CONVERGENCE AND LONG-RUN UNCERTAINTY*

CONVERGENCIA Y CRECIMIENTO DE LARGO PLAZO

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Abstract

In this paper the neoclassical convergence hypothesis is tested for the thirteen regions of Chile using cross-section techniques and the time-series based tests proposed by Bernard, A. and S. Durlauf, 1995, "Convergence in International Output", Journal of Applied Econometrics *10 (2), pp. 97-108. Cross-section analysis in combination with a Bayesian Modeling Averaging strategy supports the convergence hypothesis, despite of some instability detected in the estimated speed of convergence. When applying time-series based tests, the no convergence null hypothesis cannot be rejected at the usual significance levels. When clustering the Chilean regions into three different groups, however, evidence of cointegration within these groups is found, indicating that the regional growth process in Chile is driven by a lower number of common trends.*

Keywords: *Convergence hypothesis, economic growth, Bayesian model averaging, cointegration, Chile.*

JEL Classification: *C11, C32, O47.*

Resumen

En este trabajo se prueba la hipótesis de convergencia neoclásica con las trece regiones de Chile utilizando las técnicas de corte transversal de Bernard, A. and S. Durlauf (1995). "Convergence in International Output", Journal of Applied Econometrics *10 (2), pp. 97-108. El uso de corte transversal combinado con el promedio bayesiano de modelos apoya la hipótesis de convergencia, a pesar de cierta inestabilidad en la velocidad de convergencia. Al aplicar pruebas de series de tiempo, la hipótesis nula de no convergencia no es rechazada a niveles usuales de significancia estadística. Al agrupar las regiones de Chile en tres zonas, sin embargo, se encuentra evidencia de cointegración entre las tres zonas, sugiriendo que el crecimiento regional de Chile se debe a un bajo número de tendencias comunes.*

Palabras clave: *Hipótesis de convergencia, crecimiento económico, promedio bayesiano, cointegración, Chile.*

Clasificación JEL: *C11, C32, O47.*

1. INTRODUCTION

The economic notion of convergence refers to the role that initial conditions play in explaining the asymptotic behavior of output within a group of economies. In other words, convergence occurs in a specific set of economies when the long-run behavior of growth rates does not depend on economies' initial levels of capital. This property is a particular implication of the neoclassical growth model, which predicts convergence of the economies toward a stationationary distribution. According to this neoclassical framework, any observed difference in output per capita across economies sharing the same microeconomic fundamentals should vanish in the long term.

New theories of growth have questioned the convergence hypothesis theoretically and empirically. In terms of theory, the presence of nonconvexities in the production function has been shown to generate multiple equilibria and multiple steady states. In terms of empirics, new-growth theories emphasize that the output per capita gap between first and third-world economies has not decreased. Therefore, there is no strong evidence that poorer economies grow faster and catch up to richer ones. The different implications of the neoclassical and new theories of growth have led to a literature aimed at testing the convergence hypothesis.

This paper can be placed within the context of that literature as having both an empirical and a methodological objective. First, the present paper is aimed at testing the convergence hypothesis for the thirteen Chilean regions over the last four decades. To do that, definitions and tools from Barro and Sala-i-Martin (1995) and Bernard and Durlauf (1995, 1996) are used. Second, this paper proposes the use of a Bayesian Model Averaging (BMA) framework to control for some instability detected in the

estimated speed of convergence. Finally some comments and extensions are suggested, specially in the direction of explaining the conflicting results between cross-section and time-series based tests.

Cross-section and time-series tests of the convergence hypothesis for the thirteen Chilean regions have been reported in previous papers. Vernon (2002) summarizes the literature testing the convergence hypothesis using a cross-section approach for the Chilean regions. These results reject the null of no convergence at usual significance levels. Furthermore, they suggest that convergence within Chilean regions depends upon levels of mining activity within the regions.

Oyarzún and Araya (2001) test the convergence hypothesis using time-series techniques and the definition of convergence in output provided by Bernard and Durlauf (1995). The authors follow a univariate methodology to test the null hypothesis of no stationarity in the log difference of per capita regional GDP. Results indicate that the null hypothesis of no convergence cannot be rejected, but that three distinctive groups of regions exist. There are two groups that diverge from each other but where regions within them converge. There is also a third group composed of regions that do not converge with any other region of the country. The same basic conclusion is obtained when the test is performed by controlling for structural breaks. In this case the composition of the three groups is modified but the main conclusion still holds. In summary, the null of no convergence cannot be rejected at usual significance levels using time-series techniques.

While the two main conclusions about convergence by Vernon (2002) and by Oyarzún and Araya (2001) are confirmed in this paper, there are some relevant differences between them. First, this paper attempts to highlight sources of uncertainty that are found and disregarded in the aforementioned convergence analyses. Second, when using time-series techniques to identify convergence groups within Chilean regions, a multivariate approach is adopted to avoid conflicting results in determining cointegration relationships within groups.

A final point is worth mentioning. Thus far, convergence tests have been presented as tools to test the neoclassical growth theory. However, when testing convergence within different regions of the same country, convergence results may also have policy implications. In fact, a successful growth process for a whole economy might be changing the income distribution across regions and therefore making the gap between the richest regions and the poorest regions either larger or smaller. These implications may lead to policy changes according to the beliefs and utility functions of the government or policy makers.

2. DATA

Regional econometrics in Chile faces the challenge of data availability. From a cross-section perspective Chile is divided into thirteen different regions, and these regions are the smallest unit for which GDP data are available. Such a small number of economies may be a problem when thinking about asymptotic properties of econometric estimators. From a time-series perspective, data limitation stems from

the the fact that the current division of Chile into thirteen regions started in the mid-1970s. Therefore, less than 30 years of per capita GDP data are directly available for each region. However, population and GDP regional data for the period 1960-1979 are constructed based upon the previous division of Chile. These data were kindly provided by the National Institute of Statistics. GDP series for the 1960-1979 period are estimations and are assumed to be consistent with the Central Bank 1980-1998 figures. As a result, this paper uses annual real GDP per capita from 1960 to 1998 for each of the thirteen Chilean regions.

Chile has a particular geographic structure: it is long and narrow with regions distributed almost uniformly from north to south (see map in the Appendix). Region 1 is the furthest north, Region 12 is furthest south and Region 13 corresponds to the Metropolitan Region of Santiago in the middle of the country. Three geographical groups for cointegration are considered: The North comprises Region 1, Region 2 and Region 3; the Central comprises Regions 4, 5, 6, 7, 8, 11 and Metropolitan Region (hereafter, MR); and the South comprises Regions 9, 10 and 12.

3. CROSS-SECTION ANALYSIS

As previously discussed, the economic notion of convergence refers to the role that initial conditions play in explaining the asymptotic behavior of output within a group of economies. Convergence occurs in a specific set of economies when the long run behavior of their growth rates does not depend on their initial levels of capital. This notion is clearly stated by Durlauf (2003) when he defines convergence as the condition

$$\lim_{k \to \infty} \mu(g_{i,t+k} \mid S_{i,t}, \theta, \rho) \text{ does not depend on } S_{i,t},$$

where $g_{i,t}$ denotes the growth rate of output per capita in economy i at time t, $S_{i,t}$, denotes human and physical capital in economy i at time t, θ denotes technology, ρ preferences and μ is a probability measure.

In this section, the convergence prediction of the neoclassical growth model is tested within all the thirteen Chilean regions. Because the analysis is made over regions of the same country instead of different countries, some considerations regarding the neoclassical implication of convergence are worth mentioning. First, although regions may have differences in tastes and technology, these differences are possibly smaller than those across countries. This is because a country's policymakers usually share the same culture, language and education. This, in turn, creates similar preferences. Moreover, regions within a single country share the same political framework and have the same, or at least similar, institutional and legal systems. Hence, the homogeneity assumption that usually is made for cross-country analysis is more likely to hold true across regions of the same country. Consequently, convergence is more likely to hold true within regions than across countries. Regardless of the homogeneity argument given above, there is an important consideration in regional analysis that at first glance

could potentially undermine convergence results. When modeling regions within a country, the closed-economy assumption for these units is not likely to hold true. Indeed, according to Barro and Sala-i-Martin (1995), mobility of production factors tends to be higher across regions than across countries.

However, Barro and Sala-i-Martin (1995) also show that economies that are open to capital inflows have similar dynamic properties to those of closed economies as long as a fraction of the stock of capital is fixed. While the speed of convergence is higher in the presence of capital mobility, this speed varies in a small range for usual sizes of the fraction of capital that is not fixed. The same authors show that migration also tends to increase the speed of convergence. Therefore, regardless of the fact that regions within a nation are relatively open economies in terms of capital and labor mobility, the neoclassical implication of convergence still holds.

