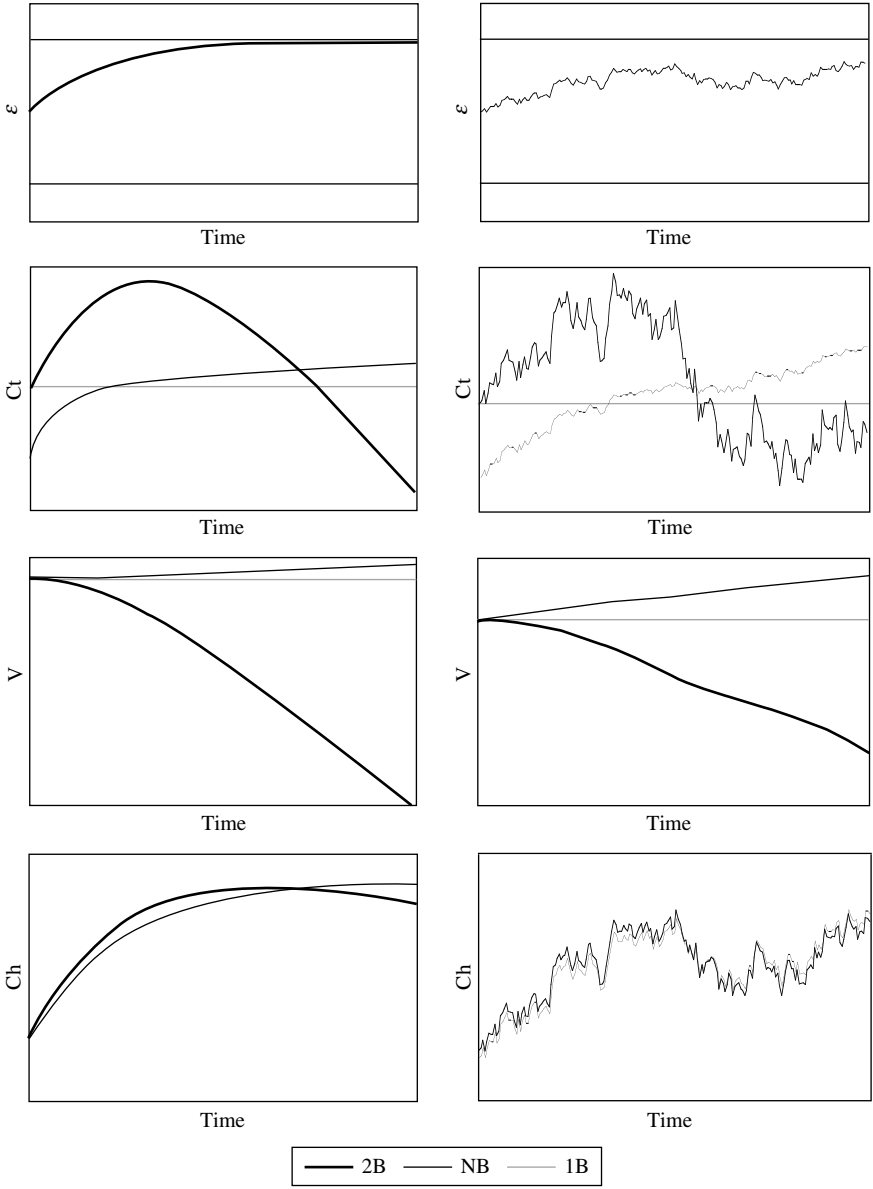


FIGURE 12

THE ECONOMY UNDER AN UPWARD PATH OF THE RER



come from the fact that under this regime, there is always a positive probability of facing a devaluation, but there does not exist a possibility of a revaluation. In that case, individuals reduce the demand for money, as well as the consumption of both types of goods, compared to the hard peg regime. These changes allow for wealth accumulation even in the case in which the real exchange rate departs from its lowest admissible level.

