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DOES LINEARITY IN THE DYNAMICS OF INFLATION GAP AND UNEMPLOYMENT RATE MATTER?*

¿IMPORTA LA LINEALIDAD EN LA BRECHA INFLACIONARIA Y LA TASA DE DESEMPLEO?

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Abstract

This paper test the null hypothesis of linearity against a specific form of nonlinearity in the Data Generating Process (DGP) of the unemployment rate and the difference between the inflation rate (measured as the twelve months variation of CPI and CPIX1) and the inflation target, using twenty years of data (1990-2009) and time series models. The rejection of the null implies that the series has more than one regime or state. The regime switching process could explain the recent boom/bust of inflation observed during these years, or the unemployment rate after the Asian crisis, for instance. The main results are: it is not possible to reject linearity in the deviation of inflation from the inflation target. During the last twenty years, inflation has converged smoothly to the target without any regime switching. The speed of convergence to the target has been constant over the years and inflationary shocks have been dissolved with the usual degree of persistency. Finally, strong evidence is found against linearity in the unemployment rate. On the contrary, it fluctuates with high probability between states or regimes through time.

Keywords: *Inflation, unemployment, SETAR models, regime switching, nonlinear models.*

JEL Classification: *E3, E4, E5.*

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Resumen

Este artículo testea la hipótesis nula de linealidad, contra una forma específica de no-linealidad en el Proceso Generador de Datos (PGD) de la tasa de desempleo y la diferencia de la inflación (medidas como la variación en 12 meses del IPC e IPCX1) y la meta inflacionaria, usando veinte años de datos (1990-2009) y modelos de series de tiempo. El rechazo de la nula implica que la serie tiene más de un régimen o estado. Este cambio de régimen puede explicar el reciente auge/caída de la inflación durante estos años, o la tasa de desempleo después de la crisis asiática, por ejemplo. Los principales resultados son: no es posible rechazar la linealidad en la desviación de la inflación con respecto a la meta. Durante estos últimos veinte años la inflación ha convergido suavemente hacia la meta sin cambiar de régimen. La velocidad de convergencia hacia la meta ha sido constante a través de los años y los shocks inflacionarios han sido disueltos con la persistencia habitual. Finalmente, se encuentra fuerte evidencia en contra de la linealidad para la tasa de desempleo. Ésta fluctúa entre los regímenes con alta probabilidad a través del tiempo.

Palabras Clave: *Inflación, desempleo, modelos SETAR, cambios de regímenes, modelos no lineales.*

Clasificación JEL: *E3, E4, E5.*

1. INTRODUCTION

This paper presents an empirical and reduced form approach, to improve our understanding of the dynamics of inflation and unemployment. Specifically, I focus in the linearity assumption that is usually taken as granted because of its simplicity. This paper tests the hypothesis of linearity in the DGP[1] of the unemployment and inflation rates (measured as the twelve months deviation of CPI and CPIX1[2] from target) against a specific form of nonlinearity, a SETAR[3] model. Unlike traditional ARIMA models, these models allow different regimes for the endogenous variable across time. Therefore, the rejection of the null could help to understand the recent boom/bust of inflation or the unemployment rate after the Asian crisis as regime switching processes. To this end, I used twenty years of economic data and the linearity test presented by Hansen (1996a, 1999).

The main results are: it is not possible to reject linearity in the deviation of inflation from inflation target (IT), which is 3% in Chile. During the last twenty years, the convergence of inflation to IT has been smooth without any regime switching

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1 Data Generating Process.
2 Core inflation.
3 Self-Exciting Threshold Autoregressive.

or parameters instability. This implies that the velocity at which inflation converges to its target has been constant over the years and that inflationary shocks have been dissolved with the usual degree of persistence. The recent boom/bust of inflation can be attributed to a series of positive/negatives shocks randomly drawn from the innovations distribution. There is no doubt that the Central Bank has played a key role in this process, or at least it has not done anything to cause an opportunistic change in the speed of adjustment. However, I cannot assert if this is a desirable outcome for the monetary authority. Finally, I find strong evidence against linearity in the unemployment rate. The statistical evidence shows that the unemployment fluctuates between states more frequently which means that economic crises have very persistent effects over this variable and can easily carry from one regime to another.

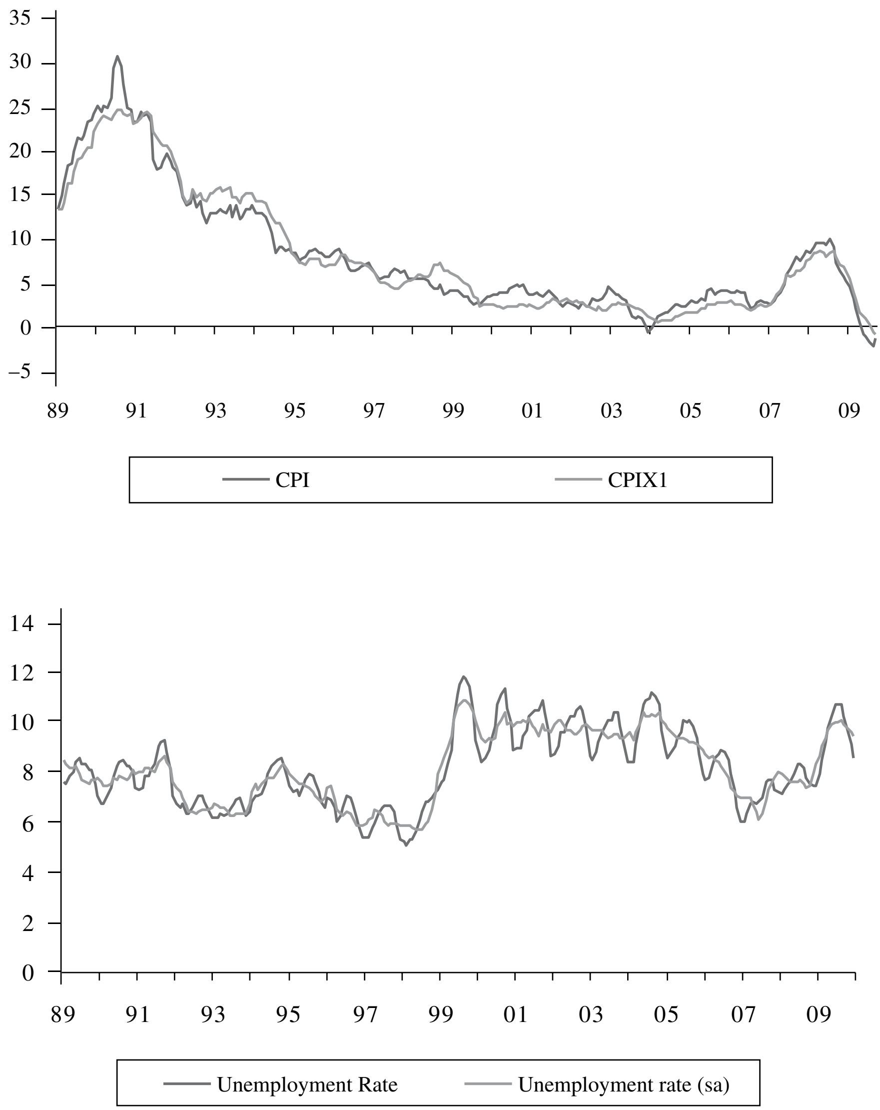
During the last twenty years, the Chilean economy went through many economic cycles. The first half of the nineteen nineties was a period of solid economic growth. This could be attributed to many factors, such as the return to democracy, access to new international markets or even the autonomy of the Central Bank that allowed a suitable macroeconomic environment, among other reasons. Figure 1 shows the twelve-month variations of CPI and CPIX1 and the unemployment rate, for the 1990-2009 period. The unemployment rate had a very particular evolution. Until 1999 it was stable between 6% and 8%. Also, during this period, the Central Bank implemented an inflation targeting scheme with great success. Inflation was under control following a clear downward trend. These economic data show an ideal scenario to the Chilean economy: convergence of inflation towards inflation targeting along with solid economic growth has enabled a lower unemployment rate. However, all changed in the late nineties with the arrival of the Asian crisis that had a strong impact over the Chilean economy.