3.1. Convergence Within Homogeneous Regions

The following equation summarizes the neoclassical growth model's transitional dynamic:

$$\gamma_{\hat{k}} = sf(\hat{k}) / \hat{k} - (x + n + \delta) \tag{1}$$

where $\hat{k}$ is the quantity of capital per unit of effective labor defined as $\hat{k} = K / [A(t) \cdot L]$, where, in turn, K is the total stock of capital in the economy, L is the size of the labor force, and $A(t)$ represents the level of technology at time t. Labor is assumed to evolve with the population at a constant rate n, similarly, technology also evolves at a constant rate x, while s and δ are assumed to be constant. They represent the saving rate and the depreciation rate of the economy respectively. Finally, f is defined as $f(k) = F(\hat{k}, 1)$ where F denotes a neoclassical production function with labor augmenting technological progress[1], and $\gamma_{\hat{k}}$ represents the growth rate of $\hat{k}$.

Assuming a Cobb-Douglas production function, $F(K, A(t) \cdot L) = K^{\alpha}[A(t) \cdot L]^{1-\alpha}$, equation (1) becomes:

$$\gamma_{\hat{k}} = s\hat{k}^{-(1-\alpha)} - (x + n + \delta), \tag{2}$$

and the steady-state level of capital per unit of effective labor, $\hat{k}^*$, is given by the condition $\gamma_{\hat{k}}(\hat{k}^*) = 0$. Solving for $\hat{k}^*$ yields the following expression:

$$\hat{k}^{* -(1-\alpha)} = (x + n + \delta)s^{-1}, \tag{3}$$

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[1] A neoclassical production function with labor augmenting technological progress is modeled as $F(K, A(t) \cdot L)$.

which finally yields

$$\hat{k}^* = \left(\frac{s}{x+n+\delta}\right)^{\frac{1}{1-\alpha}}. \tag{4}$$

A first order Taylor expansion around $\hat{k}^*$ for both functions $\gamma_{\hat{k}}$ and $\log(\hat{k})$ yields the following two equalities:

$$\gamma_{\hat{k}}(\hat{k}) \cong \gamma_{\hat{k}}(\hat{k}^*) + \gamma'(\hat{k}^*)(\hat{k}-\hat{k}^*), \tag{5}$$

$$\log(\hat{k}) \cong \log(\hat{k}^*) + \frac{\hat{k}-\hat{k}^*}{\hat{k}^*}. \tag{6}$$

Using the fact that $\gamma_{\hat{k}}(\hat{k}^*) = 0$ and substituting $\hat{k}-\hat{k}^*$ from (6) in (5); the following expression is obtained

$$\gamma_{\hat{k}}(\hat{k}) \cong \hat{k}^* \gamma'(\hat{k}^*) \log(\frac{\hat{k}}{\hat{k}^*}).$$

From (2) it is possible to compute

$$\gamma'(\hat{k}^*) = -(1-\alpha)s\hat{k}^{*-(1-\alpha)-1}$$
$$= -(1-\alpha)s(x+n+\delta)s^{-1}\hat{k}^{*-1}$$
$$= -(1-\alpha)(x+n+\delta)\hat{k}^{*-1}.$$

Therefore, the growth rate of capital per unit of effective labor can be approximated as follows

$$\gamma_{\hat{k}}(\hat{k}) \cong \lambda \log(\frac{\hat{k}}{\hat{k}^*}), \tag{7}$$

where $\lambda = (1-a)(x+n+\delta)$ is called the speed of convergence in the steady state or just the speed of convergence.

Denoting $\hat{y} = f(\hat{k}) = \hat{k}^\alpha$, it is possible to derive similar expressions for the output per effective units of labor. In fact,

$$\gamma_{\hat{y}} = \frac{d\log(\hat{y})}{dt} = \alpha \frac{d[\log(\hat{k})]}{dt} = \alpha\gamma_{\hat{k}},$$

likewise,

$$\log(\frac{\hat{y}}{\hat{y}^*}) = \alpha \log(\frac{\hat{k}}{\hat{k}^*}).$$

Therefore, equation (7) is also true for the output per effective units of labor

$$\gamma_{\hat{y}}(\hat{y}) \equiv \frac{d\log(\hat{y})}{dt} \cong -\lambda \log\left(\frac{\hat{y}}{\hat{y}^*}\right). \tag{8}$$

Equation (8) plus the initial condition $\hat{y}(t_0) = \hat{y}_0$ is a differential equation in $\log(\hat{y}(t))$. The solution may be expressed as:

$$\log(\hat{y}(t)) = (1 - e^{-\lambda(t-t_0)})\log(\hat{y}^*) + e^{-\lambda(t-t_0)}\log(\hat{y}(0)). \tag{9}$$

Equation (9) involves the unobservable level of technology. Recalling that $\log(\hat{y}(t)) = \log(y(t)) - \log(A(t))$, where $y(t) = Y(t)/L(t)$ represents the observable variable output per capita, it is possible to rewrite (9) as:

$$\log(y(t)) - \log(A(t)) = (1 - e^{-\lambda(t-t_0)})\log(\hat{y}^*)$$
$$+ e^{-\lambda(t-t_0)}\log(y(t_0)) - e^{-\lambda(t-t_0)}\log(A(t_0)). \tag{10}$$

Adding $-\log(y(t_0))$ from both sides of this expression, and then adding $\log(A(t_0)) - \log(A(t_0))$ from the right-hand side yields:

$$\log(y(t)) - \log(y(t_0)) = \log(A(t)) - \log(A(t_0)) + (1 - e^{-\lambda(t-t_0)})\log(\hat{y}^*) +$$
$$(1 - e^{-\lambda(t-t_0)})\log(A(t_0)) - (1 - e^{-\lambda(t-t_0)})\log(y(t_0)). \tag{11}$$

Dividing by $(t-t_0)$ and recalling that $g_{t_0,t}$, the average output per capita growth rate in period $[t_0,t]$, can be approximated as:

$$g_{t_0,t} = \frac{\log(y(t)) - \log(y(t_0))}{t-t_0},$$

equation (11) becomes:

$$g_{t_0,t} = x + \frac{(1 - e^{-\lambda(t-t_0)})}{t-t_0}\log(\hat{y}^*) - \frac{(1 - e^{-\lambda(t-t_0)})}{t-t_0}$$
$$\log(y(0)) + \frac{(1 - e^{-\lambda(t-t_0)})}{t-t_0}\log(A(0)),$$

or equivalently,

$$g_{t_0,t} = a_{t_0,t} - \frac{(1-e^{-\lambda(t-t_0)})}{t-t_0} \log(y(0)), \tag{12}$$

where:

$$a_{t_0,t} = x + \frac{(1-e^{-\lambda(t-t_0)})}{t-t_0} \log(\hat{y}^*) + \frac{(1-e^{-\lambda(t-t_0)})}{t-t_0} \log(A(0)).$$

Note that the derivation of equation (12) could alternatively be done using the Ramsey model instead of the Solow version of the neoclassical growth model.

When testing the neoclassical growth model's convergence implication, equation (12) is commonly augmented with an error term. Basically, the empirical literature tests the convergence hypothesis using the following regression:

$$g_{t_0,t}^i = \alpha_{t_0,t} + \beta_{t_0,t} y_0^i + \varepsilon_{t_0,t}^i, \tag{13}$$

where t is a fixed period of time, t_0 is a fixed starting point, $g_{t_0,t}^i$, represents the average growth rate for each economy i in the period under analysis, y_0^i is the log of the initial output per capita of economy i and, $E[\varepsilon_{t_0,t}^i | I_0] = 0$.

Under this framework, convergence is associated with a negative $\beta_{t_0,t}$ coefficient, treating $\beta_{t_0,t} \geq 0$ as the no convergence null hypothesis.

Alternatively, it is possible to rewrite equation (12) as follows:

$$g_{t_0,t}^i = a_{t_0,t} - y_0^i \left[\frac{1-e^{-\lambda(t-t_0)}}{t-t_0} \right] + \varepsilon_{t_0,t}^i, \tag{14}$$

where in this case convergence is associated with a positive λ coefficient, which is the already mentioned speed of convergence. Notice that the $\beta_{t_0,t}$ parameter in equation (13) corresponds to the $-(1-e^{-\lambda(t-t_0)})/(t-t_0)$ term in equation (14). Therefore, while λ is a parameter independent of time[2], $\beta_{t_0,t}$ changes with the length of the period of analysis and tends to zero as $T = t - t_0$ approaches to infinity, as long as $\lambda \geq 0$.

To test for convergence within the Chilean Regions both regressions (13) and (14) were carried out for several subsamples of the whole time span 1960-1998. Tables 1 and 2 display these results.

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[2]   Recall that $\lambda = (1-\alpha)(x+n+\delta)$, so the speed of convergence in steady state is constant and independent of time.

TABLE 1

TEST OF ABSOLUTE β-CONVERGENCE

	1960-1972		1973-1985		1986-1998		1960-1985		1973-1998		1960-1998	
	α^*	β^*	α^*	β	α^*	β	α^*	β^*	α^*	β^*	α^*	β^*
Coefficient	0.077	-0.012	0.061	-0.010	0.123	-0.014	0.059	-0.009	0.099	-0.013	0.082	-0.011
Std. deviation	0.025	0.005	0.036	0.007	0.069	0.012	0.016	0.003	0.038	0.007	0.022	0.004
R^2	0.349		0.167		0.098		0.476		0.248		0.407	

Equation: $g_{i,T} = \alpha + \beta y_{i,0} + \varepsilon_{i,T}$.
[x]*: Significant at 5% level.