Finally, Figures 13 and 14 show how the welfare function behaves under the permanent fixed exchange rate regime and the band scheme respectively, when the variance of the process that moves the international price of the tradable good increases. In Figure 13 we observe the benefits of a lower variance in terms of expected welfare. The welfare function shifts upwards as the dispersion parameter σ gets lower, an intuitive result for a risk averse individual. In the case of Figure 14, it can be observed that the welfare function under a band scheme move counterclockwise as the variance of shocks decreases. A careful observation of the figure shows that the welfare function approaches the welfare function of the hard peg regime when the variance decreases. A lower variance reduces the probability of hitting the band in a reasonable time span and of a necessary correction of the nominal exchange rate.

5. SLUGGISH ADJUSTMENT IN NON-TRADABLE GOODS PRICE

As explained before, some degree of price stickiness is necessary for allowing the real exchange rate to depart from its long-run equilibrium level. The assumption that

FIGURE 13

THE VARIANCE OF THE SHOCK AND EXPECTED WELFARE UNDER NO RER TARGETING

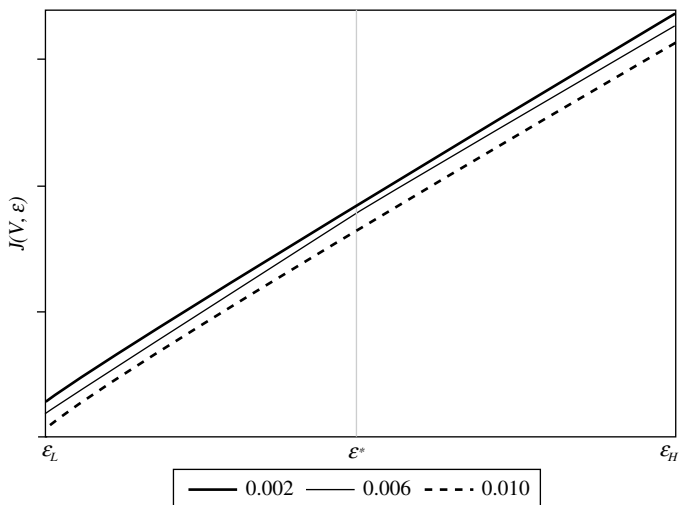
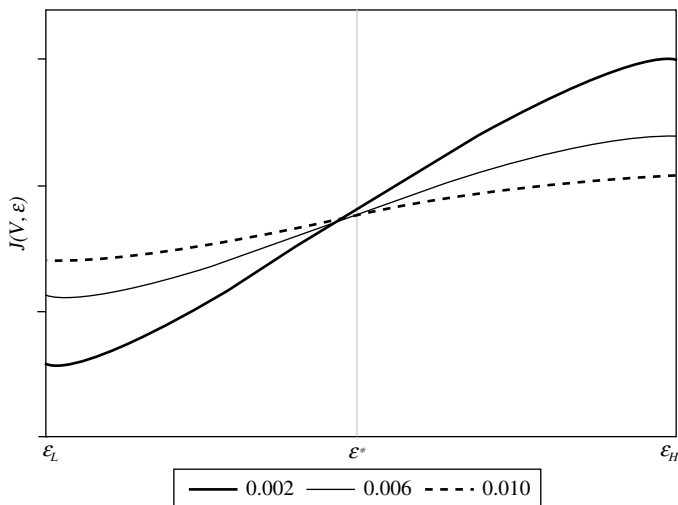


FIGURE 14

THE VARIANCE OF THE SHOCK AND EXPECTED WELFARE: A BAND ON THE RER



the price of non-tradable goods was fixed we used previously allowed us to understand the effects of the targeting rules. This assumption yielded a great simplification.

In this section, a sluggish adjustment replaces the fixed non-tradable goods price assumption and analyzes how the benchmark model of Section 2 changes when we allow for some degree of price flexibility. This specification is sufficient to guarantee that the targeted real exchange rate is reached in expected terms in the long run, even without intervention in the foreign exchange market.