Figure 1 shows that the Asian crisis had a very persistent effect over the unemployment rate. In fact, it reached 10% a couple of months after the crisis began. Graphically, it is not difficult to argue that the unemployment went from one regime to another of a higher mean. After the Asian crisis the unemployment rate was systematically higher, and remained high many years after the crisis officially ended. Finally, this Figure also shows three particular episodes of inflation. The first one is the gradualist approach to reduce inflation between 1990 and 1998. The second one is the deflation process that happened around 2004; and finally the recent boom/bust of inflation, due mainly to the high price of commodities and the recently financial crisis, respectively.

The main objective of this paper is to test the linearity hypothesis in the DGP of the unemployment rate and the difference between inflation (measured as the twelve months variation of CPI and CPIX1) and inflation target (IT). To this end, I use SETAR (Self-Exciting Threshold Autoregressive) models, which are regime switching models that allow adding nonlinearities to a variable in a direct and simple way through thresholds parameters. Under the null hypothesis, the series have an AR(p) representation and a SETAR specification under the alternative. If the null hypothesis is rejected, these series will have different regimes across time. Performing the linearity test is far from trivial because the thresholds parameters of the SETAR model are not defined under the null hypothesis, meaning that the asymptotic distribution of the test is nonstandard and therefore the likelihood ratio test cannot be implemented.

FIGURE 1

INFLATION 12M VAR. (CPI AND CPIX1) AND UNEMPLOYMENT RATE
BETWEEN 1989 AND 2009



However, Hansen (1996a, 1999) proposed Monte Carlo and Bootstraps simulations that allowed approximating the asymptotic distribution of the test under the null more accurately. The implications and relevance of this particular test are explained in detail in the following paragraph[4].

Regarding prices, choosing the deviation between the inflation and IT is economically appealing. If the persistence[5] of inflation has changed over time and it takes more time to converge to the target after an inflationary shock, then by backward induction it is possible to conclude that the parameters behind the AR model of this DGP also have changed. Therefore inflation would be in a new regime, specifically one with higher persistence. This is mainly because any degree of persistence that we may define will always be a function of the parameters of the AR model behind the DGP. This is fundamental in order to have a better understanding of the recent boom/bust of inflation, since it has been argued by other authors that the persistency in inflation increased due to the high prices of commodities between 2007 and 2009. If we agree with this statement, someday commodities prices should return to their levels consistent with their long run fundamentals and therefore inflation will return to its normal degree of persistence. If the velocity at which inflation moves towards the target is different across time, then this DGP should have at least two regimes and the null hypothesis of linearity should be rejected in favor of the SETAR representation. However, there are no clear reasons to expect the rejection of the null hypothesis. For instance, one could argue that the recent episode of boom/bust of inflation was only a series of shocks randomly selected from the innovations distribution, which were dissolved with the usual degree of persistence[6].

In addition, due to the close economic and statistical relationship between inflation and unemployment, at least in the short run, I run the same analysis using the unemployment rate. The graphical evidence provided by Figure 1 strongly suggests that unemployment has been fluctuating between several regimes during the last twenty years. For instance, before the Asian crisis unemployment rate was between 6% and 8%, but after the crisis it reached 10%. In the late 2007 it decreased and in 2009 it increased again due mainly to the financial crisis. These movements argue in favor of the idea that the unemployment rate has been fluctuating between several regimes more frequently. Economic crises do have a strong impact over the unemployment. It could be argued that public policies implemented during the Asian or financial crisis

[4] An important caveat of this paper is that I am using regime switching models instead of structural breaks models. The main difference in these approaches is that in structural break models, the break is permanent, whereas in regime switching models there is always a chance to reverting to an old regime. For instance, the deviation of inflation from target could fluctuate between two regimes: one of high and another of low or normal persistence. I find this characteristic of regime switching models more suitable to understand the behavior of unemployment and inflation during the last twenty years in Chile.

[5] I do not want to define any particular measure of persistence, because as I will explain later, is not really necessary or relevant. The key point to notice is that persistence will always be a function of the parameters of an AR model. If they remain constant over time (without any regime switching), then persistency will as well.

[6] For instance, if there are five positive consecutives shocks, then it is clear that inflation will increase and it will take it more time to converge to the inflationary target of the Central Bank, however there is no prior reason to believe that the persistence has changed over this process.

were not strong enough to prevent unemployment from switching regime. However it is not possible to assure if this was the ultimately goal of these policies, because there is no explicit target for this variable, neither from the Central Bank or the government authority[7]. Due to the graphical evidence, I should be able to reject the null hypothesis with a high level of significance. I use the seasonally adjusted rate in order to avoid any problem with the high seasonality that this variable displays which could induce a false rejection of the null. Also, I do not use the detrended unemployment rate in order to avoid the statistical problems of these filters.

The use of nonlinear models has been growing systematically over time. In general, these models have been oriented to the field of finance, because they are able to explain the chaotic behavior of volatility in assets returns, in which a linear structure is unable to characterize the volatility clusters that are observed over time. However, recently there have been an important numbers of papers that use nonlinear models to explain some macroeconomics variables. This comes from a very appealing idea, from a theoretical and empirical point of view. For instance, the existence of asymmetric adjustments costs, exchange rate bands, structural thresholds or multiple equilibriums could generate dynamics that ARIMA models are not able to capture efficiently. A nonlinear model may offer some flexibility that could help to explore these complex dynamics, at least hypothetically.

