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WELFARE ANALYSIS OF AN OPTIMAL CARBON TAX IN CHILE

ANALISIS DE BIENESTAR PARA UN IMPUESTO OPTIMO A LAS EMISIONES DE CARBONO EN CHILE

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Abstract

We analyze a dynamic stochastic general equilibrium model which includes a negative externality that arises from fossil fuels burning. The carbon released to the atmosphere by electricity producers is the main driver of climate change. We adapt the optimal tax derived by Golosov et al. *(2011) to a small open economy to force polluters to internalize their damages. The results show that the tax benefits outweigh their costs; yet welfare gains seem to be marginal under plausible parameters. We calculate the optimal carbon tax for Chile and the tax effectiveness achieved, which is around 10 percent. The results remain robust to variations in the utility function, changes in parameters that determine the externality and alternative degrees of commitment to reduce emissions.*

Keywords: *DSGE, climate change, CO_2 emissions, optimal taxation, carbon tax.*

JEL Classification: *E32, Q54, Q58, H210.*

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Resumen

En este trabajo analizamos un modelo de equilibrio general dinámico que incluye una externalidad negativa que surge por la quema de combustibles fósiles. El carbono liberado a la atmósfera en el sistema eléctrico es el principal impulsor del cambio climático. Adaptando el impuesto óptimo derivado por Golosov et al. *(2011) para una pequeña economía abierta, se fuerza a que los productores internalicen los daños que producen. Si bien los beneficios del impuesto sobrepasan a sus costos, las ganancias en bienestar resultan ser marginales. La efectividad de la aplicación del impuesto óptimo alcanza aproximadamente el 10% del potencial para Chile. Los resultados se mantienen robustos a cambios en la función de utilidad, cambios en los parámetros que determinan la magnitud de la externalidad y alteraciones al grado de compromiso para reducir las emisiones.*

Palabras clave: *DSGE, Cambio climático, emisiones de CO_2, impuesto óptimo, impuesto al carbono.*

Clasificación JEL: *E32, Q54, Q58, H210.*

1. INTRODUCTION

It is the view of many that –previous to the industrial revolution– our planet had stable environmental conditions fully compatible with the biodiversity. The emerging literature, notably the report made by a group of experts on climate change, IPCC (2007), emphasizes negative effects of "climate change"[1]. Since then, the policy agenda on the climate change gained momentum. Loosely speaking, the main hypothesis is that the speed of climate change is linked directly to the intensity of human activity[2].

The central issue at hand is that the agents that pollute (i.e., those responsible for the negative externality) do not take into account the adverse effects on others.

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[1] The dynamics of climate change closely relates to the greenhouse effect. Its mechanic is relatively simple: the sun's thermal radiation is reflected by the earth's surface and "absorbed" by atmospheric gasses while it is re-radiated in all directions. The balance of this dynamic process means in practice that the surface temperature is higher (or lower) than the effective temperature. Thus, the "Earth's natural greenhouse effect" is a sort of buffer, making life on earth possible.

[2] The largest and most direct human influence is through direct carbon emissions of greenhouse gases (GHGs) into the atmosphere due to burning of fossil fuels. Besides rising temperatures, the dynamic of the carbon cycle process is also becoming increasingly unbalanced due to interference in the ecosystems of natural areas by human activity, which obstructs the ecosystems' ability to remove carbon from the atmosphere. This may all trigger instability in the climate, with potentially severe social and economic consequences.

As such, externalities are a known market failure which could be corrected with the implementation of proper policy instruments[3]. Golosov *et al.* (2011) proposes a global model where the implementation of an optimal tax turns the equilibrium efficient. However, as global warming affects every economy, their implementation is not general for the case of a small open economy like Chile. Thus, we examine the global scope of the externality in a different setting or model.

Developed countries have responded to this global problem coordinating their policies. In particular, they have signed the Kyoto Protocol, which is a multilateral treaty that sets binding obligations in order to coordinate the reduction of greenhouse gasses. Besides capping emissions, the treaty allows for the trading of pollutant emission units. As a result of the trading process, the price for the marginal unit of gas emitted is determined. A brief review of the Kyoto Protocol data shows that it has been successful for limiting emissions of European industrialized countries[4]. However, the Kyoto Protocol is not binding for developing countries. It turns out to be that developing countries carbon dioxide (CO_2) emissions have shown an upward tendency in the last 30 years. Figure 1, based on Energy Information Administration (EIA) data, illustrates that Chile has more than doubled its emissions, surpassing other comparable economies.

A recent study by CEPAL (2009) quantifies expected losses due to climate change for many countries, including Chile. In brief, it could lose up to 1.1% of its annual GDP in over roughly one century (until the year 2100).

The application of Pigouvian taxes are standard in markets that are regulated. For example, we find these taxes in banking liquidity regulation devised to contain systemic risk externalities and preserve credit quality (see e.g., Perotti and Suarez, 2011)[5]. Posterior to the study of CEPAL (2009) –therefore not necessarily incorporated in its calculations– several countries comparable to Chile implemented "green" taxes aiming at reducing CO_2 emissions. The most recent case is Australia, which in 2011 established a tax of U.S. $ 24.7 per ton of CO_2 emitted. Furthermore, since 2009, Denmark, Finland, Norway, Sweden and Canada have applied some kind of carbon tax on emissions. France also announced that they would implement the tax in the 2012's budget, but so far they have not followed through. Finally, New Zealand has established a market for trading emission allowances. In 2012, Chile considered "green" taxes in the update of the taxation law, but eventually they were not included. Again "green" taxes are being considered in the law update this year. The proposal under consideration focus on two damages: (i) a "Green" tax of $ 5 per ton of CO_2

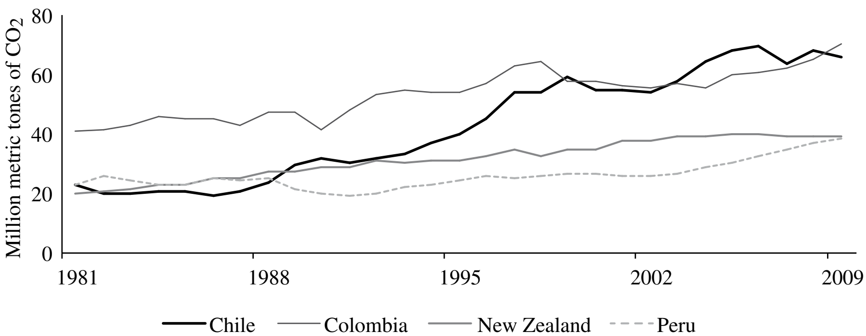
[3] Broadly, the standard textbook covers two set of instruments; both reach the goal of implementing the efficient allocation of resources. First, the government may cut quantities of emissions via imposing caps. Second, the government may increase subsidies or taxes to change relative prices.

[4] Details are plotted in the webpage of the United Nations Framework Convention for Climate Change: http://unfccc.int/files/inc/graphics/image/jpeg/total_excl_2013.jpg.

[5] In the next section we discuss on the advantages and disadvantages of choosing taxes vs. limiting quantities.