5.1. Modeling Non-Tradable Goods Prices

As explained in section 2.4, for a given level of wealth of the whole economy, there is a unique level of consumption of tradable goods that satisfies the transversality condition and the equilibrium in the external sector (equation [29]). For that level of consumption of tradable goods, there is one and only one level of the real exchange rate that results in the long-run full employment level of activity in the non-tradable goods market. That level of the real exchange rate ϵ^F is determined by the marginal condition [24]. As the relative price between tradable and non-tradable goods decreases, non-tradable goods become more expensive and then, in a demand determined market, the level of production of non-tradable goods is lower than the long-run equilibrium level. This result needs the crucial assumption that the price level of these goods is not fully flexible. In previous sections, zero flexibility was assumed and it stayed fixed. However, it seems intuitively appealing to accept that although non-tradable

goods prices can not adjust instantaneously, at least they can adjust with some lag. To be more specific, we can assume that the price of the non-tradable goods adjusts deterministically toward their long-run equilibrium level at a rate that is proportional to the gap between the actual and the long-run equilibrium level of the real exchange rate, that is

$$dH = \kappa H (\varepsilon - \varepsilon^F) dt \quad [67]$$

Equation [67] means that, if the relative price of the non-tradable goods in terms of tradable goods is “too high”, then the proportional rate of change of the price of the non-tradable goods is negative and eventually it converges to a level compatible with full employment. Notice that this rate of inflation in domestic goods prices is not constant over time and gets smaller as the economy approaches its long-run equilibrium.

In accordance with the definition adopted for the real exchange rate, in an economy under a hard peg nominal exchange rate regime, such as the one described in previous sections, the proportional rate of change of the real exchange rate should be determined by the difference between the proportional changes in tradable and non-tradable goods prices. Using Ito’s Lemma,

$$d\varepsilon = \kappa \varepsilon (\varepsilon^F - \varepsilon) dt + \sigma \varepsilon dz \quad [68]$$

This expression adds into equation [11] a deterministic component to the price level of non-tradable goods. Let us recall that the price of the tradable goods follows a completely stochastic path described by the process in equation [3]. In that case, it was assumed that the drift parameter was equal to zero, meaning that the “deterministic” inflation rate equals zero in the rest of the world. Given that expected dz equals zero, expression [68] means that the expected proportional change in the real exchange rate is just the correction of the price level of the non-tradable goods toward its equilibrium level. In an extreme case, equation [11] can be interpreted as the limiting case when $\kappa \rightarrow 0$. That is, the rate of adjustment to the equilibrium level is extremely low.

Technically, equation [68] has the form of a geometric mean-reverting stochastic process for the real exchange rate. This kind of processes is frequently used in derivatives and option pricing theory and theory on investment under uncertainty. The literature on exchange rate bands has also been extended to allow for mean-reversion. In that case, the reverting process applies to the value of the fundamentals that determine the exchange rate. In particular, Delgado and Dumas (1992) model fundamentals following an arithmetic-mean-reverting process and concentrate on explaining the effects of varying the width of the band. Special attention is devoted to analyzing the smooth pasting conditions for finding the solution to the resulting stochastic differential equation for the exchange rate. Other applications for exchange rate bands, both nominal or real, can be found in Froot and Obstfeld (1991), Miller and Weller (1991), Tristani (1994) and Knot *et al.* (1999). In all cases, it is recognized that the tractability of the problem can be seriously compromised and that intuitive

interpretation of the results becomes more difficult when mean-reversion is added.¹⁸ Often numerical simulations are needed. Even more, closed-forms solution are not always possible and it is sometimes necessary to use numerical methods for solving the differential equation.

5.2. Solution to the Model Under a Permanent Hard Peg

To start with, the consumer's problem is the same as before with a unique change in the law of motion for the real exchange rate that behaves according to equation [68], instead of equation [11]. Notice that changes in the nominal price of the non-tradable goods do not affect the real money stock that individuals hold. As a consequence, neither the return on money holdings nor the individual's budget constraint change (see equations [5] and [10].)