It is not common to find applications of nonlinear models to the Chilean economy. The linearity hypothesis is usually taken as granted in most of the empirical research because of its simplicity. Pincheira (2008) tries to asses if the inflation has showed different degrees of persistency. He studied the evolution of the persistence for several measures of inflation using AR models. To this end, he focused on the mean life of inflationary shocks that are obtained from the impulse response function (IRF) of traditional AR models in different time windows. His results show that the punctual estimation of inflationary persistence is greater than before. Thus inflation is more likely to remain above his historical mean value after a shock. However, no formal test is presented to assure that these differences are statistically significant between samples; the confidence intervals are needed to discard this possibility. Chumacero (2004) presents several automatic estimation routines for AR, SETAR and Neuronal Networks (NN) models. The lag order of the AR and SETAR models is determined by the Hannan-Quinn information criterion; and one to three hidden units are considered for the NN along with the logistic activity function. The author performs an out/in-sample evaluation of each model. The main conclusion is that nonlinear models tend to provide superior forecast than linear models in the short term. For other countries, Feng and Liu (2002), also compares the predictive power of SETAR and ARMA models for the GDP of Canada. The evaluation is performed for several forecast horizons. The Diebold and Mariano test reject the equal forecast accuracy hypothesis in favor of the SETAR models. Ferrara y Gugan (2005) also estimated SETAR models for the European Industrial Production index. However, they do not address the statistical significance of the thresholds parameters.

7 Unlike inflation unemployment is not directly targeted.

The rest of the paper is organized as follows. The next Section discusses the main characteristics of AR and SETAR models and the linearity test. The third Section presents the econometric results. Finally, the last Section concludes.

2. METHODOLOGY

This section presents a brief description of the traditional methodology used in time series to model the DGP of a series. Subsequently, SETAR models and Hansen test (1996a, 1999) are presented and discussed.

2.1. Lineal Autoregressive Model (AR)

Following Tsay (2005), a time series variable will be linear if it can be written as,

$$y_t = \mu + \sum_{i=0}^{\infty} \psi_i \varepsilon_{t-i} \tag{1}$$

Where μ is the mean of y_t, ψ_i is a real number with $\psi_0 = 1$, and $\{\varepsilon\}$ is a sequence of independent and identically distributed (iid) random variables with mean zero and from a well defined distribution. It is assumed that the distribution of ε_t is continuous and $E(\varepsilon_t) = 0$. The process is linear because it has an MA representation. A time series process will be nonlinear if it does not satisfies the above relationship. The traditional linear autoregressive model AR(p) is usually defined as:

$$y_t = \beta_0 + \sum_{k=1}^{p} \beta_k y_{t-k} + \varepsilon_t \tag{2}$$

Where is the error term and it is assumed that $\varepsilon_t \sim N\left(0,\sigma^2\right)$, with implies that the OLS and maximum likelihood estimators are the same. Finally, several criteria may be used to select the proper lag order[8].

2.2. SETAR (Self-Exciting Threshold)

SETAR models are special cases of more general nonlinear models introduced by Tong (1977) and developed with more depth in Tong and Lim (1980) and Tong (1983). The main idea is that the parameters of an autoregressive specification could change depending on a weak observable exogenous variable.

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[8] The search for a good criterion still is an active field of research in econometrics. For a deeper discussion between the Akaike and Bayesian information criterion, refers to Yang (2003) "Can the Strengths of AIC and BIC Be Shared?"

A SETAR model of m regimes is denoted as $SETAR(m, p_1, p_2, ..., p_m)$, where p_j is the lag order of regime j and is defined as:

$$y_t \begin{cases} \alpha_1 + \sum_{k=1}^{p_1} \beta_{k,1} y_{t-k} + \varepsilon_t & \text{if } q_{t-1} \leq \gamma_1 \\ \alpha_2 + \sum_{k=1}^{p_2} \beta_{k,2} y_{t-k} + \varepsilon_t & \text{if } \gamma_1 < q_{t-1} \leq \gamma_2 \\ \vdots & \\ \alpha_m + \sum_{k=1}^{p_m} \beta_{k,m} y_{t-k} + \varepsilon_t & \text{if } q_{t-1} > \gamma_m \end{cases}$$

Let us define $\{\gamma\}_{i=1}^{m}$ as the threshold parameters and $q_{t-1} = (y_{t-1}, y_{t-2}, ..., y_{t-k})$ as a known function of the data. Usually $q_{t-1} = y_{t-d}$ where d is referred as the delay parameter[9]. This allows a simple way to introduce nonlinearities to the DGP of a series. However, the threshold parameters must be restricted in order to assure that each regime contains a minimum number of observations, usually between 10% and 15% of the sample. It is important to point out that if m takes the unit value then the SETAR(1) becomes an AR(p) model.

To ease of exposition, I assume that $p_1 = p_2 = ... = p_m$ and the SETAR(m,p) model is redefining according to Hansen (1999) as:

$$y_t = \beta_1 ' X_{t-1} I_{1t}(\gamma, d) + ... + \beta_m ' X_{t-1} I_{mt}(\gamma, d) \tag{3}$$

Where is a univariate time series and:

$$q_{t-1} = y_{t-d} \tag{4}$$

$$X_t = \begin{pmatrix} 1 & y_{t-1} & y_{t-2} & \cdots & y_{t-p} \end{pmatrix} \tag{5}$$

$$I_{jt}(\gamma, d) = I(\gamma_{j-i} < y_{t-d} \leq \gamma_j) \tag{6}$$

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[9] In this framework, the function that defines the regime is $q_{t-1} = y_{t-d}$. However, this is only one of the three functions considered by Koop and Potter (1997) which are the following:

1. $q_{t-1} = y_{t-d}$
2. $q_{t-1} = y_{t-1} - y_{t-d}$
3. $q_{t-1} = \frac{y_{t-1} - y_{t-d}}{d}$

Here $I_{jt}(\gamma,d)$ denotes the indicator function which takes the unit value if the internal condition holds and zero otherwise. Finally, define d and $\gamma=(\gamma_1,...,\gamma_m)$ with $\gamma_1<\gamma_2<...<\gamma_m$ as the *delay* and *threshold parameters*, respectively. The observation y_t belong to the regime j if and only if: $\gamma_{j-1}<y_{t-d}\leq\gamma_j$.