FIGURE 1

SMALL OPEN ECONOMIES CO_2 EMISSIONS: 1980-2009



issued by energy firms and (ii) other gases with particles, oxides of nitrogen (NOx) and sulfur dioxide (SO_2), are taxed with $ 0.1 per ton[6].

This paper aims at providing a formal exercise to evaluate a green tax policy for Chile by: (i) explicitly introducing an externality of the sort described above in a model of minimal scale and (ii) analyzing the welfare improvement as a result of the implementation of a tax that is optimal.

The strategy we pursue is to include the externality explicitly in a stylized structural model (a DSGE model) because climate change and corrective measures are implemented under uncertainty, involve dynamic behavior of agents and take a long time for the effects to be evaluated. The model assumes that Chile is a small open economy that trades with the rest of the world and cannot control other countries' level of emissions (though we assume they are disciplined by the Kyoto Protocol). The global amount of CO_2 released into the atmosphere each period feeds the carbon cycle. Since the stock of CO_2 affects productivity, the rational regulator seeks to put a ceiling on the amount of emissions. This can be implemented via an optimal tax as in Golosov *et al*. (2011). They showed that the structure of this tax depends on time discounting rate (i.e. how much agents care about future generations), the scope of the externality damage and the carbon cycle structure (depreciation, etc.).

The stylized model assumes that the electricity sector is the only one that produces the negative externality. The assumption is coherent with data available from the World Resources Institute. Based on data from 2005 (illustrated in the figure of Appendix A.3), Energy accounts for 66.5% of the total CO_2 emissions, agriculture, 13.8%, Land use change, 12.2%, Industrial processes, 4.3%, Waste, 3.2%. Then, the

[6] Message of the President of Chile 24-362 to the Chamber of Deputies on April 1st 2014. An extensive account of the countries that apply carbon taxes is in http://www.carbontax.org/progress/where-carbon-is-taxed/ .

economic impact of the tax will depend on two opposite forces. First, a higher tax increases the relative price of energy, demand and production drops and gas emissions are lowered. As consumption and energy consumption fall, welfare should diminish. Second, in the opposite direction, the reduction of total energy produced abates emissions, moderating the stock of CO_2 dynamics in the midterm. Consequently, it will have a positive impact in GDP, leading to increases in consumption and welfare. As a speculative result, the net effect in welfare is ambiguous[7]. The assessment of the effectiveness of the Pigouvian tax on welfare is done with the aid of simulations from a stylized model calibrated with Chilean data. To the best of our knowledge, this is the first paper in the subject that focuses on small open economies. We take the case of Chile as an example.

The rest of the paper is organized as follows: Section 2 reviews the literature. Section 3 presents the model with its decentralized solution. Section 4 presents the planner's problem and derives the optimal tax. Section 5 discusses the calibration of key parameters. Section 6 reports the results. Section 7 concludes.

2. LITERATURE REVIEW

The issue of climate change and the market failure problem described in the previous section traditionally belonged to a field known as "environmental economics". Several leading institutions are working on environmental economics programs whose agenda undertakes theoretical or empirical studies focusing on the costs and benefits of alternative environmental policies to deal with air pollution, water quality, toxic substances, solid waste, and global warming[8].

In the early development of the field, we find the seminal contribution by Weitzman (1974) who built a partial equilibrium model where the interesting case is under uncertainty. The discussion revolves around which is the most efficient instrument to first best: a cap on quantities vs set taxes. The economic setting is static and simple. The equilibrium results from the intersection between the marginal benefits and marginal cost curves (of reducing one unit of CO_2 emission)[9].

Nordhaus (2007) pioneered the literature on climate change that uses dynamic general equilibrium models. He chooses carbon taxes as the policy instrument and argues that the main reason is that taxes capture the dynamic of Greenhouse Gas (GHG) economic damages in a simple way. Moreover, a carbon tax directly compensates for the present discounted value of the reduction in future utility flows due to the higher

7 In the absence of energy consumed by agents and its positive effect on welfare, the optimal Pigouvian tax should follow the classical Chamley's rule: to maximize welfare the tax rate should be zero.

8 For example, prominent programs are at the National Bureau of Economic Research (NBER), the World Bank and the European Commission.

9 More in detail, the marginal benefit curve draws in the space \$/quantity of Greenhouse Gas (GHGs) and quantity of GHGs. It is confronted with the marginal abatement cost (MAC) curve, which is the locus of points showing the options to reduce pollution available for an economy.

warming caused by the marginal emission. The author also provides reasons for not choosing a cap and trade system (like the advocated by the Kyoto Protocol).

Recently, Golosov *et al.* (2011) based on Nordhaus and Boyer (2003) build a dynamic stochastic general equilibrium (DSGE) model for the global economy with a climate change externality that impacts negatively on the technical progress of the world economy. This assumption is equivalent and also good approximation to the quadratic cost term that Nordhaus assumes, which is a function of global temperature[10]. Thus, in the setting of Golosov *et al.* , there will be a functional form that links the CO_2 stock to damages, which are proxied by a proportional reduction of GDP. The amount of CO_2 in the economy is, in part, determined by the structure of the assumed carbon cycle. Nordhaus and Golosov suppose a quite simple process for the carbon cycle. Under the assumptions of Golosov, they find a closed form expression for the Pigouvian tax that is optimal in the sense that it allows the economy to reach the first best. The strategy is to equalize the solutions of the central planner problem and the decentralized economy. The optimal tax that results is a factor of the GDP evolution, in particular, a function of parameters and exogenous processes such as: the discounting rate, the externality damage and of the assumed carbon cycle process.

Riveras (2009) builds a stylized DSGE model to provide an explanation of the negative supply shock in the electricity sector that hit the Chilean economy in 2006 when Argentina blocked unilaterally gas pipelines.

Heutel (2012) estimates an inelastic relationship between the cyclical components of CO_2 emissions (which produce a negative externality in production) and US GDP. He calibrates a stylized DSGE model and finds that the optimal "green" policy allows carbon emissions to be procyclical, that is increasing (decreasing) during expansions (recessions). However, the policy intervention (either the tax rate or quota) dampens the procyclicality. The paper also examines the decentralized economy equilibrium. In the footnote 13 on page 249, the author suggests that he followed an alternative modeling choice in which "utility" is diminished by pollution, rather than in production. The results remain the same. For that reason, we introduce an externality that diminishes the supply.

Our contribution relates closely with the most recent papers, for example Golosov *et al.* From the latter, we take a similar approach to model the climate change externality and we follow them in the derivation of the optimal tax. Finally, we borrow some minor assumptions (in the sense the results remain robust) such as, a log utility function, among others. Rivera's paper provides us with a nice structure for modeling the electric system adapted to the Chilean economy. Regarding the Nordhaus global integrated economy model, the main difference is on the focus. While our analysis centers attention on Chile as a small open economy leaving the rest of the world very simple, Nordhaus's aim at constructing a big scale model (closed economy). Besides, we focus on the welfare analysis using the utility function whereas Nordhaus employs an "ad hoc" loss function. Finally, the study by Weitzman is mentioned to reflect its important influence in the policy debate regarding the optimal instrument. For practical reasons, we use the tax instrument in line with recent papers.