TABLE 2

TEST OF ABSOLUTE β-CONVERGENCE USING NLLS

	1960-1972		1973-1985		1986-1998		1960-1985		1973-1998		1960-1998	
	α^*	λ^*	α	λ	α	λ	α^*	λ^*	α^*	λ	α^*	λ^*
Coefficient	0.077	0.012	0.061	0.011	0.123	0.015	0.059	0.011	0.099	0.016	0.082	0.015
Std. deviation	0.025	0.006	0.036	0.008	0.069	0.015	0.016	0.004	0.038	0.010	0.022	0.007
R^2	0.349		0.167		0.098		0.476		0.248		0.407	

Equation: $(1/T)\log(y_{i,t}/y_{i,t-T}) = \alpha - [(1-e^{-\lambda T})/T]\log(y_{i,t-T}) + \varepsilon_{i,T}$.
[x]*: Significant at 5% level.

It can be seen from Tables 1 and 2 that when testing convergence using the whole period (1960-1998), the null hypothesis of no convergence is rejected as the β parameter is negative and significant. The associated speed of convergence of 1.5% is shown in the last column of Table 2 and is consistent with the intra-region estimates found by Barro and Sala-i-Martin (1995) for the US, by Canova and Marcet (1995) for European regions and by Easterly *et al.* (1996) for developing countries.

It is interesting to mention that the speed of convergence is discouraging for policy purposes because with an estimated parameter of 1.5%, closing 50% of the gap between the richest and the poorest regions[3] will take over 28 years. Another way to support this argument is to compute the half-life of the process, that is, the time t for which $\log(y(t))$ is halfway between $\log(y(0))$ and $\log(y^*)$. With an estimate for λ of 1.5% the half-life is 46.2 years.

When partitioning the whole period into three parts (1960-1972; 1973-1985; 1986-1998) evidence of convergence is only significant for the first group. The numerical estimates of the β parameter, however, are quite similar and negative. Furthermore, tests of stability for the β parameter cannot reject the null hypothesis of β being the same across all three periods, as shown in Table 3.

TABLE 3

TEST OF STABILITY OF THE β PARAMETER

Restriction	Test F
$\beta_1 = \beta_2$	0.09
$\beta_2 = \beta_3$	0.07
$\beta_3 = \beta_4$	0.13
$\beta_1 = \beta_5$	0.04

5% Critical value: 4.3.
β_i: β estimated in column i of Table 1 and 2.

Therefore, the convergence hypothesis still seems supported by the data. The analysis could end here and the conclusion of convergence would be relatively strong. However, when periods 1975-1985 and 1992-1998 are analyzed, the convergence hypothesis is challenged. Table 4 shows the results of these estimations. In both subsamples the null of no convergence cannot be rejected, as the numerical estimates of the β parameter are positive. Furthermore, the stability test rejects the hypothesis of stability of the β parameter between periods 1975-1985 and 1986-1998[4].

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[3]   Regions 2 and 9 in 1998.
[4]   *F*-stastistic: 10.91.

TABLE 4

EXCEPTIONS IN TESTING FOR UNCONDITIONAL β-CONVERGENCE

|	1975-1985		1992-1998	
	α	β	α	β
Coefficient	−0.003	0.003	0.033	0.003
Std. deviation	0.028	0.005	0.103	0.018
R^2	0.036		0.002	

Equation: $g_{i,T} = \alpha + \beta y_{i,0} + \varepsilon_{i,T.}$
[x]*: Significant at 5% level.

The instability of the β and λ parameters can also be seen by estimating both parameters for all possible subsamples of the data. Figures 1 and 2 summarize these estimations using frequencies histograms for both parameters.

FIGURE 1

HISTOGRAM OF β COEFFICIENTS
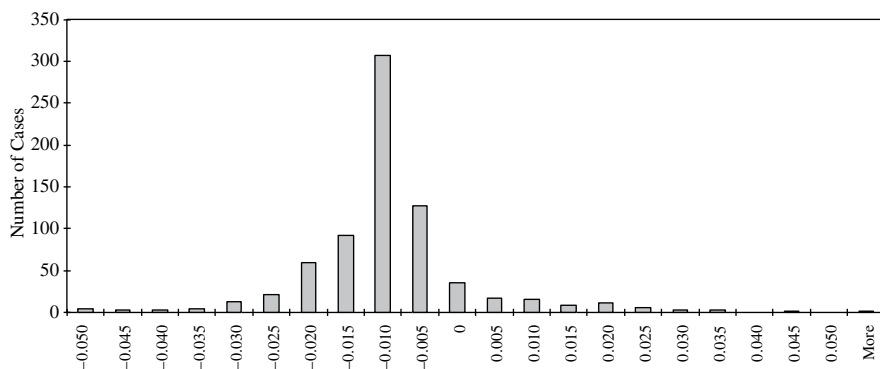
The instability in estimates of the speed of convergence is consistent with the results of Barro and Sala-i-Martin (1995) when analyzing patterns of convergence across US regions and Japanese prefectures, and also with the results of Vernon (2002) when analyzing convergence for the thirteen Chilean Regions.

The standard procedure when testing convergence runs either regression (13) or (14) over the longest horizon available. The second step is to subdivide the first period of analysis into smaller subperiods and to test for convergence in the different subsamples. Afterwards, the different estimates of the speed of convergence are compared. Finally, if instability across different periods is found, explanations of this instability are given based upon evidence of structural changes or regional heterogeneity.

FIGURE 2

HISTOGRAM OF THE SPEED OF CONVERGENCE
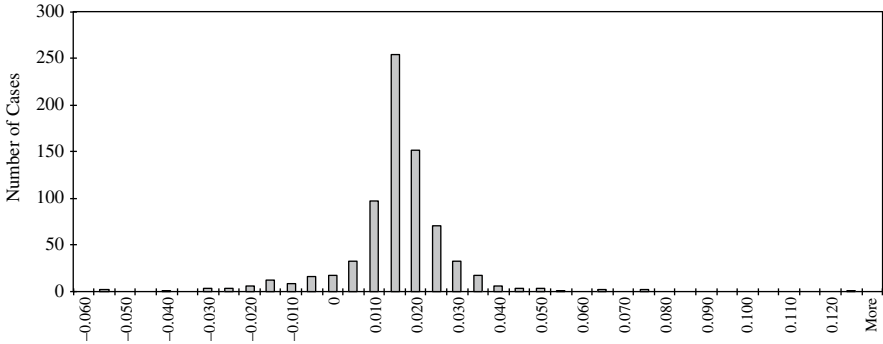
Aside from regional heterogeneity and structural changes there might be also some other factors adding to the observed instability of the estimates. In fact, testing convergence running regression (13) or (14) for a given possible horizon provides estimates of the speed of convergence that, at least in principle, depend on the horizon of analysis T according to the following expression:

$$\hat{\lambda}(T) = -\frac{\log(\hat{\beta}T + 1)}{T}, \tag{15}$$

where $\hat{\beta}$ represents the OLS estimates of β in (13).

Equation (15), however, does not always yield accurate or "correct" estimations of the speed of convergence λ. Indeed, equation (14) is the solution of the differential equation resulting from the log-linearization of the growth rate of the economy around its steady state. Accurate estimations of λ will be achieved only when output levels of the economy are in a small neighborhood of the steady state. This issue is usually addressed by estimating the speed of convergence for the longest horizon available given the data. This strategy stems from the deterministic neoclassical growth model. In fact, equation (9) implies that as the horizon T increases, economies get closer to the steady state level.

To deal with unknown sources of instability, a cross-section analysis is carried out in combination with a BMA strategy. For this purpose, the idea is to take the framework presented by Brock and Durlauf (2001) in which the authors used a BMA approach to control for model uncertainty in growth regressions.

As Brock and Durlauf (2001) argue, the standard econometric approach in the growth literature is based upon the choice of a particular model M, which is considered a good approximation of the "true model". Given a data set D and the chosen model M, estimates of the parameters β of interest and their variances can be obtained. The

analogous Bayesian strategy involves the calculation of the posterior density of the parameter $\mu(\beta \mid D, M)$.

Brock and Durlauf (2001) analyze the problem of model uncertainty, which basically originates in the ignorance of the researcher about the true model. Under this type of uncertainty, any estimate of the parameters of interest β is conditioned to the particular choice of a model M. Therefore, despite of the fact that the researcher is interested in the density $\mu(\beta \mid D)$, she is only able to uncover $\mu(\beta \mid D, M)$.

To remove the model uncertainty problem, Brock and Durlauf (2001) propose the definition of a space of possible models Π. Integrating out the dependence of $\mu(\beta \mid D, M_m)$ on the particular model $M_m \in \Pi$ leads to the unconditional density $\mu(\beta \mid D)$. To do this, Bayes theorem provides the following expression

$$\mu(\beta \mid D) = \sum_{M_m \in \Pi} \mu(\beta \mid D, M_m) \mu(M_m \mid D),$$

which reduces:

$$\mu(\beta \mid D) \propto \sum_{M_m \in \Pi} \mu(\beta \mid D, M_m) \mu(D \mid M_m) \mu(M_m)$$

where $\mu(D \mid M_m)$ is the likelihood of the data given the particular model $M_m \in \Pi$, and $\mu(M_m)$ represents the prior density defined over M. Basically these results show that the posterior density of the parameter β is a weighted average of the conditional densities of the parameter for different assumptions about the true model.