Let $\theta=(\beta_1,...,\beta_m,\gamma,d)$ be the vector that groups all the parameters of the model. Then the OLS estimators may be obtained through the minimization of the following objective function:

$$\hat{\theta}=\arg\min\sum\left(y_t-\beta_1{}'X_{t-1}I_{1t}(\gamma,d)-...-\beta_m{}'X_{t-1}I_{mt}(\gamma,d)\right)^2 \tag{7}$$

The parameters are estimated sequentially by concentration. To illustrate this, consider the following SETAR(2,p) model:

$$y_t\begin{cases}\alpha_1+\sum_{k=1}^{p}\beta_{k,1}y_{t-k}+\varepsilon_t & \text{if } \; y_{t-1}\leq\gamma\\[2ex] \alpha_2+\sum_{k=1}^{p}\beta_{k,2}y_{t-k}+\varepsilon_t & \text{if } \; y_{t-1}>\gamma\end{cases}$$

This model has $(2+2p+2)$ parameters to estimate and the estimation procedure can be summarized as:

1. Let $Z(y_t)=sort(y_t)$[10].
2. Define the delay parameter d_j.
3. Define γ_j^i as the $\alpha\%T-1+i$ element of $Z(y_t)$[11].
4. Conditional to γ_j^i and d_j, split the sample and obtain the OLS estimators of β_j^i and SSR_j^i (sum squared resid).
5. Define the estimator candidate as $\hat{\theta}_j^i=\left(\beta_j^i,\gamma_j^i,d_j\right)$.
6. Repeat (2) - (5), for $i=1,2,...,(T-2\alpha\%T)$.
7. Repeat (2) - (6), for $j=1,2,...,w$[12].
8. Choose $\hat{\theta}_j^i$ such that minimizes the SSR.

[10] Ordered values from lowest to highest of y_t. Ergo, $Z(y_t)$ contains all the relevant values that the threshold may take.

[11] Where T is the sample size and α is the minimum percentage of observations for each regime.

[12] Where w, is the maximum value allowed for the search of the delay parameter, it should be at least or equal to the lag order of the SETAR model.

Once the parameter vector that minimizes the SSR is obtained, each regime can be estimated independently and statistical inference on the parameters can be carried out in the traditional way.

To estimate of SETAR models of higher orders the procedure is quite similar. For example, to estimate a SETAR(3), start by estimating a SETAR(2) and save the threshold parameters estimate $\left(\hat{\gamma}_1, \hat{d}\right)$. Since the parameter space for d is discrete the estimator of $\hat{d}$ is super consistent. In addition $\hat{\gamma}_1$ will be a consistent estimator of one of the two thresholds parameters[13]. Then, conditional on these estimations the second threshold parameters are estimated (two step procedures). It is possible to gain asymptotic efficiency if additional iterations are carry out to estimate the first threshold again. More details can be found on Bai (1997) and Bai and Perron (1998)[14].

2.3. Linearity Test

Hansen (1996a, 1999) presents two methodologies to test the hypothesis of linearity. This section presents and discusses the application of these tests. First, suppose that the econometrician is interested to test whether a series fluctuates between k or j regimes $(j > k)$:

$$H_0 \; : \; Model \;\; SETAR\left(k, p_1, ..., p_k\right)$$

$$H_1 \; : \; Model \;\; SETAR\left(j, p_1, ..., p_j\right)$$

A direct test is to reject for high values of the following statistic:

$$F_{kj} = n\left(\frac{S_j - S_k}{S_k}\right) \tag{8}$$

Where, $S_h, h = k, j$ are the sums of squared residuals (SSR) for each model, respectively. This is the traditional likelihood ratio test and it has an asymptotic $\chi^2(k)$ distribution under the assumptions of normality and independency in the error term. Under the traditional hypothesis testing framework, the SSR is calculated for

13 The key point noticed by Bai (1997) and Bai and Perron (1998) is that if the true model is a SETAR(3), but the SETAR(2) is estimated instead, the estimator of $\hat{\gamma}$ of the SETAR(2) will be a consistent estimator of one of the two thresholds parameters of the SETAR(3).

14 However, in this paper I follow a different strategy. I estimate the SETAR(3) considering all the possible combinations of thresholds parameters, and then I choose the one that minimizes the SSR. The main disadvantage of my strategy is that it is more time consuming and it will be extremely time consuming in the bootstrap simulation needed for the linearity test. However, it enables me to get more accurate estimations of the thresholds parameters.

the unrestricted and restricted version of the model. Under the restricted model, the parameters of interest are equal to the values of the alternative hypothesis because we want to asses if this assumption implies a statistically significant loss of likelihood; then the test is performed in the usual way. However, in this test, *j–k* threshold parameters do not take the zero value under the null hypothesis because they are not defined under this hypothesis. These parameters are called *nuisance parameters* and the testing procedure is non-standard and F_{kj} will not converge to a χ_k^2 distribution[15]. The main idea explained in Hansen (1996a) is to calculate the statistic of the test over a grid of values for the nuisance parameters, and then take the supremum or some average of the statistic.

Having this on mind, it is possible to derive a very simple linearity test. Recall that when *k* takes the unit value the SETAR(k) becomes an AR(*p*) model. Therefore, the linearity test is straightforward and it can be written using the following notation:

$$H_0 : Model\ \ AR(p)$$

$$H_1 : Model\ \ SETAR\left(j, p_1, ..., p_j\right)$$

Taking a closer look to both hypotheses it is possible to notice that, under H_0 the DGP has a linear representation which is the traditional AR(*p*) model. However, under H_1 has a nonlinear, specifically a SETAR representation. Under the alternative hypothesis there are: *j*–1, threshold parameters that are not going to be defined under the null.

Hansen (1996a) proposed the following Monte Carlo simulation that allowed calculating the "correct" asymptotic distribution of F_{kj}. Also, he showed that this distribution converges weakly in probability to the null distribution of F_{kj}.

1. Generate μ_t^i draws from a $N(0,1)$.
2. Define $y_t^{i*} = \mu_t^i$.
3. Estimate the specification under H_0 and H_1 using y_t^{i*} as dependent variable.
4. Compute the SSR of each model: $\left(S_k^{2i}, S_j^{2i}\right)$.
5. Compute the following statistic: $F_{kj}^{i*} = n\left(\dfrac{S_j^{2i} - S_k^{2i}}{S_k^{2i}}\right)$.

[15] Moreover, Hansen (1999) showed that the use of the χ_k^2 distribution may leads to false rejections of the null hypothesis, because this distribution concentrate a large amount of mass near the zero value, whereas the correct asymptotic distribution of F_{kj} does not necessarily do so.