However, the non-constant drift component for real exchange rate behavior does affect the expression for the value function [16] and its differentiation according to the differential operator used in [17]. In particular and by application of Ito differentiation, the last expression will include an extra term reflecting the effect of this new deterministic component in the behavior of the real exchange rate. More specifically, the function $L[e^{-\rho t} J(v, \varepsilon)]$ now has the following form

$$L[e^{-\rho t} J(v, \varepsilon)] = e^{-\rho t} \left[-\rho J + f v J_v + \frac{1}{2} v^2 \sigma^2 J_{vv} + \right. \\ \left. + \kappa \varepsilon (\varepsilon^F - \varepsilon) J_\varepsilon + \frac{1}{2} \varepsilon^2 \sigma^2 J_{\varepsilon\varepsilon} - \frac{1}{2} v \varepsilon \sigma^2 J_{v\varepsilon} \right] \quad [69]$$

The procedure for finding the solution follows the steps developed in Section 2.3. The first order conditions are the same as in [19]-[22]. Consequently, the marginal conditions that must be fulfilled at each point in time do not change.

However, the Hamilton-Jacobi-Bellman equation now includes an extra term related to the actual real exchange rate gap, causing the ratio G/v of equation [40] to be replaced by

$$\bar{G} = G \left[1 - \frac{1}{1 - \beta \gamma} \kappa (\varepsilon^F - \varepsilon) \phi \right] \quad [70]$$

It has to be noticed that now the optimal policy regarding the level of G as a proportion of wealth is no longer constant and varies with the size of the misalignment of the real exchange rate with respect to its long-run equilibrium level. In particular, the lower the observed real exchange rate is, the lower the ratio G/v . That is the

¹⁸ See Bartolini and Dixit (1991) and Metcalf and Hassett (1995).

short-run consequence of introducing a reverting-to-equilibrium process in the non-tradable-goods price level.

To derive the long-run effect it is necessary to find the value of the parameter A of the guess function [35], which results in

$$\bar{A} = A \left[1 - \frac{\phi}{1 - \beta\gamma} \kappa (\varepsilon^F - \varepsilon) \right]^{\gamma-1} \tag{71}$$

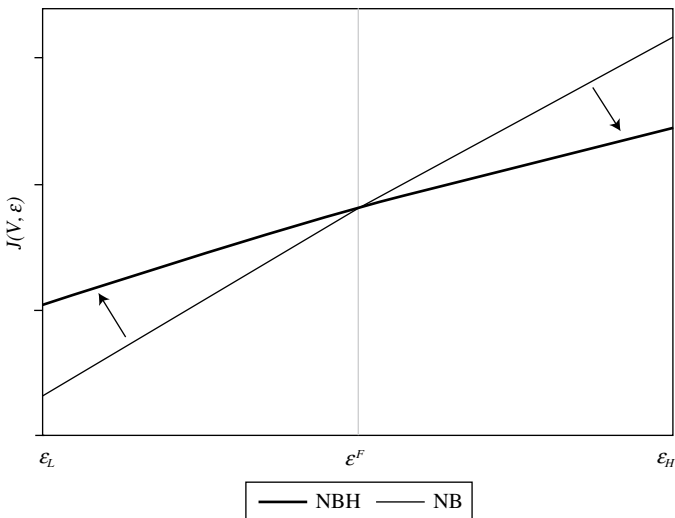
This parameter is no longer constant and changes with the size of the misalignment of the real exchange rate.

As in the previous section, we proceed by using simulations of the value function to evaluate the effects of the sluggish adjustment on the non-tradable goods price level. Figure 15 shows the result for a permanent fixed exchange rate regime with and without adjustment on the level of the non-tradable goods prices. Introducing non-tradable goods price sluggishness makes the welfare function move clockwise. For any level of the real exchange rate, the level of welfare attained is closer to its long-run equilibrium level.

For a real exchange rate that is below its long-run level, the economy with fixed non-tradable goods prices performs worse than one that allows at least a gradual adjustment

FIGURE 15

EXPECTED WELFARE UNDER FIXED AND SLUGGISH ADJUSTMENT IN NON-TRADABLE GOODS PRICE (THE BENCHMARK)



to equilibrium. In other words, the ability of the price of the non-tradable goods to adjust, such that its long-run level of production can be reached, results in reduced (increased) levels of consumption in the short-run when the real exchange rate is below (above) its long-run level, but with higher (lower) expected discounted welfare in the long run.