[10] The quadratic term is a cost that is a function of the gap between the current temperature and the average temperature in the pre-industrial era.

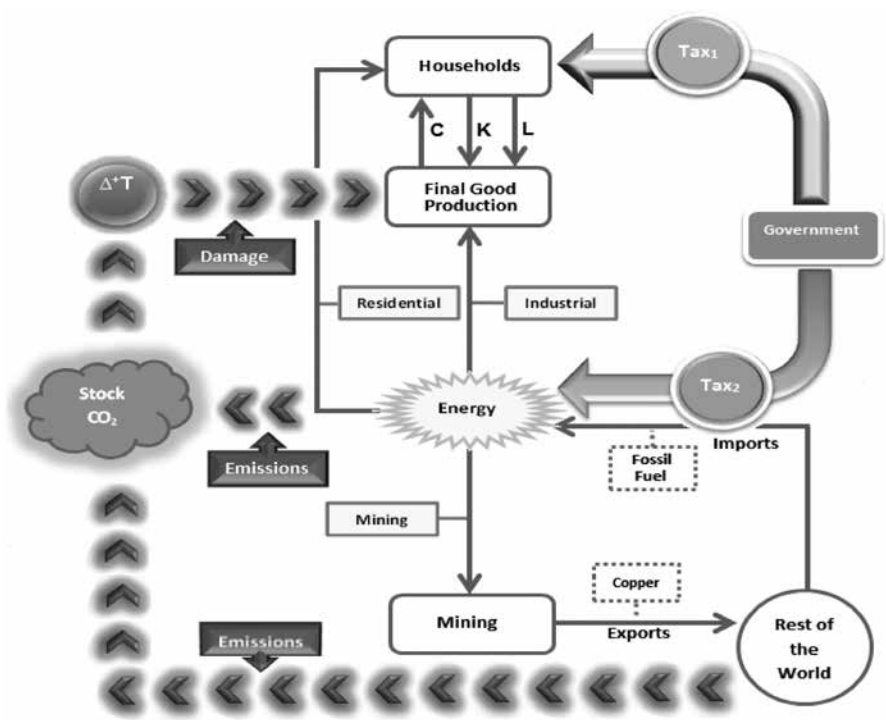
3. THE MODEL

The model represents a stylized economy with five different agents. First, households maximize utility based on consumption (goods and residential electricity), investment and labor decisions. Second, different firms supply final goods, copper (mining) and electricity. Third, there is a government. Finally we add the rest of the world.

Figure 2 illustrates the relationships among the economic sectors that our model embodies. Each box and circle represents a sector of the model while the lines denote trade flows between them. The red lines represent the economic effects caused by CO_2 emissions, coming from domestic and foreign sources. To cope with this problem, the local government that applies a "green tax" to control the amount negative externality produced by the burning of fuel. As in Riveras (2009), the energy inputs are imported and an exporting sector balances the Chilean international trade flow. The rest of the

FIGURE 2

MODEL'S DIAGRAM



section explains the decentralized equilibrium. For the formal competitive equilibrium definitions refer to Appendix A.1.

3.1. Households

The representative household is rational and maximizes a utility function. It is strictly concave and monotonic in each of its arguments: consumption, c_t, the amount of electricity used, e_{rt}, and leisure, $1–l_t$ (with total disposable time normalized to one), subject to a budget constraint that basically assures that outflows are lower or equal than income. We can state the consumer problem, where c_t, e_{rt}, l_t, and investment, i_t, are choice variables, as:

$$\max_{c_t, e_{rt}, l_t, i_t} \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t, 1-l_t, e_{rt})$$

$$st\ c_t(1+\tau_{ct}) + p_t e_{rt} + i_t \le r_t k_t + w_t l_t + \Pi_{gt} + \Pi_{mt} + \Pi_{et} + T_t \tag{1}$$

$$k_{t+1} = i_t + (1-\delta)k_t \tag{2}$$

$$k_0 > 0 \tag{3}$$

where β is the discount factor, τ_{ct} is the tax on consumption. Notice that the consumption price index is set as the *numèraire* and p_t is the residential electricity price relative to consumption goods. We assume that the price of investment equals the price of consumption goods. Sources of income are: (i) income from renting out the capital to goods producers (the rate of return on capital times the units of capital), (ii) wage income (the wage rate w_t times the hours supplied), (iii) profits withdrawn from the firms that households owe (Π_{gt}, Π_{mt}, and Π_{et}), and (iv) net *lump sum* transfers from the government, T_t. Finally, notice that capital depreciates at the rate δ.

The standard consumption Euler equation results from first order conditions (FOC):

$$U'_{c_t} = \beta \mathrm{E}_t \left[U'_{c_{t+1}} \frac{1+\tau_{ct}}{1+\tau_{c,t+1}} \left(1 + r_{t+1} - \delta\right) \right]$$

which states that one unit of additional consumption today is valued the same as the next period consumption that might be purchased with that savings. It implies that consumption is evenly allocated over time. In addition, there are two intratemporal FOC that yield:

$$\frac{U'_{e_{rt}}}{U'_{1-l_t}} = \frac{p_t}{w_t}$$

$$\frac{U'_{c_t}}{U'_{1-l_t}} = \frac{1+\tau_{ct}}{w_t}$$

These ratios state that the marginal rate of substitution between goods must equal their relative prices for an optimal allocation in every period.

3.2. Energy producer firms

The representative energy producer maximizes period by period real profits, which are defined as real energy sales (quantity produced times the price net of tax) minus costs. Energy, E_t, is sold to industrial and mining firms and to households. For simplicity, the variable input is imported fossil fuel (e.g., diesel, coal and natural gas). The production process is simplified in the sense that we assume that fuel must be burned in order to produce energy. The burning process yields an undesirable collateral effect: an amount of pollution, particularly CO_2 is released to the atmosphere. This is a negative externality which is not taken into account by the private energy producer. Specifically, the firm solves:

$$\max_{M_t, E_t} \Pi_{et} = (p_t - \tau_{et})E_t - q_t M_t$$

$$st\ E_t = F_e(M_t) \tag{4}$$

$$\ln q_t - \ln \overline{q} = \rho_q \left(\ln q_{t-1} - \ln \overline{q}\right) + \varepsilon_{qt}, \quad \varepsilon_{qt} \sim N(0, \sigma_q^2) \tag{5}$$

where $F_e(\cdot)$ is a strictly concave production function and has all desirable properties and depends on fuels, M_t, that costs q_t (imports price relative to consumption good price). As the economy is small, $ln\ q_t - ln\ \overline{q}$ follows an exogenous *AR(1)* process.