6. Repeat (1) - (5), for $i = (1,2,\ldots, n)$.

7. The Monte Carlo P-value is obtained by counting the numbers of time that F_{kj}^{i*} is greater than F_{kj}.

Later, Hansen (1999) proposed the following bootstrap procedure that allowed calculating the empirical distribution of F_{kj}:

1. Estimate the model under H_0.

2. Save $\hat{\theta}$ and $\hat{\varepsilon}$.

3. Generate $\hat{\mu}^i$ by random draws of $\hat{\varepsilon}$ with resampling.

4. Using $\hat{\theta}$ and $\hat{\mu}^i$ of (2) and (3), generate y_t^{i*}.

5. Using this new data, estimate the model under H_0 and H_1.

6. Compute the SSR of each model: $\left(S_k^{2i}, S_j^{2i}\right)$.

7. Compute the following statistic: $F_{kj}^{i*} = n\left(\frac{S_j^{2i} - S_k^{2i}}{S_k^{2i}}\right)$.

8. Repeat (3) - (6), for $i = (1,2,\ldots, n)$.

9. The P-value may be obtained by counting the numbers of times that F_{kj}^{i*} is greater than F_{kj}.

Hansen tests (1996a, 1999) are simple procedures that allow for making of valid inference procedures over nuisance parameters (in this case, the threshold parameters). However, if conditional heteroscedasticity exists in the error term, these simulations could induce a false rejection of the null. Hansen (1999) proposed to re-escalate and normalize the residuals of the model in order to account for this possibility.

3. ECONOMETRIC RESULTS

This section presents a brief description of the data and the econometrics estimations of AR and SETAR models for each variable. Finally, the P-values of the linearity test performed over each DGP are reported.

3.1. The Data

Let us define π and π^* as the deviation of the 12 months variation of the CPI and CPIX1 against inflation target (IT) and *ur* as the seasonally adjusted (SA) unemployment rate. Using basic statistics it is possible to characterize each variable before doing any estimation. Table 1 presents a statistic summary for each variable[16].

TABLE 1

STATISTICS 1989M4-2009M12

	π	π^*	ur
Mean	0.331	0.150	8.178
Median	0.070	–0.351	7.979
SD	1.984	2.039	1.429
Min	–5.266	–4.060	5.738
Max	6.853	5.511	10.905
Kurtosis	1.729	–0.100	–1.253
Skewness	0.769	0.764	0.012
Jarque Bera	55.09	24.10	15.96
JB P-value	0.000	0.000	0.000

The summary statistic of π reveals that the mean is higher than the median, however both statistics are near the zero value, whereas the standard deviation is almost of 2%. The MIN and MAX deviations[17] from IT correspond to 11-2009 and 08-2008 respectively. The summary statistic for π^* also reveals that the mean is higher than the median, but with different signs. The standard deviation is also 2%. The MIN and MAX deviation from IT correspond to 12-2009 and 11-2008, respectively. Finally, the statistics for *ur* reveals that the mean is higher than the median, the standard deviation is almost 1.5% and the MIN and MAX correspond to 05-1998 and 09-1999, respectively.

The kurtosis[18] of π and π^* shows that they have a leptokurtic and mesokurtic distribution, respectively. Whereas *ur* has a platykurtic distribution (lower and wider peak around the mean with thinner tails.). The skewness is positive for the three variables which imply that the right tail has more mass than the left one. These statistics reveals that these variables show potential deviations from normality. This is confirmed by the Jarque-Bera (JB) statistic which is high for the three variables and therefore the null hypothesis of normality is rejected at any traditional significance level. Figure 2 shows the empirical distribution of each variable.

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16 The Jarque-Bera (JB) statistic is defined as:

$$JB = \frac{n}{6}\left(S^2 + (K-3)^2\right) \tag{9}$$

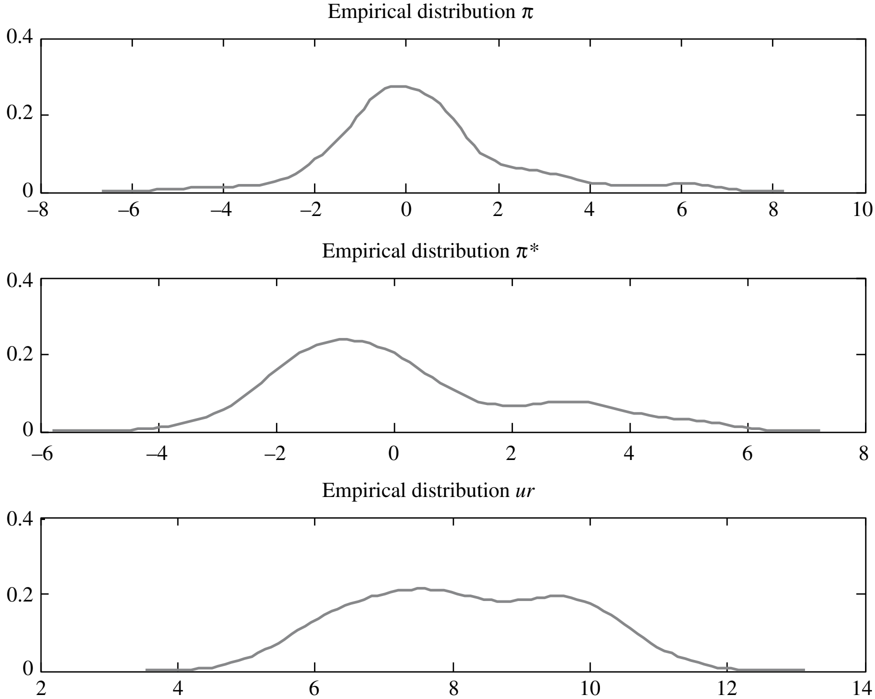
Where S and K are skewness and kurtosis, respectively. The JB statistic is used to test the null hypothesis of normality. It has an asymptotic χ^2 distribution with two degrees of freedom.

17 The MIN and MAX deviation of inflation (CPI and CPIX1) from IT refers to the period in which it was well below and above the target, respectively.

18 Where kurtosis refers to excess of kurtosis.

FIGURE 2

EMPIRICAL DISTRIBUTION (EPANECHNIKOV KERNEL)



Econometricians should be aware of these results when they are making inference over the coefficients to get the right test. Bootstrap simulations should be performed instead. Recall that, OLS estimators do not require any assumption regarding the distribution of the variable, neither do the simulations used in the linearity test. The results and estimations reported in the following sections should not be affected by these departures from normality.