Some attention may be necessary to understand the short-run effect. Suppose that the observed real exchange rate is below its long-run level, meaning that its price is too high in terms of tradable goods and the level of production will be below its long-run level. Accordingly, downward pressure on the price level of those goods should exist. That is what equation [67] states. Given that the non-tradable goods price level is expected to go down, the real exchange rate starts to rise (see equation [68]). This means the relative price of the non-tradable goods in terms of the tradable goods decreases. As individuals expect non-tradable goods to become cheaper in the future, they optimally decide to delay their consumption with the aim of maximizing present discounted utility. This result is along the same lines as Calvo *et al.* (1995) when they analyze the effect of a transitory change in the rate of nominal devaluation. The process works the other way around when the real exchange rate is above its long-run level, creating incentives for present consumption and reducing welfare in the long run.

5.3. Conjectures on the Effects of Targeting Rules

As explained before, when no barriers are imposed on the real exchange rate and the government lets it float freely under a permanent fixed nominal exchange rate regime, the particular solution given in the previous section is the appropriate solution for the differential equation given by the dynamic programming problem. In the case the government decides to prevent the real exchange rate from deviating too far from a certain predetermined level, the complete solution for the differential equation at hand should be the sum of the particular and the complementary solution that results from solving the homogeneous part of that differential equation.¹⁹ That requires finding the roots that solve the homogeneous equation. In the case non-tradable goods price is allowed to revert to its long-run equilibrium level, those roots are both real and of different sign

$$\bar{\omega}_{1,2} = \frac{1}{2} \left[(1 + \gamma) - \frac{2\kappa(\varepsilon^F - \varepsilon)}{\sigma^2} \pm \sqrt{\bar{q}} \right] \quad [72]$$

$$\text{where } \bar{q} = \left[(1 + \gamma) - \frac{2\kappa(\varepsilon^F - \varepsilon)}{\sigma^2} \right]^2 + 4\gamma(1 + \gamma) + \frac{8(\rho - f\gamma)}{\sigma^2} \quad [73]$$

¹⁹ Note that under this set up the dynamics of the real exchange rate also changes so that [69] is replaced by $d\varepsilon = \kappa \varepsilon (\varepsilon^F - \varepsilon) dt + \sigma \varepsilon dz + \varepsilon dN_L$, where dN_L is the regulator introduced in section 3.1.

²⁰ We assume that $\bar{\omega}_1 < 0$ and $\bar{\omega}_2 > 0$.

Following the steps developed in Sections 3 and 4, the constants \bar{K}_1 and \bar{K}_2 are determined by the smooth-pasting conditions mentioned in Sections 3.2 or 4.1.²¹ The tractability of the problem at hand becomes complicated and closed-form solutions cannot be derived. For these reasons, we use the result of the previous section to infer the possible effects of a lower bound or a band policy under sluggish adjustment of the non-tradable goods price.

When only a lower bound is established, a natural question that arises regards the level of the policy variable $\underline{\varepsilon}$. In Section 3.3, it was determined that the existence of the lower bound policy is a welfare improving policy in the long run. In particular, those long-run welfare gains are bigger as $\underline{\varepsilon}$ becomes higher. More specifically, one might be interested in the case in which the government targets a level higher than the long-run equilibrium level.

Under the assumption of a fixed level of price of the non-tradable goods in Section 3, the answer to the last question is unambiguous. No limit on the real exchange rate lowest admissible level exists. The higher that level is, the bigger the long-run welfare gain is. However, the conclusion changes when reversion to long-run equilibrium is allowed in the non-tradable goods price level. In this case, the government forces the real exchange rate to be above its long-run equilibrium level and consequently it imposes a lowest positive inflation rate in the non-tradable goods sector. That lowest limit is given by the expression

$$\pi = \frac{dH}{dt} \frac{1}{H} = \kappa \left(\bar{\varepsilon} - \varepsilon^F \right) \text{ where } \bar{\varepsilon} > \varepsilon^F > 0 \quad [74]$$

As the non-tradable goods price level present a trend to rise over time (in expected terms), hence the real exchange rate follows the inverted path with downward pressure always present. Permanent inframarginal devaluations will be needed to prevent the real exchange rate from perforating the band, resulting in an expected path for the domestic price of the tradable goods higher than if the target did not exist and in a lower desired level of real cash balances by the public. This result is along the lines of the traditional literature on real exchange rate targeting mentioned in Section 1.1 in the sense that a policy that tries to impose a higher than the long-run equilibrium level of the real exchange rate leads to price level instability.