Notice that the relative price q_t represents for this economy, the real exchange rate (RER). This is because the price index of imported fossil fuel in pesos relative to the consumption price index p_t^c resembles the usual RER definition. As we discussed, since the economy cannot borrow or lend in the financial markets to smooth out consumption it must be the case that the trade balance is in equilibrium[11]. The energy price p_t is augmented by the tax rate τ_{et} to correct the negative effect of the atmospheric externality. The FOC yields:

[11] Following Heathcote and Perri (2002) we define the exchange rate as a ratio of the prices of imported fossil fuels and a domestic aggregate. Formally, if p_t^m describes the import prices, the tradable good, and p_t^c the domestic prices, the non-tradable good, *numeràire* in this case, then the real exchange rate is $q_t = \frac{p_t^m}{p_t^c}$. We will calibrate this parameter so that in steady state the trade balance is in equilibrium.

$$(p_t - \tau_{et})F'_{e,M_t} = q_t \tag{6}$$

which states that energy firms imports until the value of the marginal production net of tax equals the input price.

3.3. Mining sector firms

Mining firms maximize real profits by using electricity to produce copper. All other inputs such as land, labor, etc. are gathered in a fixed production factor and finally normalized to 1.

$$\begin{aligned} \max_{e_{mt}, y_{mt}} \ \Pi_{mt} &= p_t^x F_m(e_{mt}) - p_t e_{mt} \\ st\ y_{mt} &= F_m(e_{mt}) \\ \ln p_t^x - \ln \overline{p^x} &= \rho_{px}\left(\ln p_{t-1}^x - \ln \overline{p^x}\right) + \varepsilon_{px,t}, \ \ \varepsilon_{px,t} \sim N(0, \sigma_{px}^2) \end{aligned} \tag{7}$$

where $y_{mt} = F_m(\cdot)$ is a strictly concave production function that depends on energy. The cooper real price p_t^x is assumed exogenous and follows an *AR(1)* process with unconditional mean equal to $\overline{p^x}$. The first order condition is:

$$p_t^x F'_{m,e_{mt}} = p_t$$

which states that mining firms produce copper until the value of marginal exports equal the relative energy price.

3.4. Final goods firms

Producers maximize real profits by hiring capital, labor and buying industrial electricity to produce the consumption good. Besides these traditional inputs, it is noteworthy that the production function, $F_g\left(A_t, S_t, e_{gt}, k_t, l_t\right)$ is negatively affected by the externality, S_t. As long as the electricity production increases, the emissions of CO_2 also go up, as well as S_t, which triggers a fall in output of the consumption good.

Then, the producer problem is:

$$\begin{aligned} \max_{e_{gt}, k_t, l_t} \ \Pi_{gt} &= y_{gt} - p_t e_{gt} - r_t k_t - w_t l_t \\ st\ y_{gt} &= F_g(A_t, S_t, e_{gt}, k_t, l_t) \\ S_t &= L(E_t) \end{aligned} \tag{8}$$

$$\ln A_t = \rho_a \ln A_{t-1} + \varepsilon_{at}, \ \ \varepsilon_{at} \sim N(0, \sigma_a^2) \tag{9}$$

where y_{gt} is a strictly concave production function. The technical change term A_t follows an *AR(1)* process. Capital and labor real payments are r_t and w_t, respectively. p_t denotes the energy price relative to the consumption price.

Technically, the carbon cycle process means the manner in which an amount of CO_2 is released into the atmosphere by producing a unit of energy, increasing the stock levels. The stock of CO_2 in the atmosphere is represented by the $L(\cdot)$ function whose argument is the electricity produced.

Optimal conditions provide energy, capital and labor demands of the sector:

$$\frac{F'_{g,e_{gt}}}{F'_{g,k_t}} = \frac{p_t}{r_t}, \tag{10}$$

$$\frac{F'_{g,e_{gt}}}{F'_{g,l_t}} = \frac{p_t}{w_t} \tag{11}$$

These equations show that optimal demands are obtained under the equalization of marginal rate of technical substitutions between inputs to their relative input's prices.

3.5. Government

The government taxes consumption and also applies specific taxes to electricity. The latter tax pursues a normative goal which is to correct for the negative externality produced by burning fuel. Total tax income is totally rebated as lump sum transfers; hence, the budget balance is zero for all t. Formally:

$$T_t = \tau_{ct} c_t + \tau_{et} E_t .$$

3.6. Market clearance conditions

In equilibrium, at any period, the following conditions must be satisfied:

$$F_g(A_t, S_t, e_{gt}, k_t, l_t) = c_t + i_t + q_t M_t - p_t^x F_m(e_{mt}) \tag{12}$$

$$E_t = e_{gt} + e_{mt} + e_{rt} \tag{13}$$

$$k_t^s = k_t^d \tag{14}$$

$$l_t^s = l_t^d \tag{15}$$

The first equation is the final goods standard aggregate supply/demand equalization. Similarly, the remaining equations stand for energy, capital and labor market clearance conditions.

4. THE PLANNING PROBLEM

It is easier to derive the optimal Pigouvian tax considering the economy from the planner's perspective. Following Golosov *et al.* (2011), we proceed by imposing equivalence between the planner's solution and the decentralized equilibrium, and then, to seek the optimal tax that support the decentralized allocation[12]. The planner problem is:

$$\max_{c_t, e_{gt}, e_{mt}, e_{rt}, E_t, S_t, k_{t+1}, k_t, l_t} \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t, 1-l_t, e_{rt})$$

subject to equations (2), (4), (5), (7), (8), (9), (12) and (13). In this case, the planner internalizes the effect of the externality in the production function of the good sector, by cutting CO_2 emissions to accomplish the social optimum. The function characterizing the carbon cycle process is $S_t = L(E^t)$, where $E^t = \{E_{-T}, E_{-T+1}, \ldots E_T\}$. Here, $-T$ represent a date at which human emissions began, and accounts for the stock of CO_2 in the atmosphere as a function of energy produced until t.

The traditional textbook treatment establishes that the optimal tax makes the allocation of planner's problem equivalent to that of the decentralized economy. As such, if we find a mathematical expression for a tax that achieves the optimum, then it must take the form of a social cost of carbon.

The planner optimal conditions for c_t, E_t, e_{gt} and M_t are:

$$\lambda_{1t} = U'_{c_t}, \tag{16}$$

$$\lambda_{2t} = -\lambda_t^s - \mu_t, \tag{17}$$

$$\lambda_{2t} = -\lambda_{1t} F'_{g,e_{gt}}, \tag{18}$$

$$\lambda_{1t} q_t = \mu_t F'_{e,M_t}. \tag{19}$$

where $\beta^t \lambda_{1t}, \beta^t \lambda_{2t}$ and $\beta^t \mu_t$ are shadow prices for restrictions (12), (13) and (4), respectively, while λ_t^s is the derivative of L with respect to E_t. By combining (16), (17), (18), and substituting them into (19), we obtain:

$$(F'_{g,e_{gt}} U'_{c_t} - \lambda_t^s) F'_{e,M_t} = U'_{c_t} q_t \tag{20}$$

$$(F'_{g,e_{gt}} - \Lambda_t^s) F'_{e,M_t} = q_t \tag{21}$$

[12] Employing Welfare Theorems, planner and decentralized solutions must coincide. Therefore, the optimal tax is the social cost of carbon.

where $\Lambda_t^s = \dfrac{\lambda_t^s}{U'_{ct}}$. Equation (21) shows that after making a correction due to the externality damage, the marginal value of imports must equal the marginal cost of imported inputs.