3.2. Results with AR Models

The linear autoregressive model (AR) for each series is defined as:

$$\pi_t = \sum_{i=1}^{p} \beta_{i,\pi} \pi_{t-i} + \varepsilon_{t,\pi} \tag{10}$$

$$\pi_t^* = \sum_{i=1}^{p} \beta_{i,\pi^*} \pi_{t-i}^* + \varepsilon_{t,\pi^*} \tag{11}$$

$$ur = \beta_0 + \sum_{i=1}^{p} \beta_{i,ur} ur_{t-i} + \varepsilon_{t,ur} \tag{12}$$

First, in the AR specifications of π and π^* the constant term is suppressed for two reasons: it was not statistically different from zero when it was included in previous estimations and I assume that the unconditional mean of both series is 0%, meaning that the Central Bank will achieve the inflation goal of 3% in the long run or that inflation will converge to the IT. Finally, the lag order for each specification is chosen according to the *Bayesian Information Criterion* (BIC) which is reported in Table 1 of the appendix section. According to these results, two lags are selected for π and π^* whereas five lags are chosen for *ur*. Once the lag order of each model is chosen, the estimation is performed through OLS. Table 2 reports the estimations along with the standard error (S.E.) in parentheses[19].

TABLE 2

OLS ESTIMATORS OF AR(p) MODELS

Coef.	π	π^*	*ur*
β_0			0.171 (0.065)
β_1	1.155 (0.086)	1.366 (0.070)	1.538 (0.083)
β_2	–0.223 (0.085)	–0.414 (0.071)	–0.411 (0.136)
β_3			–0.450 (0.093)
β_4			0.575 (0.096)
β_5			–0.273 (0.062)
β_6			
T	247	247	244
σ^2	0.447	0.236	0.029

19 Newey West standard errors.

Table 2 shows that all parameters are statically different from zero at the 5%. The unconditional means for π and π^* are zero and 8.3% for *ur*. The following section presents the estimation results for nonlinear models and a comparison between the fit and the residuals between the linear and nonlinear specifications.

3.3. Results with SETAR Models

Using the lag order selected for each variable in the previous section, I estimate two SETAR models for each variable (with one and two thresholds). The estimation is performed following the procedure outlined early. Results are reported in Tables 3 through 5 for π, π^* and *ur* respectively.

TABLE 3

DEVIATIONS OF INFLATION (CPI) w.r. INFLATION TARGET

	SETAR(2)		SETAR(3)		
	$y_{t-1} \leq 1.92$	$y_{t-1} > 1.92$	$y_{t-1} \leq 0.81$	$0.81 < y_{t-1} \leq 1.92$	$y_{t-1} > 1.92$
β_1	1.189 (0.109)	1.046 (0.128)	1.122 (0.116)	1.483 (0.218)	1.046 (0.128)
β_2	–0.264 (0.099)	–0.106 (0.136)	–0.179 (0.100)	–0.646 (0.193)	–0.106 (0.136)
T	207	40	170	37	40
σ^2	0.390	0.760	0.305	0.743	0.760

Newey West standard errors.

TABLE 4

DEVIATIONS OF INFLATION CORE (CPIX1) w.r. INFLATION TARGET

	SETAR(2)		SETAR(3)		
	$y_{t-2} \leq 0.43$	$y_{t-2} > 0.43$	$y_{t-2} \leq 0.43$	$0.43 < y_{t-2} \leq 2.51$	$y_{t-2} > 2.51$
β_1	1.239 (0.079)	1.557 (0.114)	1.239 (0.079)	1.775 (0.095)	1.215 (0.195)
β_2	–0.321 (0.071)	–0.589 (0.118)	–0.321 (0.071)	–0.839 (0.106)	–0.252 (0.195)
T	163	84	163	42	42
σ^2	0.209	0.271	0.209	0.215	0.289

Newey West standard errors.

TABLE 5

UNEMPLOYMENT RATE, SEASONAL ADJUSTED (*ur*)

	SETAR(2)		SETAR(3)		
	$y_{t-3} \leq 9.39$	$y_{t-3} > 9.39$	$y_{t-3} \leq 7.03$	$7.03 < y_{t-3} \leq 9.39$	$y_{t-3} > 9.39$
β_1	0.215 (0.092)	3.450 (0.666)	0.763 (0.329)	0.296 (0.145)	3.450 (0.666)
β_2	1.709 (0.079)	0.926 (0.093)	2.000 (0.124)	1.585 (0.081)	0.926 (0.093)
β_3	–0.675 (0.152)	0.005 (0.117)	–1.441 (0.266)	–0.372 (0.121)	0.005 (0.117)
β_4	–0.378 (0.138)	–0.303 (0.107)	0.145 (0.241)	–0.526 (0.133)	–0.303 (0.107)
β_5	0.648 (0.129)	0.323 (0.103)	0.415 (0.266)	0.562 (0.154)	0.323 (0.103)
β_6	–0.332 (0.071)	–0.303 (0.087)	–0.236 (0.152)	–0.286 (0.076)	–0.303 (0.087)
T	170	74	66	104	74
σ^2	0.025	0.024	0.026	0.022	0.024

Newey West standard errors.

Table 3 shows that the parameters estimates of (γ, d) of π are: (1.92;1) and (0.81,1.92;1) for the SETAR(2) and (3), respectively. The previous lag of π defines today's regime. Each of these specifications contains a regime of almost 80% of the total data available and these regimes are the ones that are closer to zero (the unconditional mean of π and the main inflationary goal of the Central Bank of Chile). Table 4 shows similar results for π^*. The parameters estimates for (γ, d) are: (0.43;2) and (0.43,2.51;1) for the SETAR(2) and (3), respectively. The second lag of π^* defines today's regime. The reasons for the difference between the delay parameter of π and π^* are not the focus of this investigation, but further research should be conducted in order to analyze this result. Finally, Table 5 reports that the estimates of (γ, d) for *ur* are: (9.39;3) and (7.03,9.39;3) for the SETAR(2) and (3). The third lag of *ur* defines today's regime.

The signs and magnitudes of the coefficients estimated for π and π^* are very similar between the OLS and SETAR specifications. The parameters β_1 and β_2 between the specifications are alike except in the second regime of the SETAR(3). But this is not the case for *ur*, Table 5 shows that the signs and magnitudes of the parameters differ substantially between the OLS and SETAR specification[20].

I finish by showing Figure 3 that shows the fit and threshold estimates of the SETAR models and Figure 4 the OLS and SETAR residuals of each model, respectively.

[20] I am not interested to test if these differences are statistically significant or not. My main point is to test if the functional form of the model is linear or not.