Leamer (1978) provides expressions for the conditional expectation and variance of β given the set of data D:

$$E(\beta \mid D) = \sum_{M_m \in \Pi} \mu(M_m \mid D) E(\beta \mid D, M_m),$$

and:

$$\text{var}(\beta \mid D) = E(\beta^2 \mid D) - (E(\beta \mid D))^2 = \sum_{M_m \in \Pi} \mu(M_m \mid D) \text{var}(\beta \mid D, M_m) +$$
$$\sum_{M_m \in \Pi} \mu(M_m \mid D)[E(\beta \mid D, M_m) - E(\beta \mid D)]^2, \tag{16}$$

where:

$$\mu(M_m \mid D) = \frac{\mu(D \mid M_m) \mu(M_m)}{\displaystyle\sum_{M_m \in \Pi} \mu(D \mid M_m) \mu(M_m)}.$$

Therefore, the conditional variance of β given the set of data D in (16) is broken down into two additive components: an intra-model variance and an across-models variance.

The BMA technique used in this paper is slightly different from the strategy to remove model uncertainty in growth regressions.

In this case, the space Π represents all the combinations of horizons T. That is to say:

$$\Pi = \{[g_{t_0,T}],\ T > 0 \ for \ given \ t_0\}.$$

Therefore, each regression is estimated for all possible horizons T and instead of providing different estimates of λ for each interval $[t_0, t_0 + T]$, the posterior distribution of λ given the data D is provided. In particular, the posterior expected value and variance of the parameters β and λ are reported.

For numerical implementation of the BMA technique, some approximations are commonly found in the literature. The Laplace approximation described by Volinsky *et al.* (1997) is adopted in this paper. This approximation is shown in the following equation:

$$\log(\mu(D \mid M_m)) \approx l - d_k \log(n), \tag{17}$$

where d_k represents the number of β parameters to estimate and l denotes the log-likelihood evaluated in the estimated parameters. (17) is called the Bayesian information criterion (BIC) approximation showed by Hoeting *et al.* (1999).

When BMA is applied the no convergence hypothesis is rejected at a 5% significance level[5], and the computed speed of convergence of 1.20% implies a half-life around of 50 years, not very encouraging for policy purposes. These results are displayed in Table 5.

TABLE 5

ESTIMATIONS OF β-CONVERGENCE USING A BMA APPROACH

	λ	β
Coefficient	0.012	–0.010
Std. deviation	0.004	0.003

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[5]   A uniform prior was used.

Two observations are mentioned:

-   First, as always in Bayesian analysis, results might be sensitive to the choice of the prior distribution. It would be interesting to study how robust are the conclusions to small disturbances in the prior distribution.
-   Second, Figure 3 shows the sequences of estimates of the speed of convergence λ, obtained from the estimation of equation (14) for different horizon values.

The goal of this figure is to give graphical evidence of the connection between the instability of the estimates of the speed of convergence and the length of the horizon T. It can be seen that the instability of the estimations reduces when the horizon of analysis increases.

FIGURE 3

THE EVOLUTION OF THE SPEED OF CONVERGENCE ESTIMATES

It is clear from Figure 3 that parameter instability is related to the horizon T. The picture indicates that the longer the horizon the smaller is the parameter instability. This fact might be considered when thinking about a prior. A uniform prior gives the same weight to all the estimations. However, different beliefs about the source of instability could lead to different priors. If one believe that short horizons suffer from short-term fluctuations, then a prior that penalizes short horizons should be preferred.

3.2. Convergence Within Heterogeneous Regions

It was mentioned before that the availability of data is a serious limitation in any empirical analysis involving regional economies in Chile. In this regard, for instance, there is no regional data available for capital stock. Therefore it is impossible to check whether the proportions of capital stock are constant across regions over the sample

period. If one want to relax the assumption of regional homogeneity, then there are some alternative approaches that can be followed.

The next table shows the decomposition of regional output into three different sub-sectors: Services, Construction and Manufacturing, and Natural Resources.

TABLE 6

DECOMPOSITION OF REGIONAL OUTPUT

Region	Services	Natural Resources	Construction and Manufacturing
Region 1	56%	14%	30%
Region 2	23%	61%	15%
Region 3	30%	59%	12%
Region 4	34%	47%	19%
Region 5	49%	21%	30%
Region 6	25%	55%	20%
Region 7	33%	29%	38%
Region 8	40%	14%	46%
Region 9	55%	24%	22%
Region 10	47%	29%	24%
Region 11	56%	27%	17%
Region 12	36%	43%	21%
Metropolitan Region	72%	3%	25%
Mean	36%	37%	26%
Std. deviation	12%	18%	11%

Given the data availability, shares are computed as regional averages during 1985-1997.

Table 6 shows a heterogeneous decomposition of output within the thirteen Chilean regions. In fact, the three sectors used in the output decomposition show a high dispersion across regions, with Natural Resources displaying the highest variance. This is a key issue because, unlike services, manufacturing and construction, natural resources might be a source of heterogeneity across regional production functions. This is because the relative amount of natural resources might factor into regional productivity. Under the neoclassical perspective, the existence of heterogeneity in the production function might be linked to the existence of several steady states.

Irrespective of the possible existence of regional heterogeneity, the question of convergence still holds and can be tested using cross-section tests. The significance of a natural resource component in the equation used to test convergence may be analyzed by cross-section tests using the following equation

$$g_{i,T} = \alpha + \beta y_{i0} + \delta X_i + \varepsilon_{i,T}, \qquad (18)$$

where X is now a vector containing information about the share of Natural Resources in regional output. This variable is aimed at detecting differences in productivity across regions. This exercise was carried out in the case of the thirteen Chilean Regions and the results are displayed in Table 7.

TABLE 7

ESTIMATIONS OF β-CONVERGENCE USING A BMA APPROACH

	$g_{i,T} = \alpha + \beta y_{i,0} + \delta X_i + \varepsilon_{i,T}$		$g_{i,T} = \alpha + \beta y_{i,0} + \varepsilon_{i,T}$
	β	δ	β
Coefficient	–0.0120	0.0230	–0.0100
Std. deviation	0.0024	0.0080	0.0030

Table 7 shows that the coefficient associated with the variable of natural resources δ is statistically significant and positive. The convergence parameter β is statistically significant, negative and with lower variance than the estimate obtained when the natural resources variable is excluded. This result suggests convergence, conditional on the distribution of natural resources across regions.

A time-series approach to test the convergence hypothesis in Chile follows next.

4. TIME-SERIES APPROACH

Bernard and Durlauf (1995) provide definitions of convergence implied by the neoclassical growth model in a stochastic framework. In order to formulate these definitions they assume that individual logarithms of output series satisfy

$$a(L)y_{it} = \mu_i + \varepsilon_{it}, \tag{19}$$

where $a(L)$ has a unit root and ε_{it} is a mean zero stationary process. This formulation is wide enough to allow for either linear deterministic or stochastic trends in the series.

Formally, Bernard and Durlauf (1995, 1996) provide the following definitions:

DEFINITION 1 *Convergence in Output*

Economies i *and* j *converge if the long-term forecasts of (log) per capita output for both economies are equal at a fixed time* t,

$$\lim_{k \to \infty} E(y_{j,t+k} - y_{i,t+k} \mid \Psi_t) = 0$$

This definition is also extended for an arbitrarily finite number of economies as follows:

DEFINITION 2 *Convergence in Multivariate Output*

Economies p = 1, ..., n *converge if the long-term forecasts of (log) per capita output for all economies are equal at a fixed time* t,

$$\lim_{k \to \infty} E(y_{1,t+k} - y_{p,t+k} \mid \Psi_t) = 0 \text{ for all } p \neq 1.$$

An appealing property of the time-series approach to testing for convergence stems from the fact that this perspective explicitly addresses the long-run behavior of the economies under analysis. On the contrary, cross-section based definitions do not focus in the long-run behavior but rather look at particular transition periods.

Bernard and Durlauf (1995) claim that if $y_{j,t+k} - y_{i,t+k}$ is a mean zero stationary process then these definitions of convergence will be satisfied. Therefore both definitions can be tested using unit root or cointegration tests. Basically, in order for economies *i* and *j* to converge their output per capita should be cointegrated with cointegrating vector [1,-1].

Furthermore, if the output series are trend stationary then the definitions imply that the time trends for each country must be the same.