Turning to the decentralized economy, we replace the first order condition of the final good with respect to energy into (6) and obtain an expression similar to (21). Then, both problems yield similar results as long the optimal tax is $\tau_{et} = \Lambda_t^s$. In order to obtain an expression for Λ_t^s we turn first to the λ_t^s term, which takes the form:

$$\lambda_t^s = \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \xi_{t+i} \frac{\partial S_{t+j}}{\partial E_t} \tag{22}$$

This equation shows that the marginal costs of emissions equals the expected present value of damages. Taking the optimal condition for S_t in the planning problem, and considering $\beta^t \xi_t$ as the shadow price of restriction (8) in the planning problem, gives:

$$\xi_t = -\lambda_{1t} \frac{\partial F_g}{\partial S_t} \tag{23}$$

Combining (23) with (16), replacing into (22) and also taking into account that $\lambda_t^s = \Lambda_t^s U'_{ct}$, yields:

$$\Lambda_t^s = -\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \frac{U'_{c_{t+i}}}{U'_{c_t}} \frac{\partial F_g}{\partial S_t} \frac{\partial S_{t+i}}{\partial E_t}$$

This is the tax necessary to internalize the externality damage that decentralized markets are not able to price. It considers the present value of the expected damage in the good sector as a consequence of fossil fuel burning in the energy markets and its effect on the dynamics of the marginal utility.

4.1. Additional Assumptions

To better characterize the optimal tax, we make explicit assumptions about key functions:

Assumption 1: The utility function is separable and takes the logarithmic form

$$U(c_t, 1-l_t, e_{rt}) = \theta_c \ln c_t + \theta_l \ln(1-l_t) + \theta_r \ln e_{rt} \tag{24}$$

This choice simplifies the analysis since with this utility function we are able to characterize the optimal tax in closed form; as a function of exogenous variables.

Assumption 2: The final goods firm's technology is

$$F_g(A_t, S_t, e_{gt}, k_t, l_t) = (1 - D_t(S_t))\widetilde{F_g}(A_t, e_{gt}, k_t, l_t)$$

where $1 - D_t(S_t) = \exp\left(-\gamma\left[S_t - \overline{S}\right]\right)$, $S_t = S_t^{ch} + S_t^{rw}$ and $\widetilde{F_g}\left(A_t, e_{gt}, k_t, l_t\right) = A_t e_{gt}^{\alpha_g} k_t^{\eta} l_t^{\nu}$

We assume a constant return to scale Cobb-Douglas production function. The technological change term follows a Hicks neutral process, which is diminished by an exponential damage structure, D_t[13]. The latter depends on: (i) the potential damage of carbon concentration captured by γ and (ii) total carbon deviations from Kyoto 1990 emission goal, $S_t - \overline{S}$, which is a sum of the amount emitted by Chile, S_t^{ch} and by the rest of the world, S_t^{rw}.

Assumption 3: Mining and energy production functions are also Cobb-Douglas (with the only additional assumption that each sector has a fixed input normalized to one):

$$F_m(e_{mt}) = e_{mt}^{\alpha_m}$$
$$F_e(M_t) = M_t^{\alpha_e}$$

Here, mining and energy production functions are assumed to have decreasing returns to scale. The parameters represent each input's share in mining and energy value added, respectively.

Assumption 4: External CO_2 levels follow an *AR(1)* mean reverting process that depends on the degree of commitment to the Kyoto Protocol, $\lambda \in (0,1)$:

$$\ln S_t^{rw} = (1 - \rho_{rw})\ln[\lambda\overline{S} + (1-\lambda)S_{t^*}^{rw}] + \rho_{rw}\ln S_{t-1}^{rw} + \varepsilon_{rwt}, \quad \varepsilon_{rwt} \sim N(0, \sigma_{rw}^2)$$

where $\lambda = 1\ (= 0)$ means full (no) commitment to the Kyoto Protocol and S_t^{rw} represents actual CO_2 levels. Then, the unconditional mean of the process is a linear combination between potential damages and the Kyoto goal.

13 As referred in the literature review, Heutel (2012) suggests that quantitative results (the model's equilibrium and the optimal tax) are similar if the externality diminishes the utility function instead of the production. Assumption 2 makes explicit the damage in the production. The main reason for this choice is that it is intuitively easier to calibrate our production function with data on output and emissions. Moreover, there is previous literature which already measured the impact of CO_2 emissions on output (e.g., Nordhaus, IPCC, etc.). In contrast, the alternative of introducing pollution in the utility function, poses difficult challenges. First, parameters in the utility function are usually difficult to identify from aggregate data. In other words, it is difficult to answer what is the elasticity derived from a variation of 1% in the externality? Second, there are several families of utility functions. Different assumptions translate into changes in optimality conditions and ultimately on optimal taxes. Hence, our results may underestimate the welfare effect.

Assumption 5: As in Engström (2012), the function L has the following carbon cycle depreciation structure:

$$S_t = \sum_{i=0}^{t+T}(1-d_i)E_{t-i} = \sum_{i=0}^{t+T}\varepsilon(1-\varphi)^i E_{t-i} \tag{25}$$

This is the amount of carbon that is left in the atmosphere in $t + i$ periods. The rate of removal of CO_2 from the atmosphere is denoted by φ and ε is the airborne fraction, which is the share of emitted CO_2 that remains in the atmosphere[14]. Removal might be due to uptake by oceans or the terrestrial biosphere for example. The airborne fraction is calibrated as a constant along the business cycle[15].

4.2. Optimal tax characterization

Making use of Assumptions 1-5 above, the carbon tax can be described as:

$$\Lambda_t^s = \mathsf{E}_t \sum_{i=0}^{\infty} \beta^i \frac{c_t}{c_{t+i}} \gamma Y_{t+i}(1-d_i)$$

Dividing by the GDP to express the tax as a fraction of the product, one obtains:

$$\hat{\Lambda}_t^s \equiv \frac{\Lambda_t^s}{Y_t} = \mathsf{E}_t \sum_{i=0}^{\infty} \beta^i \frac{c_t}{Y_t} \frac{Y_{t+i}}{c_{t+i}} \gamma(1-d_i) \tag{26}$$

Assuming constant saving rates[16] and replacing the $(1-d_i)$ term of (25) into (26) gives:

$$\hat{\Lambda}_t^s = \sum_{i=0}^{\infty} \beta^i \gamma(1-d_i) = \frac{\gamma\varepsilon}{1-\beta(1-\varphi)} \tag{27}$$

Then, using logarithmic utility functions, linear depreciation structure, and constant saving rate assumptions we can express the optimal tax as a function of exogenous variables. Three factors determine the optimal tax, discounting, damages and depreciation. In comparison with Golosov *et al.* (2011), we slightly simplify

14 This equation comes from an auto-regressive process in stock emissions $S_t = (1-\varphi)S_{t-1} + \varphi E_t$. We do not follow Nordhaus (2008) and Golosov *et al.* (2011), because we prefer to use a simpler approach. This simplification produces a reasonable approximation.

15 Although there are several studies that have reported an apparent increasing trend in the airborne fraction, Knorr (2009) claims that this trend is statistically insignificant.