FIGURE 3

FITTING AND THRESHOLDS ESTIMATES FOR SETAR MODELS

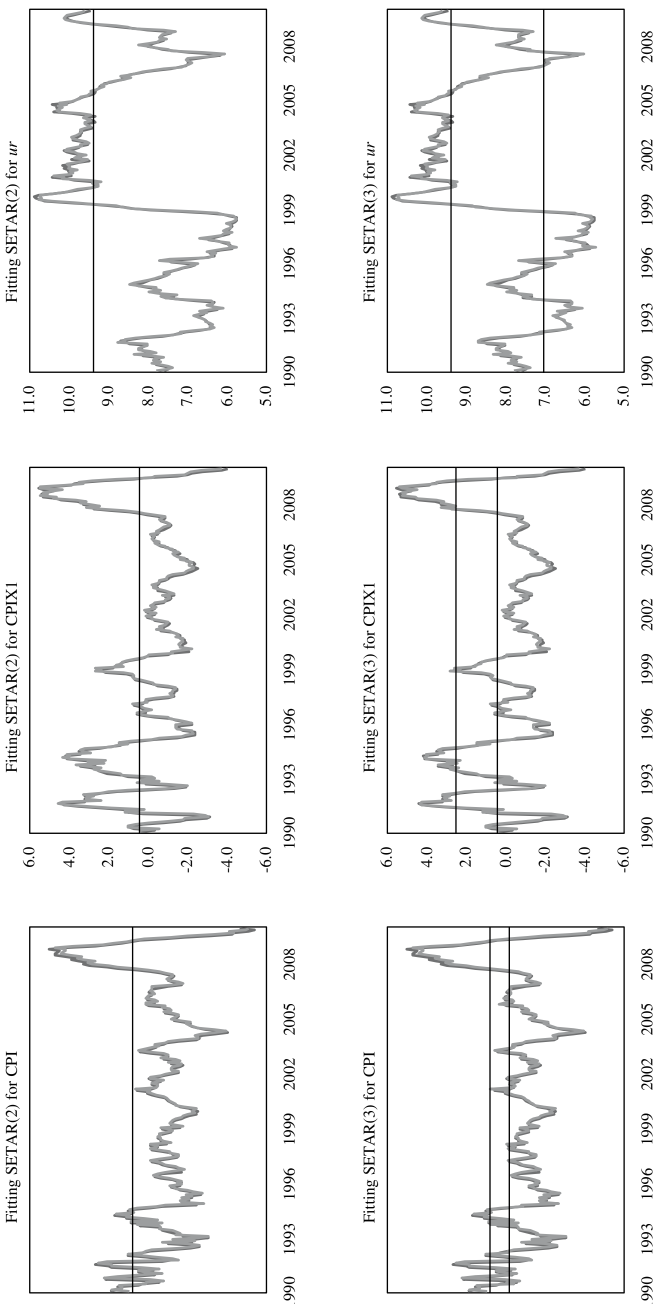


FIGURE 4

OLS AND SETAR RESIDUALS

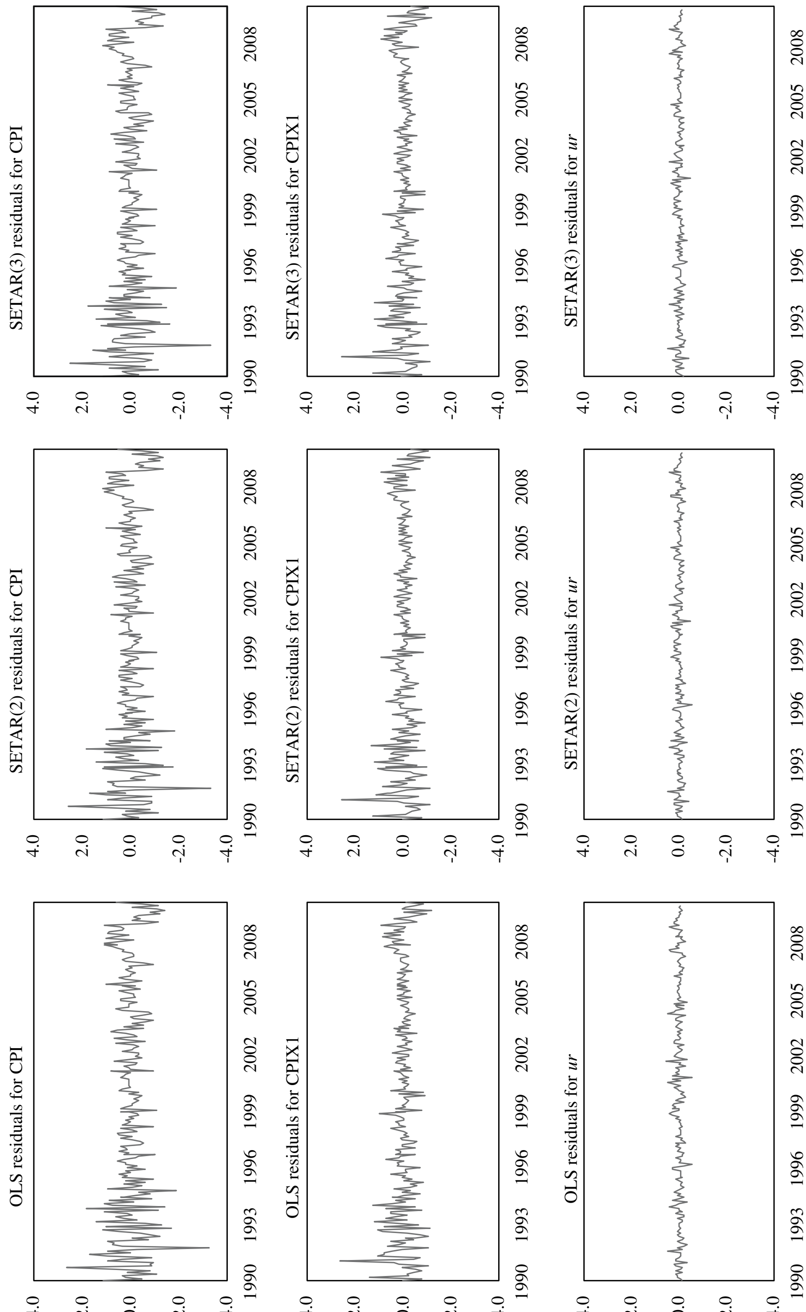


Figure 3 shows that that all variables tend to be under the threshold in the SETAR(2) model, this is especially true for *ur*. But considering the SETAR(3), it is interesting to notice that the regime between the threshold parameters does not contain an important fraction of the data. In fact, it acts more like a transition regime. Finally, Figure 4 reveals that the residuals from each (linear and nonlinear) specification are quite similar.

3.4. Linearity Test

SETAR models will always provide a better fit to the data than AR models because they have more parameters to adjust in order to reduce the SSR of the model. However this does not imply a statistically significant improvement of the fit. A test between these two models needs to be performed in order to answer this question and decide which one is better to characterize the dynamics of these variables over the last twenty years. I do this with a linearity test. Under the null hypothesis the DGP has an AR(p) representation and under the alternative a SETAR representation[21].