If economies are not converging, they might still be responding to the same permanent shocks, but with different weights. The following definitions capture this idea:

DEFINITION 3 *Common Trends in Output*

Economies i *and* j *contain a common trend if the long-term forecasts of (log) per capita output are proportional at a fixed time* t,

$$\lim_{k \to \infty} E(y_{j,t+k} - \alpha y_{i,t+k} \mid \Psi_t) = 0$$

DEFINITION 4 *Common Trends in Multivariate Output*

Economies p = 1, ..., n *contain a single common trend if the long-term forecasts of (log) per capita output are proportional at a fixed time* t. *Let* $\overline{y}_t = [y_{2t}, ..., y_{pt}]$, *then:*

$$\lim_{k \to \infty} E(y_{1,t+k} - \alpha^T \overline{y}_{t+k} \mid \Psi_t) = 0.$$

These definitions also have testable counterparts in the cointegration literature considering a general, and not a particular, cointegration vector between two countries.

Unit root analysis and the Johansen cointegration method are used to carry out cointegration tests. In order to apply the latter technique, it is assumed that the vector of regional outputs per capita admits a finite vector autoregressive representation as follows:

$$\Delta y_t = \Gamma(L)\Delta y_t + \Phi y_{t-1} + \mu + \varepsilon_t,$$

where:

$$\Gamma_i = -(A_{i+1} + ... - A_k), \ i = 1,...,k-1,$$

and:

$$\Phi = -(I - A_{i+1} + ... - A_k),$$

where Φ could also be expressed as $\Phi = \alpha\beta^T$ whith α and β, $p \times r$ matrices of rank $r \le p$. Coefficient β is called the matrix of cointegrating vectors.

According to Definition 2, convergence requires the existence of $p-1$ cointegrating vectors of the form $[-1,1]$. If the no convergence hypothesis for the whole group of thirteen regions is not rejected, a natural second step is looking for subsets of regions where the convergence hypothesis may hold. Finally, and also in the non-convergence scenario, a third step would be related to the determination of common trends in the stochastic behavior of regional output per capita, notion that is linked to the existence of a cointegration relationship between the economies under analysis, but not necessarily to the particular cointegration condition required for Definition 1 and 2 to hold.

4.1. Empirical Results On Unit Roots

First, assumption (19) is checked. This goal is addressed by testing for the presence of stochastic trends in each of the thirteen series via the Augmented Dickey-Fuller (ADF) unit root test.

The ADF test assumes that the log of each output per capita series follows a pth order autoregressive process AR(p):

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + ... + \phi_p y_{t-p} + u_t,$$

which could be equivalently expressed as follows:

$$\Delta y_t = \phi_1^* y_{t-1} + \phi_2^* \Delta y_{t-2} + ... + \phi_{p-1}^* \Delta y_{t-p+1} + u_t, \tag{20}$$

where u_t is a white noise and:

$$\phi_1^* = \phi_1 + \ldots + \phi_p - 1$$

The ADF test checks the null of existence of a unit root, $\phi_1^* = 0$, against the alternative of stationarity, $\phi_1^* < 0$.

The previous model may be extended to include some deterministic components like trends and drifts. If the objective is to test the null hypothesis of a stochastic trend against the alternative of a deterministic trend, then an appropriate formulation of the model is:

$$\Delta y_t = \phi_1^* y_{t-1} + \sum_{i=2}^{p-1} \phi_i^* \Delta y_{t-i} + \mu + \gamma t + u_t. \tag{21}$$

Under this formulation both the null and alternative hypotheses are nested. It might be the case, however, that the constant and the deterministic trend are nuisance parameters that lower the power of the test. To overcome this problem, Perron (1988) proposed a sequential testing algorithm summarized in the following table, see Harris (1995).

TABLE 8

TESTING PROCEDURE USING THE DICKEY FULLER TEST

	Step and Model	Null Hypothesis	Statistic	Critical Values
(1)	$\Delta y_t = \mu_c + \gamma_c t + (\rho_c - 1)y_{t-1} + u_t$	$\rho_c - 1 = 0$	τ_t	Fuller, Table 8.5.2 b3
(2)	$\Delta y_t = \mu_c + \gamma_c t + (\rho_c - 1)y_{t-1} + u_t$	$\gamma_c = \rho_c - 1 = 0$	Ξ_3	Dickey & Fuller
(2a)	$\Delta y_t = \mu_c + \gamma_c t + (\rho_c - 1)y_{t-1} + u_t$	$\rho_c - 1 = 0$	t	Standard Normal
(3)	$\Delta y_t = \mu_b + (\rho_b - 1)y_{t-1} + u_t$	$\rho_b - 1 = 0$	τ_u	Fuller, Table 8.5.2 b2
(4)	$\Delta y_t = \mu_b + (\rho_b - 1)y_{t-1} + u_t$	$\gamma_b = \rho_b - 1 = 0$	Ξ_1	Dickey & Fuller
(4a)	$\Delta y_t = \mu_b + (\rho_b - 1)y_{t-1} + u_t$	$\rho_b - 1 = 0$	t	Standard Normal
(5)	$\Delta y_t = (\rho_a - 1)y_{t-1} + u_t$	$\rho_a - 1 = 0$	τ	Fuller, Table 8.5.2 b1

Critical values are given in Fuller (1976) and Dickey and Fuller (1981).

Table 8 outlines the procedure proposed by Perron in the case that the Dickey Fuller test is performed. A natural extension for the ADF test requires a specification of the model based upon formulation (21) and different critical values. τ_t, τ_u and τ denote the "t-Statistic" for the simple hypothesis of a unit root for different specifications of the model. These statistics follow their respective DF distribution rather than the usual t distribution. Ξ_3 and Ξ_1 denote the " F-Statistics" for the joint hypothesis of unit root and no deterministic trend and unit root and no drift respectively. They also follow particular DF distributions.

Perron's procedure starts testing the simple hypothesis of a unit root under formulation (21). If the null hypothesis is not rejected using the most general specification, possibly due to the lower power of the test, testing moves to more restricted formulations. Testing stops either when the null is not rejected in step (5) or when the null hypothesis is rejected in one of the previous stages. Intermediate steps (2a) and (4a) are performed only if the joint hypothesis in (2) and (4) are rejected respectively.

Visual inspection of the regional output per capita series indicates that a formulation that includes a deterministic trend and a drift is plausible when testing the null of a unit root on each of the thirteen series.

FIGURE 4

REGIONAL OUTPUT PER CAPITA, 1960-1998
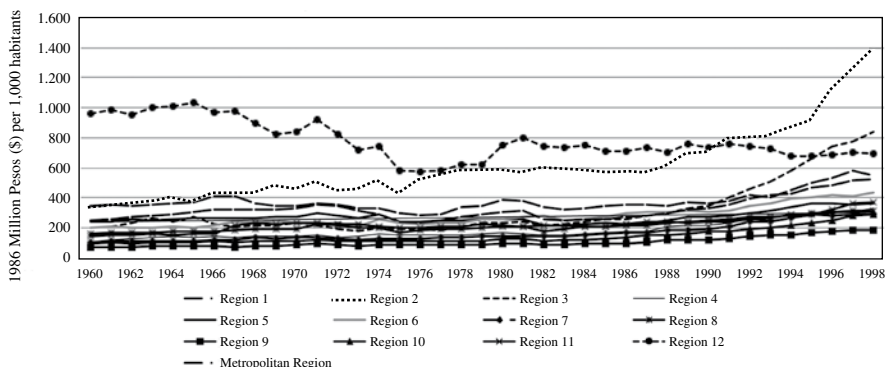
When testing whether the series are integrated of order 2, a specification with deterministic trend seems unnecessary. The drift is still considered due to the fact that sample averages of the series in differences indicate a possibility of non zero constant terms[6].

When testing for a unit root, the null hypothesis stating the existence of a unit root cannot be rejected for any of the thirteen regions at 5% significance level. Yet, when testing whether the series are integrated of order 2 (we will call this two unit roots), the null hypothesis stating the existence of one unit root in the differenced series was rejected in all cases at the same level of significance. Tables 10 and 11 show these results using the ADF test with Perron's procedure.

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[6]   The difference of a series in logarithm is an approximation for the annual growth rate of the series. That is why even small numbers like 0.02 are not necessarily considered zero. Basically 0.02 represents a 2% per capita growth rate.

TABLE 9

AVERAGE DIFFERENCE OF LOG OUTPUT

Region 1	1.2%
Region 2	3.7%
Region 3	3.8%
Region 4	2.9%
Region 5	1.0%
Region 6	2.0%
Region 7	2.7%
Region 8	1.8%
Region 9	2.5%
Region 10	3.0%
Region 11	2.5%
Region 12	–0.8%
Metropolitan Region	2.0%

The test results suggest that the data support the assumption of each of the thirteen regions following an integrated process of order 1 (I(1) process). This assumption enables the researcher to search for cointegration vectors on every non-empty subset of the thirteen regions.

4.2. Time-Series Convergence Analysis

According to Definition 2, convergence requires the existence of $p-1$ cointegrating vectors of the form $[-1,1]$. Therefore, not rejecting the null hypothesis of a unit root in the difference of output per capita for any pair of regions is interpreted as not rejecting the null of no convergence for the whole set of regions.

Table 12 shows the results of the ADF test carried out to detect the presence of stochastic trends in the difference of regional output. Pairwise comparisons are made between every region against region number 2. The existence of a unit root cannot be rejected at usual levels of significance in all cases, showing that the data are consistent with the null hypothesis of no convergence.