16 Though this assumption might look strong, it is empirically validated if one observes long data sets.

the assumption regarding the depreciation structure of the carbon cycle following Engström (2012). This simplification has almost no effect on the results that we will present. To gain further insight, Figure 3 shows how the optimal tax responds to changes in those arguments.

The figure illustrates the sensitivity of the optimal tax to variations of its parameters, keeping the other calibrations fixed. As seen in the figure, the optimal tax depends negatively on the discount rate, this is, if the discount rate is low, the discount factor will be high meaning that future periods are important and it is optimal to tax more carbon emissions. Also, it depends positively on the damage parameter: higher expected damage raises the social cost of carbon, so it is better to increase the tax. Similarly, carbon-cycle parameters indicating that carbon stays in the atmosphere longer (a higher airborne fraction), yields a higher tax. Finally, increases in the removal rate capacity of CO_2 result in a lower carbon tax.

5. MODEL CALIBRATION

This section explains the calibration of parameters used in the model simulations. To do this, we use a sample with official economic variables of the Chilean economy between 1980 and 2012. Energy ratios were calculated using data from Instituto Nacional de Estadísticas (INE) with a sample of similar size, supplemented with data on CO_2 emissions from the EIA[17].

Table 1 shows the calibration of key parameters of the model along with references that support the values chosen.

TABLE 1

PARAMETERS CALIBRATION

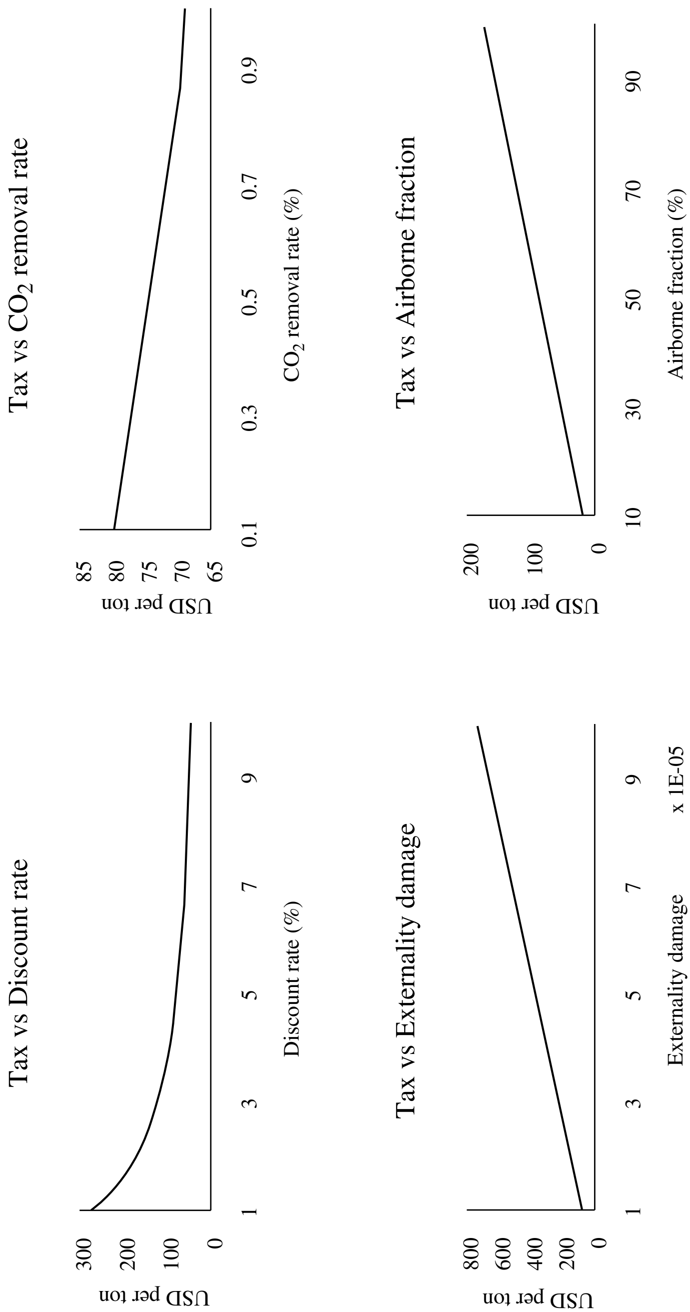
Parameter		Calibration					Source
Discount factor	β	0.95					Bodenstein (2006)
Depreciation rate	δ	0.06					Bergoeing *et al.* (2001)
Firm product elasticities	α_g	0.02	η	0.488	ν	0.51	Riveras (2009)
Mining product elasticity	α_m	0.2					Sectoral GDP, NA
Energy product elasticity	α_e	0.5					Sectoral GDP, NA
Technology shock	ρ_a	0.83	σ_a	0.002			NA, CBCh
Copper price shock	ρ_{px}	0.87	σ_{px}	0.06			London Metal Exchange
Imports price shock	ρ_q	0.71	σ_q	0.07			Imports deflator, CBCh
Foreign CO2 shock	ρ_{rw}	0.48	σ_{rw}	0.015			CO2, EIA

Note: θ_c is set to 1, θ_l and θ_r are constants that replicate the ratio of energy sector value added to total VA. η and ν are capital and labor shares in final good's production function, respectively.
Source: National Accounts (NA) and Central Bank of Chile (CBCh). Annual data: 1980-2012.

[17] For more details, energy statistics are available at www.eia.gov.

FIGURE 3

OPTIMAL TAX ARGUMENTS



The estimation of mining and energy product elasticities are adjusted to match the first moments of each variable. For exogenous processes (e.g. the copper price) the persistence and standard errors of shocks are estimated by fitting an *AR(1)* model[18].

The next three sections will describe in detail the evidence that supports the calibration of key parameters directly associated with climate change. Those related with the degree of commitment to the Kyoto Protocol and the damage function externality parameters are estimated. Others are directly taken from previous literature, such as the ones involving the carbon cycle structure.

5.1. Kyoto Protocol parameters

Global emission levels are a linear combination of actual CO_2 stock levels and the Kyoto 1990 baseline goal (Assumption 4). Accordingly, parameters λ, S_{t*}^{rw} and $\bar{S}$ are calibrated. Beginning with $\lambda \in (0,1)$, it measures the degree of commitment to the Kyoto Protocol by foreign countries. We simulate a deterministic path of emissions that achieves the goals of the Kyoto protocol in 2012 (year in which the Protocol evaluation period ended). That path is our counterfactual because it embodies full commitment to Kyoto. It is confronted with the real commitment, whose source is countries' emissions data from the EIA (2005-2010). In other words, we are able to compare the actual path of emissions versus a time series of emissions coherent with Kyoto. Then, λ is the average ratio between these series. If the optimal is close (far) to the actual path, the commitment is high (low). Our most likely estimate is the foreign commitment degree is $\lambda = 0.6$.

Second, actual stock levels of CO_2 are set to $S_{t*}^{rw} = 852$ GtC (gigatons of carbon)[19]. This value is consistent with 2013 reported emissions. As for the baseline stock levels, we assume it constant, $\bar{S} = 746$, consistent with global CO_2 concentration in 1990. The year 1990 was chosen as a baseline because that was the year the United Nations (UN) first launched negotiations on climate change (Kyoto).