I perform two linearity tests; however the null hypothesis is the same in both of them. Under the null hypothesis the DGP has an AR(p) representation. In the first test, the alternative is a SETAR(2) whereas in the second one it is a SETAR(3). This last one is conducted in order to discard or validate a more complex form of nonlinearity. I do not perform the test to decide between a SETAR(2) and a SETAR(3) model because I am interested in the linearity hypothesis and not to identify the exact form of nonlinearity. Both tests may be written in short form as:

$$H_0 : Model \;\; AR(p)$$

$$H_{1a} : Model \;\; SETAR(2p, p)$$

$$H_{1b} : Model \;\; SETAR(3p, p, p)$$

The results are reported in Tables 2 and 3 of the appendix section. The F_{kj} statistic is constructed following the procedures explained in the methodological section. Also, three P-values are reported. The first one is the p-value associated with the χ_k^2 distribution. The second and the third ones are the associated with the "correct" asymptotic distribution (Hansen 1996a) and the empirical distribution (Hansen 1999) of F_{kj}. This enables me to assess whether the null hypothesis is rejected or not and

[21] Even if the null is rejected, it is not clear that SETAR models will provide a more accurate forecast than traditional AR(p) models, because the test procedure is an in-sample rather than an out-sample exercise. This is an open agenda that has not been fully developed. Chumacero (2004) presents interesting results for two Chilean variables.

what are the possible implications of using the χ_k^2 distribution. The main results for the first test are the following:

1. It is not possible to reject the linearity hypothesis for π. The asymptotic and bootstrap P-values are greater than 5%.
2. For π* the evidence is not so clear. The null hypothesis could be rejected at 10%, but it is not rejected at the traditional 5% level.
3. The linearity hypothesis over *ur* is rejected at any traditional significance level. The asymptotic and bootstrap P-values are near the zero value.
4. Finally, the use of the χ^2 distribution, may lead to a false rejection of the null for π*.

Finally, the main results for the second test are the following:

1. It is not possible to reject the linearity hypothesis for π. The asymptotic and bootstrap P-values are greater than 5%.
2. Once again, the evidence for π* is not so clear. The null hypothesis could be rejected at 10% level. But it is not rejected at the traditional 5% level.
3. The linearity hypothesis over *ur* is rejected at any traditional significance level. The asymptotic and bootstrap P-values are near the zero value.
4. Finally, the use of the χ^2 distribution may lead to a false rejection of the null for π* at the 5% level for all variables.

The bottom line of these results is that the linearity hypothesis is not rejected for π and π*. The asymptotic and bootstrap P-values are greater than 5%. However, if a 10% size is assumed, then it could be rejected for π*. These tests provide statistical evidence that confirms the idea that the dynamics of the Chilean inflation during these last twenty years has been smooth without any change in the velocity at which inflation moves toward the target. Finally, these results clearly show that the linearity hypothesis can not hold for *ur*. The evidence against linearity in the DGP of the unemployment rate is very strong. The null is rejected at any traditional significance level.

4. CONCLUSIONS AND FINAL REMARKS

This paper tests the hypothesis of linearity of inflation (CPI and CPIX1) and unemployment rate in Chile. Under the null hypothesis, the series has an AR(p) representation, whereas under the alternative has a SETAR specification. The main results are the following: it is not possible to reject linearity in the DGP of inflation using the CPI but it could be rejected for CPIX1 inflation at the 10% level, especially when a more complex form of nonlinearity such as SETAR(3) is considered. However, using the traditional standard level of 5% the null is not rejected. The convergence of inflation towards IT has been constant or smooth during the last twenty years without switching regime. There is no doubt that the Central Bank has played a key role in this process, or at least it has not done anything to cause an opportunistic change in the speed of adjustment. The recent boom/bust of inflation should not be considered as evidence of a different degree of persistence in the inflation process, but rather as a sequence of several shocks randomly drawn from the distribution of innovations. However, a more complex form of nonlinearity could exist that could validate the hypothesis of different degree of persistence in the inflation process. Finally, as expected, I find strong evidence against unemployment linearity. The unemployment rate has at least two different regimes. It fluctuates with high probability between states or regimes through time. Further research should be conducted in order to analyze the implications of adding nonlinearities to traditional unemployment models used to forecast this variable.

Some considerations are needed to have a better understanding of these results. First, the key assumption is that the error term of each model must be *iid*. If conditional heteroscedasticity exists, it could induce a false rejection of the null hypothesis. This is not a problem for π and π^*, because the null was not rejected. However, the null was rejected for *ur* but no ARCH/GARCH component was detected in its residual. Second, all regime switching and structural break models are valid only in the sample used. These results could fail if a different sample is used. I tried to avoid this by using the last twenty years of economic data; however the problem of choosing this particular sample still applies. A cautious econometrician could always find the correct sample to reject the null. Finally, the rejection of the null does not necessarily imply that the true DGP of the variable is a SETAR(2)/(3), it only concludes that the DGP could be better represented as a nonlinear process.

A possible application of this methodology consists to test if the Central Bank follows a linear or nonlinear Taylor Rule. There is no prior reason to expect that it should be linear especially when the Monetary Policy Rate (TPM) has a lower bound. The Central Bank could act more aggressively in specials circumstances like in the recent financial crisis, where the policy rate was lowered very quickly until reaching the lower bound. After remaining flat for several months, it started a period of monetary normalization that continued until the mid of 2011, with very different movements (in absolute value) from the ones observed during the financial crisis. This could suggest an asymmetric behavior, in the sense that the monetary authority is more willing to decrease its policy rate quickly in special circumstances, whereas it prefers to make small increases when it comes to the convergence towards the natural policy rate from below[22].

[22] The evidence of this paper can not be used to argue against or in favor, that the Central Bank of Chile has a nonlinear Taylor Rule.

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APPENDIX

TABLE 1

BAYESIAN INFORMATION CRITERION (BIC)

AR(p)	π	π^*	ur
0			3,593
1	2.075	1.546	–0.188
2	2.053	1.351	–0.527
3	2.075	1.357	–0.518
4	2.084	1.373	–0.525
5	2.095	1.391	–0.582
6	2.061	1.381	–0.559

TABLE 2

AR(p) AGAINST SETAR(2,p) TEST

	F_{kj}	χ^2	Asymptotic	Bootstrap
π	1.2	0.537	0.960	0.979
π^*	8.7	0.013	0.100	0.091
ur	50.2	0.000	0.000	0.000

TABLE 3

AR(p) AGAINST SETAR(3,p) TEST

	F_{kj}	χ^2	Asymptotic	Bootstrap
π	7.9	0.019	0.815	0.810
π^*	18.6	0.000	0.075	0.064
ur	74.3	0.000	0.000	0.000