Once convergence has been rejected for the thirteen regional economies, the next step is to check for the existence of convergence within subgroups of regions. Patterns of geographical clustering are studied. The thirteen Chilean regions are classified into three different groups: the Northern group (including Regions 1, 2 and 3), the Central group (including Regions 4, 5, 6, 7, 8, 11 and MR) and the Southern group (including Regions 9, 10 and 12). The prior belief is that these groups might have a small number of common trends within them. Basically, the regions in the north of Chile are rich in copper mines, and the mining industry has been the major economic activity in that zone[7]. Similarly, Santiago is the biggest city in the country, with more

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[7]   For instance, the mining sector represents about a 60% of total GDP in Region 2.

TABLE 10

ONE UNIT ROOT TEST

	Lags	Tests Statistics					Critical Values (5%)				
		τ_τ	Φ_3	τ_μ	Φ_1	τ	τ_τ	Φ_3	τ_μ	Φ_1	τ
Region 1	1	-0.82	1.34	-0.03	0.67	1.17	-3.53	6.77	-2.94	5.02	-1.95
Region 2	2	0.61	1.96	2.01	8.54*	3.11	-3.54	6.77	-2.94	5.02	-1.95
Region 3	1	0.20	3.72	2.14	6.32*	2.05	-3.53	6.77	-2.94	5.02	-1.95
Region 4	1	-0.88	0.79	0.48	5.13*	3.27	-3.53	6.77	-2.94	5.02	-1.95
Region 5	1	-0.89	0.96	-0.38	0.68	1.12	-3.53	6.77	-2.94	5.02	-1.95
Region 6	10	0.59	11.16*	4.30	10.63*	2.16	-3.58	6.77	-2.97	5.02	-1.95
Region 7	1	-0.98	1.28	0.80	6.79*	3.67	-3.53	6.77	-2.94	5.02	-1.95
Region 8	1	-1.70	1.62	-0.26	2.11	2.05	-3.53	6.77	-2.94	5.02	-1.95
Region 9	1	-0.96	1.66	1.02	5.03*	3.07	-3.53	6.77	-2.94	5.02	-1.95
Region 10	1	0.72	4.88	3.05	14.83*	4.30	-3.53	6.77	-2.94	5.02	-1.95
Region 11	1	-1.28	0.93	-0.29	2.84	2.38	-3.53	6.77	-2.94	5.02	-1.95
Region 12	1	-1.76	1.65	-1.64	1.61	-0.76	-3.53	6.77	-2.94	5.02	-1.95
Metropolitan Region	2	-1.49	1.43	-0.90	0.90	0.97	-3.54	6.77	-2.94	5.02	-1.95

Lag length chosen by the SIC criterion.
Critical value for the Standard Normal Distribution is −1.65.

TABLE 11

TEST FOR TWO UNIT ROOT TESTS

	Statistic		Critical Value (5%)
	Lags	τ_μ	τ_μ
Region 1	1	–4.73	–2.94
Region 2	1	–7.80	–2.94
Region 3	1	–4.72	–2.94
Region 4	1	–6.89	–2.94
Region 5	1	–6.12	–2.94
Region 6	1	–8.26	–2.94
Region 7	1	–6.21	–2.94
Region 8	1	–6.66	–2.94
Region 9	1	–6.01	–2.94
Region 10	1	–5.30	–2.94
Region 11	1	–5.04	–2.94
Region 12	1	–6.18	–2.94
Metropolitan Region	1	–3.71	–2.94

Lag length chosen by the SIC criterion.

than a third of the Chilean population[8]. Its influence over the neighboring regions may be important. Finally the southern group is characterized by similar economic activities like the exploitation of renewable natural resources.

Cointegration analysis performed over these three subgroups is consistent with the assumption of a small number of common trends within each subgroup. Based upon computation of the trace statistic, the null of "at most one cointegrating vector" is rejected at usual significance levels (5%) in the north and south[9]. The null of "at most two cointegrating vectors", however, was not rejected. This result is suggestive of the existence of only one common trend in both groups. For the central group things are not so clear. In fact results are quite sensitive to the number of lags included in the vector error correction model (VECM) representation[10]. Nevertheless, evidence from the cointegration analysis suggests that there are on the order of 3 to 6 common trends in the central group. The following tables display these results[11].

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[8] The population of Chile is 15,116,435.
[9] Is rejected against the alternative of 3 cointegrating vectors.
[10] 1 and 2 lags were tried, but in general the number of lags included was determined with the AIC and BIC criterion.
[11] It should be pointed out that convergence in output was also tested in the three subgroups, but evidence of pairwise divergence was always found.

TABLE 12

UNIT ROOT TESTS FOR CONVERGENCE

Regions	Lags	Tests Statistics						Critical Values (5%)					
		τ_τ	Φ_3	τ_μ	Φ_1	t	τ	τ_τ	Φ_3	τ_μ	Φ_1	t	τ
12-lr_2	1	-2.55	3.34	-0.91	1.49	–	1.29	-3.53	6.77	-2.94	5.02	-1.65	-1.95
12-l1	4	-1.41	1.12	-0.89	6.49	-0.89	2.50	-3.55	6.77	-2.95	5.02	-1.65	-1.95
12-l3	1	-1.49	1.71	-1.45	nn	–	-0.45	-3.53	6.77	-2.94	5.02	-1.65	-1.95
12-l4	1	-2.57	3.55	-1.79	1.80	–	0.47	-3.53	6.77	-2.94	5.02	-1.65	-1.95
12-l5	1	-2.54	3.42	-0.22	nn	–	1.98	-3.53	6.77	-2.94	5.02	-1.65	-1.95
12-l6	1	-2.46	3.20	-1.16	nn	–	2.00	-3.53	6.77	-2.94	5.02	-1.65	-1.95
12-l7	1	-2.67	3.71	-2.44	3.28	–	0.57	-3.53	6.77	-2.94	5.02	-1.65	-1.95
12-l8	1	-2.01	2.48	-0.23	nn	–	1.48	-3.53	6.77	-2.94	5.02	-1.65	-1.95
12-l9	1	-3.05	4.69	-2.35	3.20	–	0.74	-3.53	6.77	-2.94	5.02	-1.65	-1.95
12-l10	1	-2.73	3.78	-2.61	3.61	–	0.42	-3.53	6.77	-2.94	5.02	-1.65	-1.95
12-l11	1	-2.97	4.75	-0.46	nn	–	1.03	-3.53	6.77	-2.94	5.02	-1.65	-1.95
12-l12	1	-1.20	1.05	0.36	nn	–	-1.42	-3.53	6.77	-2.94	5.02	-1.65	-1.95
12-lm	1	-2.26	2.60	-0.88	nn	–	0.89	-3.53	6.77	-2.94	5.02	-1.65	-1.95

Lag length chosen by the SIC criterion.
Critical value for the Standard Normal Distribution is -1.65.
li: Log output per capita of Region I.
lm: Log output per capita of Metropolitan Region.
lr_2: Average of log output of all regions with the exception of Region 2.
nn: Not necessary.

TABLE 13

COINTEGRATION ANALYSIS IN THE NORTHERN GROUP

Regions 1, 2 and 3		Statistics		Critical Values (5%)	
Hypothesized No. CV	Lags	Trace	Max Eig	Trace	Max Eig
None **	1	37.01	20.18	29.68	20.97
At most 1 *		16.84	13.76	15.41	14.07
At most 2		3.07	3.07	3.76	3.76

Trace test indicates 2 cointegrating equation(s) at the 5% level.
Rejection based in the trace statistic.

TABLE 14

COINTEGRATION ANALYSIS IN THE SOUTHERN GROUP

Regions 9, 10 and 12		Statistics		Critical Values (5%)	
Hypothesized No. CV	Lags	Trace	Max Eig	Trace	Max Eig
None *	1	33.88	15.94	29.68	20.97
At most 1 *		17.95	15.34	15.41	14.07
At most 2		2.60	2.60	3.76	3.76

Trace test indicates 2 cointegrating equation(s) at the 5% level.
Rejection based in the trace statistic.

TABLE 15

COINTEGRATION ANALYSIS IN THE CENTRAL GROUP (2 LAGS)

Regions 4, 5, 6, 7, 8, 11 and M		Statistics		Critical Values (5%)	
Hypothesized No. CV	Lags	Trace	Max Eig	Trace	Max Eig
None **	2	207.11	71.15	124.24	45.28
At most 1 **		135.95	49.56	94.15	39.37
At most 2 **		86.39	32.90	68.52	33.46
At most 3 *		53.49	24.83	47.21	27.07
At most 4		28.66	15.70	29.68	20.97
At most 5		12.96	12.59	15.41	14.07
At most 6		0.37	0.37	3.76	3.76

Trace test indicates 4 cointegrating equation(s) at the 5% level.
Rejection based in the trace statistic.