In the case that the monetary authority establishes a lower and a upper bound in the real exchange rate, it is important to understand that the sluggish adjustment in the price of the non-tradable goods reduces the needs for nominal exchange rate realignments. The adjustment in the mentioned price level helps the real exchange rate to return to its level compatible with full employment in the non-tradable goods sector. In some way, we could say that the adjustment process works in the same direction as the band, in the sense that, as it approaches the lower (upper) bound, it increases

²¹ When deriving \bar{K}_1 and \bar{K}_2 , it has to be considered that \bar{A} , $\bar{\omega}_1$, and $\bar{\omega}_2$ are functions of the actual real exchange rate level.

the expected value of the real exchange rate. Individuals take that information into account and lessen their reaction to the existence of the bounds.

To summarize, when the price of non-tradable goods is allowed to follow a reverting process to its level compatible with long-run equilibrium, welfare gains or losses are reduced compared to the fixed price level case. This result is in line with the conventional wisdom that higher degrees of price flexibility increase welfare. However, in the short-run this generates lower (higher) levels of activity when the real exchange rate is below (above) its equilibrium level, an effect that takes into account the expected future path of the relative price between tradable and non-tradable goods. It has also been shown that targeting a level of the real exchange rate higher than its equilibrium level results in a non-negative rate of inflation with welfare reducing effects.

6. CONCLUSIONS

Targeting of the real exchange rate has been analyzed in the literature following two different lines. The most traditional one investigates managed nominal exchange rate systems (with either a fixed nominal rate or a fixed rate of devaluation) where government is concerned about loss of competitiveness due to domestic inflation rates higher than in the rest of the world and hence a low or decreasing real exchange rate. In such an environment, more often than not the policy responses are sizable devaluations or increases in the rate of devaluation. The conclusion in this branch of the literature is unambiguous in the sense that attempts to maintain a real exchange rate higher than the equilibrium level lead to accelerating inflation.

A second branch explores the competitiveness issue in the standard context of a band (target zone) on the nominal exchange rate, where the escape clause takes the form of changes in monetary policy once the band limits are reached. In an economy with sluggishness in the price of home goods, movements in the nominal exchange rate within the band correspond to movements in the real exchange rate, and, in a sense, the establishment of the bounds can be motivated by the ultimate goal of limiting extreme changes in the real exchange rate.

Our work is intended as a step toward the integration of those lines of research. We consider an economy under a fixed exchange rate system, but with bounds (a minimum level or a band) on the real exchange rate. In our analysis, the international price of the tradable goods is characterized by the continuous arrival of shocks, and these permanent shocks are the source of shocks to the real exchange rate. In a model with microeconomic foundations, we investigated the effects of targeting (imposing bounds on) the level of the real exchange rate, in an environment otherwise characterized by an "exchange rate rule" which takes the simple form of a constant level of the nominal rate. When movements in tradable goods prices cause the real exchange rate to reach the bound(s), government resorts to changes in the nominal exchange rate in order to preclude the real rate from trespassing the bound(s). The difference between our approach and the "traditional" approach as described above is, then, that here government intervenes only when a predetermined bound is reached, and that when it intervenes, it does so following a well-established rule. The modeling technique, on the other hand, is the same as in the floating exchange rate target zones literature.

The most general conclusion of our work is that when bounds are established, then welfare gains can be expected, but that those gains in expected welfare are generated at the expense of levels of consumption that go in the opposite direction to that intended by the policymaker. Another general conclusion is that the effects bounds on real variables are present even in the case in which the bound is never reached and hence the targeting policy is never exercised –the same result as in the target zones literature, in the sense that the mere existence of the bound changes the behavior of the economy.