5.2. The carbon cycle

Carbon cycle is the dynamic process in which emissions affect the CO_2 atmospheric concentration over time. In particular, burning fossil fuel generates CO_2 emissions to the atmosphere and these enter into the global carbon circulation system, where carbon is exchanged between various reservoirs (such as atmosphere, the terrestrial biosphere and different layers of the ocean). The balance determines the stock levels of carbon dioxide we observe.

That balance is crucially driven by the rate at which the CO_2 is removed from the atmosphere (see Assumption 5) which, in turn, depends on two parameters. Firstly, φ captures the rate at which carbon is absorbed by oceans, atmosphere and biosphere. Archer (2005) claims that *"…75% of an excess atmospheric carbon concentration has a mean lifetime of 300 year and the remaining 25% always stays forever"*. Given the depreciation structure assumed in (25), we follow Engström (2012) to calibrate

[18] We split cycle and trend by using the HP filter on the series in logs. Then, we fit an *AR(1)* on the cyclical component and save the estimated persistence and the standard deviation.

[19] The conversion factor is 1 ppm by volume of atmospheric CO_2 = 2.13 GtC. See Clark (1982).

$\varphi = 0.005$ which implies that after 300 years, approximately 75% of the carbon dioxide has been removed. Secondly, the airborne fraction, ε, is the fraction of carbon dioxide emissions that remain in the atmosphere. Based on a recent study by Knorr (2009), ε was set to 0.43.

5.3. The damage function

The damage function captures the relationship between the stock of CO_2 and potential damages on GDP. The damage function used in this paper is based on Golosov *et al.* (2011), which is an approximation of Nordhaus (2008) damage function (it is a quadratic function that relates output damages with temperature)[20].

To calibrate the damage parameters, we first turn to the temperature and stock CO_2 function:

$$T_t = T(S_t) = \kappa \ln\left(1 + \frac{S_t}{\bar{S}}\right) / \ln 2$$

where $\bar{S}$ is the CO_2 level in the atmosphere in 1990, fixed at 746 GtC. The κ parameter is set at 3 and was taken from Nordhaus calibration. It links carbon concentration with global mean temperature[21]. A linear and direct application to the Chilean temperature means that it will rise about 3 degrees Celsius, which implies that total stock levels of CO_2 could rise to 1492 GtC, where according to EIA, Chile accounts for a 0.3% of that number[22].

To obtain the exponential function parameter, γ, that maps carbon dioxide concentration to damages as a percent of GDP, we need an estimation of how harmful global warming will be in Chile. Recalling the CEPAL study, at the end of this century, Chile could reduce its GDP by 1.1% due to global warming impact[23]. This, along with the potential reaches of 1492 GtC concentration levels, allow us to obtain an estimation of the exponential parameter by the direct application of Assumption 2:

$$\exp\left(-\gamma[1492 - 746]\right) = 1 - 1.1\%$$

This gives an estimation of $\gamma \approx 1.5 \times 10^{-5}$, which is very close to Golosov estimations[24].

---

[20] Nordhaus assumes that the damage function is $1 - D_N = \frac{1}{1 + \theta_2 T_t^2}$, where T is the mean global increase in temperature above the pre-industrial level.

[21] This is the consensus estimate. There is considerable uncertainty on this sensitivity, see Roe and Baker (2007) for example.

[22] Cline (2007) estimates for Chile that within a century, temperature will rise by approximately 3 degrees, in line with global increases.

[23] That estimation accounts for the direct impact on GDP. It assumes a discount rate of 5% and scenario A2 (one of the worst global warming scenarios defined by the IPCC (2007), where countries intensively use fossil fuels). The scenario implies that average temperatures rise in a range between 3 and 3.3 degrees Celsius by 2100.

[24] They arrive at an estimate of this parameter of $\gamma = 2.379 \times 10^{-5}$ taking base level as pre-industrial atmospheric CO_2 concentration. If one takes 1990 levels as the base, the estimate is $\gamma = 1.853 \times 10^{-5}$, even closer to our calculations.

5.4. Calculation of an optimal carbon tax for Chile

Conditional on the calibrated parameters, the optimal tax derived in (27) provides us with a value for the optimal carbon tax in terms of Chilean GDP. To get a figure comparable with the numbers provided in the literature –calculated for 2012–, we rescale the tax by the Chilean GDP in US dollars adjusted by PPP[25]. As a result, the optimal tax that should have been applied that year was $ 11.95 per ton of CO_2 issued, the same as the tax applied in New Zealand. Golosov *et al*. obtain a tax of $ 59.6 per ton of carbon, while Nordhaus calculates $ 30 per ton of carbon. Not surprisingly, our lower estimate of the tax is partly explained by the fact that it is specific to a small open economy, which represents a small share of the world. Table 2 briefly reports the tax the following years.

TABLE 2

AVERAGE OPTIMAL "GREEN" TAX

	2012	13	14	15	16	17	18	19
Tax ($)	11.95	12.7	13.3	14.1	15.0	15.95	17.0	18.1

6. RESULTS

Before reporting the results notice that there is minor technical issue in solving the model. In the presence of externalities, Wen and Wu (2008) show that there are additional nontrivial nonlinearities that are well captured by second order (or higher) approximations to the policy function.

The first set of results are meant to show that the model's calibration proposed above produces simulated data that allow us to compute key moments that are reasonably close to the equivalent moments of the data. Moreover, we conduct an impulse response functions (IRFs) analysis where we simulate a selection of shocks calibrated in the previous section. We will concentrate on a shock in total factor productivity and on a shock to the copper price. The idea is to show the dynamic evolution of main variables of a small open economy. Next, we report results from a welfare analysis comparing three simulated model cases:

[25] The use of GDP adjusted by PPP is for comparative purposes and follows Golosov *et al*. (2011, footnote 29, p. 29). According to International Monetary Fund (2014), Chilean GDP reached $ 316.407 billion in 2012 and grew to 334.760 billion in 2013. The following years are forecasts taken from http://www.imf.org/external/pubs/ft/weo/2014/01/weodata/weorept. aspx?sy=2012&ey=2019&scsm=1&ssd=1&sort=country&ds=.&br=1&c=228&s=PPPGDP%2CPPPPC&grp=0&a=&pr1.x=83&pr1.y=17#download. In addition, to convert the tax in terms of USD per ton of carbon it is necessary to adjust by a correction factor of 3.21×10^{-7}, which allows one to express GtC in $USD per ton of carbon.

- Economy with no externality;
- Economy with externality, i.e., no carbon tax; and
- Externality with carbon tax.

The comparisons, though crude, provide interesting insights on the effectiveness of the tax policy and quantify welfare effects. Finally, we complement the results with several robustness checks: (i) we assume a more general (a CRRA) utility function, (ii) we alter the parameters that govern the effect the externality, and (iii) we lower and increase the degree of commitment to Kyoto.