TABLE 16

COINTEGRATION ANALYSIS IN THE CENTRAL GROUP (1 LAG)

Regions 4, 5, 6, 7, 8, 11 and M		Statistics		Critical Values (5%)	
Hypothesized No. CV	Lags	Trace	Max Eig	Trace	Max Eig
None *	1	126.76	42.36	124.24	45.28
At most 1		84.40	29.34	94.15	39.37
At most 2		55.07	21.61	68.52	33.46
At most 3		33.46	15.24	47.21	27.07
At most 4		18.22	9.27	29.68	20.97
At most 5		8.95	8.83	15.41	14.07
At most 6		0.12	0.12	3.76	3.76

Trace test indicates 1 cointegrating equation(s) at the 5% level.
Rejection based in the trace statistic.

While lack of power in unit root tests is a common pitfall, there are three arguments that support the inferences drawn in here: First, the unit root tests carried out in the paper cannot reject the null hypothesis of a unit root for all pair of regions considered in the analysis (all regions against Region 2). Furthermore, Oyarzún and Araya (2001) carried out ADF tests for all possible combinations of regions, (total of 78), rejecting the null of unit root in only 10 cases, a 13% of the cases. This evidence shows that the inference of no convergence drawn by using time series tests is not a specific feature of a few regions, but rather a feature of most pairs of regions. Second, for convergence to hold, pairs of regions are required not only to be stationary, but also to be mean zero. According to Table 17, this is unlikely to be true for all pairs of regions. Third, the power problem of unit root tests is usually more serious when there are structural changes in the series. However, Oyarzún and Araya (2001) confirmed the no convergence inference obtained via ADF tests using the unit root tests developed by Zivot and Andrews (1992). In these tests the alternative hypothesis includes the possibility of a level or a trend shift.

The key issue that should be pointed out when comparing the results from time series tests to those from cross-section tests, is that the latter tests place much weaker restrictions on the behavior of economies than the former ones. In summary, while cross-section tests requires that some economies converge according to Definition 1 in Bernard and Durlauf (1996), time series tests requires that every pair of economies satisfy Definition 2 in Bernard and Durlauf (1996).

Finally, there are some other more powerful techniques that might be used to test the existence of unit roots. A better test is one that tests the restriction of cointegration subject to the maintained assumption that the cointegrating vector is $[1,-1]$[12]. The implementation of this test is suggested as a point to be improved in future research.

5. COMMENTS AND EXTENSIONS

Cross-section and time-series tests yield two different conlusions regarding the convergence hypothesis within Chilean regions. While cross-section tests support the convergence hypothesis, time-series based tests provide evidence against it.This dissimilarity between cross-section and time-series tests can be understood when observing that these two approaches differ in the assumptions they place on the data. While cross-section tests assume that the data are in transition towards a stationary distribution, time-series tests assume that the economies under analysis are mainly governed by their limiting distribution. This results in time series tests that have low power when applied to economies in transition. According to this, cross-section tests appear more appropriate for economies that are in transition toward their limiting distribution, whereas time-series tests seem more appropriate when economies are governed by their limiting distribution. Algebraic details explaining the linkage between these two tests and the different assumptions they place on the data are found in Bernard and Durlauf (1996). For completeness a brief summary of their explanation follows next.

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[12] The author is thankful to Bruce Hansen for this comment.

TABLE 17

AVERAGE OF DIFFERENCES IN THE LOG OF OUTPUT PER CAPITA BETWEEN REGIONS

Region	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	MR
I	0.00												
II	−0.44	0.00											
III	0.28	0.71	0.00										
IV	0.77	1.20	0.49	0.00									
V	0.32	0.76	0.05	−0.44	0.00								
VI	0.32	0.76	0.05	−0.44	0.00	0.00							
VII	0.87	1.30	0.59	0.10	0.54	0.54	0.00						
VIII	0.56	1.00	0.28	−0.21	0.24	0.24	−0.31	0.00					
IX	1.31	1.75	1.03	0.55	0.99	0.99	0.44	0.75	0.00				
X	1.01	1.45	0.73	0.24	0.69	0.69	0.14	0.45	−0.30	0.00			
XI	0.52	0.96	0.24	−0.24	0.20	0.20	−0.35	−0.04	−0.79	−0.49	0.00		
XII	−0.73	−0.29	−1.01	−1.50	−1.05	−1.05	−1.60	−1.29	−2.04	−1.74	−1.25	0.00	
MR	0.16	0.60	−0.12	−0.60	−0.16	−0.16	−0.71	−0.40	−1.15	−0.85	−0.36	0.89	0.00

5.1. Reconciling Cross-Section and Time-Series Results

When testing the convergence hypothesis, two main empirical techniques have been usually used in the literature: time-series tests and cross-section tests. The former tests examine the long-run behavior of differences between output per capita for pairs of economies. Under this framework, convergence between two economies is understood as a mean zero stationary behavior in the difference of output per capita. These dynamic tests allow the researcher not only to test for convergence in the whole group of economies, but also to seek to identify particular clusters of economies that might be converging. Therefore, under the time-series perspective convergence needs not to be an all or nothing assertion.

Cross-Section tests study the cross-section relationship between initial output per capita and growth rates within a given period. Evidence of convergence is found when there is an inverse relationship between initial output and the growth rate.

Bernard and Durlauf (1996) propose two definitions of convergence that are implications of the neoclassical growth model. The first definition was stated in section 4 as Definition 1 and characterizes convergence as the equality of long-term forecasts of output when the forecasting horizon goes to infinity. The second definition characterizes convergence between a pair of economies as the tendency of output per capita gap's to narrow. This definition follows next.

DEFINITION 5 *Convergence as Catching Up*

Countries i *and* j *converge between dates* t *and* t+T *if the (log)per capita output disparity at* t *is expected to decrease in value. If* $y_{i,t} > y_{j,t}$.

$$E(y_{i,t+T} - y_{j,t+T} \mid \Psi_t) < y_{i,t} - y_{j,t}.$$

It is interesting to remark that both definitions are implied by the neoclassical growth model, and that Definition 1 implies Definition 3 for some *T*.

When convergence is tested using the following cross-section regression:

$$g_{i,T} = \alpha + \beta y_{i0} + \varepsilon_{i,T}, \tag{22}$$

convergence is associated with a negative coefficient β. Bernard and Durlauf (1996) show how this requirement may be seen as a restriction on the mean of output per capita differences between two series. In fact, observing that:

$$g_{i,T} = T^{-1} \sum_{i=1}^{T} \Delta y_{i,t},$$

where $\Delta y_t = y_t - y_{t-1}$. Expression (22) implies that:

$$T^{-1} \sum_{i=1}^{T} \Delta y_{i,t} - T^{-1} \sum_{i=1}^{T} \Delta y_{j,t} = \beta(y_{i,0} - y_{j,0}) + \varepsilon_{i,T} - \varepsilon_{j,T},$$

therefore, regression (22) is testing "whether the average change in the per capita output of an initially poorer country exceeds that of an initially richer country" (Bernard and Durlauf, 1996, page 167).

Furthermore, recalling that the OLS estimator of β in (22) is given by:

$$\hat{\beta} = \sum_{i=1}^{I} \phi_i \varphi_i,$$

with:

$$\phi_i = \frac{(y_{i,0} - \overline{y}_{i,0})^2}{\sum_{i=1}^{I} (y_{i,0} - \overline{y}_{i,0})^2}, \text{ and}$$

$$\varphi_i = \frac{g_{i,T} - \overline{g}_{i,T}}{y_{i,0} - \overline{y}_{i,0}}.$$

A negative value for the OLS estimate β implies that at least one pair of countries is converging according to Definition 3. It should be pointed out, however, that these cross-section tests are unable to identify groups of countries that might be converging while the whole set of economies do not converge. In this respect, the finding of a negative and statistically significant OLS estimate for β in the case of the thirteen Chilean regions, indicates that there exists a converging pair of regions. The test has no power, however, to determine whether this convergence process involves the whole set of thirteen regions or just a few. In addition, Bernard and Durlauf (1996) also show that a negative OLS estimate for β is consistent with some structural models which violate Definition 1 of convergence. This is the case when the various regions or economies under study are distributed across N long-run steady states. According to this distribution of steady states, convergence as equality of long-term forecasts at a fixed time does not hold. However, if rich economies are initially closer to their steady states than poorer economies are, then the covariance between initial conditions and the gap between steady states and initial conditions will be negative, leading to a negative OLS estimate for the β coefficient.

These two observations suggest caution when interpreting the results of cross-section tests of convergence. In this regard it can be claimed that: If the thirteen Chilean regions share the same microeconomic features, the finding of a negative β coefficient is consistent with the hypothesis of convergence in the sense that at least some pairs of them are closing the gap between rich and poor regions.

In summary, cross-section tests show some problems in capturing the implications of the neoclassical growth model enclosed in Definitions 1 and 3. While the finding of a negative β coefficient is consistent with the fact that some economies (but not necessarily all of them) satisfy Definition 3 of convergence, it is also consistent with a family of structural models that violates Definition 1 of convergence, and hence does not provide evidence on whether economies converge according to this definition.