Section 2 developed, as the benchmark case in which there is a strict exchange rate rule and no escape clauses, a stochastic general model with two goods and fixed non tradable goods prices. The fixed price assumption, which intends to capture short run effects, is introduced in order to keep the tractability of the problem.

Section 3 analyzes the case in which a lower bound on the real exchange rate is introduced. It shows that the existence of the lowest tolerance limit on the real exchange rate, which reduces the range over which the level of activity in the non tradable goods sector can fluctuate, is a welfare improving policy in the long run, in terms of expected welfare. However, the short run effect is a lower level of production vis-à-vis the benchmark case with no escape clause.

The case of a band on the real exchange rate is analyzed in Section 4. The upper bound is introduced both for reasons of symmetry, and as a representation of the government's intention of avoiding extremely high levels of production in the non tradable goods sector and its distaste for high inflation rates associated with increasing levels of the international price of tradable goods. The results show that the welfare effects depend on the initial level of the real exchange rate. For levels closer to the lower bound, the welfare consequences are the same as in the case of a simple lower bound, but they are of smaller size. On the upper edge of the band, with an initial exchange rate above the center of the band, the policy can be welfare reducing.

An interesting –even initially intriguing– result, true in both the cases of a lower bound and a band, is that although the “targeted” variable (the real exchange rate) has exhibits exactly the same behavior within the band as it would in the benchmark case of no band (since non-tradable goods prices are fixed), the same is not true of the other real variables in the economy. In other words, although the targeted variable within the band behaves identically as the benchmark case in which there is no band, the rest of the real variables in the economy behave differently, even if the targeted variable remains within the band and the escape clause is never triggered. This is interesting because in the case of a band on a floating nominal exchange rate, this exchange, which is the target, behaves differently inside the band from the case when the band doesn't exist. In our case the targeted variable (the real exchange rate) does not behave differently (which might mislead the careless observed), but the rest of the real variables do.

The assumption of sluggish adjustment of the price of non tradable goods to some long-run equilibrium value makes the problem analytically intractable when bounds on the real exchange rate are imposed. In Section 5, we analyze once again the benchmark case but with sluggish rather than fixed prices of the non tradable good. By observing the effects of price sluggishness on the benchmark case, we develop some

conjectures about the possible outcomes when a lower bound or a band is imposed. Sluggishness in price adjustments moves the economy toward outcomes closer to the full flexibility case, and reduces the occurrence of states in which a realignment of the nominal exchange rate can be necessary and from that point it plays a welfare stabilizing role. We therefore conjecture that when the price of non tradable goods is allowed to follow a reverting process to some predetermined level, welfare gains or losses are reduced compared to the fixed price case, as well as the changes with respect to consumption and portfolio allocation decisions.

We should mention some caveats and shortcomings. The assumption of a fixed arbitrary price of non-tradable goods, while providing some insights for the short-run, is clearly a shortcoming, and future work should aim at considering a model in which there is a well-defined level of non-tradable goods output (the “full employment”) level, perhaps in the context of a slightly simplified model. Another shortcoming is the assumption of the world real interest rate being the same as the fixed rate of time preference –an assumption used in many papers, which simplifies matters but does not yield a single, unique real exchange rate of equilibrium.

The ultimate purpose of our research is to contribute to the understanding of the effects of imposing escape clauses on an environment of rational expectations, and in this sense there are many areas and policies in which those effects need to be further understood and which could provide a natural agenda for future research. Closer to the themes of our study, though, some well-defined areas appear as natural candidates for immediate future research. The main one is, probably, the analysis of which is the optimal width of the band around the long-run equilibrium level of the real exchange rate and, eventually, the analysis of what the optimal policy response would be following a “regime” or structural change”, i.e., a once-and-for-all change in the equilibrium real exchange rate requiring the reposition of the band.

Another aspect to be considered in the future is a more general definition of the real money stock. In this work we have defined the real money stock in terms of traded goods.

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