Time-series tests of convergence, instead, are based on the fact that the series of differences $y_{j,t} - y_{i,t}$ does not satisfy Definition 2 of convergence if it has either a non zero mean or a unit root component. Therefore, time-series tests are linked to Definition 2 of convergence. Besides, it is important to point out that if $y_{j,t} - y_{i,t}$ is a zero mean stationary process, so it is:

$$T^{-1}\sum_{t=1}^{T}\Delta y_{i,t} - T^{-1}\sum_{t=1}^{T}\Delta y_{j,t}.$$

Furthermore, when cross-section and time series tests of convergence are carried out over the same sets of economies, they are necessarily inconsistent. This inconsistency stems from the fact that expression (6) implies that:

$$T^{-1}\sum_{t=1}^{T}\Delta y_{i,t} - T^{-1}\sum_{t=1}^{T}\Delta y_{j,t} = \beta(y_{i,0} - y_{j,0}) + \varepsilon_{i,T} - \varepsilon_{j,T},$$

so, a negative β coefficient implies that the expected value of:

$$T^{-1}\sum_{t=1}^{T}\Delta y_{i,t} - T^{-1}\sum_{t=1}^{T}\Delta y_{j,t},$$

is negative if $y_{j,0} - y_{i,0}$ is positive, whereas time series tests require the expected value of:

$$T^{-1}\sum_{t=1}^{T}\Delta y_{i,t} - T^{-1}\sum_{t=1}^{T}\Delta y_{j,t},$$

to be zero for convergence to occur.

In the case of the thirteen Chilean regions, for which the convergence hypothesis is supported by cross-section tests but it is not supported by time series tests, the next table shows that the sample average:

$$T^{-1}\sum_{t=1}^{T}\Delta y_{i,t} - T^{-1}\sum_{t=1}^{T}\Delta y_{j,t},$$

is significantly different depending on the sign of the difference of initial conditions $y_{j,0} - y_{i,0}$. This indicates that the conflicting results obtained by time-series and cross-section tests originate in the different restrictions that both test impose on the data.

TABLE 18

$T^{-1}\sum_{t=1}^{T}\Delta y_{i,t} - T^{-1}\sum_{t=1}^{T}\Delta y_{j,t}$	
$y_{i,0} - y_{j,0} > 0$	$y_{i,0} - y_{j,0} < 0$
−0.205% (0.01321)	1.649% (0.01524)
t–Statistic: −6.068	

t–Statistic reported for the test of different means.
Number of observations is 78.

Let us assume that an economy's output has two possible states depending on whether the evolution of the economy is mainly driven by transitional dynamics or by a stationary distribution. Let us define s the state variable that takes the value 1 if the set of economies is evolving mainly according to its transitional dynamics, and 0 if it is evolving mostly according to a stationary distribution. In this context we would like to test the following no convergence null hypothesis:

$$H_0 : \Theta(s) = 0, \tag{23}$$

against the alternative:

$$H_A : \Theta(s) < 0,$$

where:

$$\begin{cases} \Theta(s) = \dfrac{\beta}{std(\beta)} & \text{if } s = 1 \\ \Theta(s) = \sup\{\phi_1^1, ..., \phi_1^k\} & \text{if } s = 0 \end{cases}$$

where k represents the number of economies and ϕ_1^i represents the coefficient denoted by ϕ_1^* in equation (20) for each economy $i = 1,...,k$.

We recall that under a stationarity regime, convergence requires:

$$\phi_1^i < 0 \ for \ all \ i = 1,...,k,$$

but under the null of no convergence we expect:

$$\phi_1^i = 0 \ for \ some \ i = 1,...,k.$$

Therefore if $\sup\{\phi_1^1,...,\phi_1^k\}$ is significantly negative we should be able to reject the no convergence hypothesis.

Let us consider an estimate of the state variable that we will denote $\hat{s}$. Let us now define by $0 \le \hat{q} \le 1$ the law of $\hat{s}$ when the null is true. That is to say:

$$P_{H_0}(\hat{s} = 1) = \hat{q}.$$

Therefore the distribution of an estimate of $\Theta(s)$ under the null of no convergence is given by:

$$
\begin{aligned}
& P_{H_0}(\hat{\Theta}(\hat{s}) \le z) \\
& = P_{H_0}(\hat{\Theta}(\hat{s}) \le z \mid \hat{s} = 1)P_{H_0}(\hat{s} = 1) + P_{H_0}(\hat{\Theta}(\hat{s}) \le z \mid \hat{s} = 0)P_{H_0}(\hat{s} = 0) \\
& = P_{H_0}(\frac{\hat{\beta}}{std(\hat{\beta})} \le z)\hat{q} + P_{H_0}(\sup\{\hat{\phi}_1^1,...,\hat{\phi}_1^k\} \le z)(1 - \hat{q}).
\end{aligned}
\tag{24}
$$

We propose, as an extension of this paper, to test the no convergence hypothesis with a test like (23) using bootstrap critical values from the distribution in (24).

6. CONCLUSIONS

Cross-section and time-series based tests of convergence were carried out to detect convergence within the thirteen Chilean regions. Usual cross-section analysis supports the convergence hypothesis for most of the periods studied. Moreover, the evidence suggests that convergence is conditional on the share of natural resources in the production function. Evidence of instability in the estimates of the speed of convergence was found, however, and the null hypothesis of no convergence was not rejected for some periods.

While instability may be caused by regional heterogeneity, structural changes or problems with the data, this paper claims that there might be also another important source of uncertainty in the estimations. This is uncertainty about the regime

(transitional or stationary) governing output evolutions. In this regard, two different approaches are followed. First a Bayesian Model Averaging (BMA) technique is implemented to obtain a posterior distribution of the speed of convergence. When applying the BMA technique, the null hypothesis of no convergence was rejected at a 5% significance level, and the computed speed of convergence was 1.2%. This approach might be useful when the data under analysis spans a period during which the leading dynamic regime may change. In addition to the cross section approach, time-series based tests were carried out. When applying these tests, the null hypothesis of no convergence across all thirteen regions was not rejected, but evidence of subgroup cointegration was found.

Indeed, when the thirteen Chilean regions were clustered into three different groups, North, Central and South, significant evidence of cointegration within the groups was found. In fact, for the North and South groups, the test suggested the existence of one common trend for the three regions that make up each group. In the case of the Central group, the test indicate between three and six common trends.

The implementation of both, cross-section and time-series tests allows coverage of two opposite situations: economies in transition dynamics and economies in stationary distribution. Because cross-section and time-series tests place different implications on the data, one can claim that under the assumption that the Chilean regions are in transition towards a stationary distribution or steady state, the convergence hypothesis is supported by the data. However, if one assumes that the Chilean regions already achieved their limiting distribution, the convergence hypothesis is not supported by the data.

Finally, it is important to remark that the findings reported in this paper share three consistencies with the empirical growth literature. First, the associated speed of convergence of 1.2% found for the Chilean regions is consistent with the intra region estimates of Barro and Sala-i-Martin (1995) for the US, of Canova and Marcet (1995) for European regions and of Easterly *et al.* (1996) for developing countries. Second, the no convergence result found when the time-series test was carried out is consistent with the results of Bernard and Durlauf (1995) and Nahar and Inder (2002) in the sense that time-series based tests of convergence tend to accept the null of no convergence quite frequently. Third, the opposite results provided by the cross-section and time-series approach are consistent with the incompatibility of these tests as shown by Bernard and Durlauf (1996).

APPENDIX A

Chilean Regions

FIGURE A1
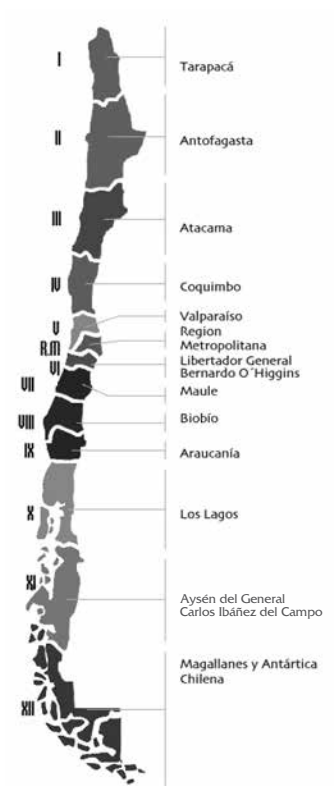
Chile is divided into thirteen regions as can be seen in the map. They usually are grouped into three zones according to their geographical distribution:

Zone 1

Contains the regions of Tarapaca (I), Antofagasta (II), Atacama (III) and Coquimbo (IV). This is a zone of big contrasts between the desert and the fertile valleys. In these regions mining is the main economic activity. Besides, agriculture, trade, tourist and manufacturing are strongly developed.

Zone 2

Includes the Valparaíso, Libertador General Bernardo O'Higgins, Maule, and Metropolitan regions (V, VI y VII, MR respectively). This zone concentrates the most of administrative, political and economic activity of Chile. Agriculture, manufacturing, trade, financial services are among the most developed sectors.

Zone 3

Contains the Bío-Bío (VIII), Araucanía (IX), Los Lagos (X), Aysen (XI) and Magallanes (XII) regions. Agriculture, forestry, fishing, manufacturing, trade, are the most developed sectors.

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