6.1. Implied model's moments vs data

In Table 3 we report moments of key variables, specifically, their persistence and volatility. The information contained in the table allows us to compare the unconditional moments of the data with the moments calculated out of simulated data from the model. Therefore, a close match of the moments delivers a good assessment of the model's goodness-of-fit. Overall, the evidence gives ample support to our stylized model and the parameterization chosen.

TABLE 3

MODEL'S MOMENTS CALIBRATION VS DATA

Calibration	Data		Model	
Variables	Volatility (%)	Persistence	Volatility (%)	Persistence
GDP	2.4	0.7	2.1	0.7
Consumption	2.9	0.6	2.6	0.6
Investment	8.1	0.8	8.3	0.7
Labor	1.4	0.7	1.4	0.6
Exports	3.8	0.1	4.0	0.1
Imports	7.4	0.7	7.2	0.7

Note: Annual data 1980-2012.
Data detrended with HP Filter.

In the table, we underscore some empirical facts commonly cited in RBC literature. In particular, for developing economies Agénor *et al.* (2000) established some stylized facts: (i) investment is more volatile than GDP; for Chile, investment is around 3.5 times more volatile than output, (ii) labor is less volatile than GDP; in Chile, it turns out that it is about one half more volatile than output, (iii) consumption is more volatile than GDP; in Chile it is a little more volatile than output. Exports and imports are expected to exhibit higher volatility than GDP. The exports and imports moments of the data are quite close to those of the model. Notice, however, that our model is highly stylized because we assume that Chilean exports are well proxied by copper exports, while Chilean imports are only fuel to be burned in order to produce energy.

Table 3 contains results consistent with Bergoeing and Suarez (2001), Bergoeing and Soto (2002) and Restrepo and Soto (2006). They find that investment and consumption (public and private) are highly volatile, while labor is not as much. Exports and imports are more volatile than GDP.

In brief, our stylized model fares quite well in reproducing key moments of Chilean data. This completes a first validation stage for our model. Next, we examine impulse responses.

6.2. Impulse and response functions

This section analyzes the dynamics of key variables of the model when they are subject to different shocks. We analyze how the propagation mechanism operates.

The first shock is a productivity shock in the final goods firms and the second is in copper price. The first is standard in the literature so we have many references, while the second is of interest for the Chilean economy (see e.g., Medina and Soto, 2007).

Recall from the previous section that the persistence of the shock is 0.83 and the standard deviation of the technology is 0.2%, the size of the initial shock.

The Figure 4 illustrates the effect of a productivity (TPF) shock. The TPF shock (denoted by the continuous line) in the final goods firms increase inputs efficiency leading to a drop in marginal costs and prices. With lower prices, agents want to consume more; therefore, there is an increase in demand. Firms expand production by hiring more labor and investment to increase capital. To attract workers in the final goods sector, firms increase the wages paid and workers are reallocated. Firms also demand more energy for industrial purposes, rising prices and energy traded. The carbon tax goes up as it follows the economic expansion (consumption, GDP, etc.); consequently, the tax collection increases as well. More consumption improves welfare, which is partially offset by less residential energy consumption and leisure.

Additionally, we also plot in the figure the effect of a transitory and a permanent shock. When it is transitory, the effect tends to disappear quickly in the analyzed period (20 years), while when it is permanent it tends to prevail in time. These results suggest two boundaries for the effects that our model predicts.

In relation to the copper price, the persistence is 0.87 and the standard deviation is 6% (initial shock). These numbers are close to the ones reported in García (2011) and Heresi (2011), when using annual data.

The Figure 5 illustrates a copper price shock. It increases copper's sector production; therefore, exports. This shifts energy demand upwards, rising its price and total energy traded in equilibrium. The production of electricity adjusts quickly due to the flexibility in importing fuel from abroad. As energy is an input in goods production, it increases marginal costs and prices, leading to a reduction in sales. The firm substitutes costly energy by hiring more labor and investing more. Moreover, consumers will reduce energy consumption. Thus, these two effects accumulate and explain a drop in output. Since the representative agent owns all firms, the increase in

FIGURE 4

PRODUCTIVITY SHOCK

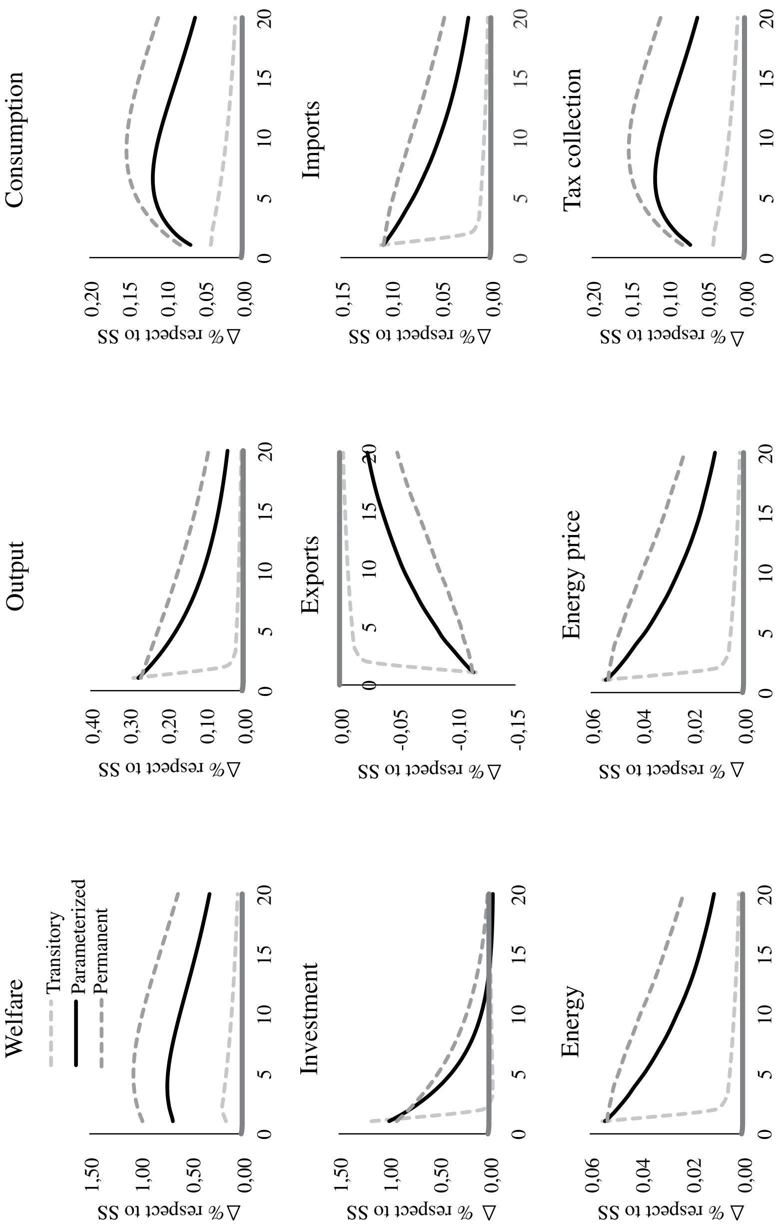
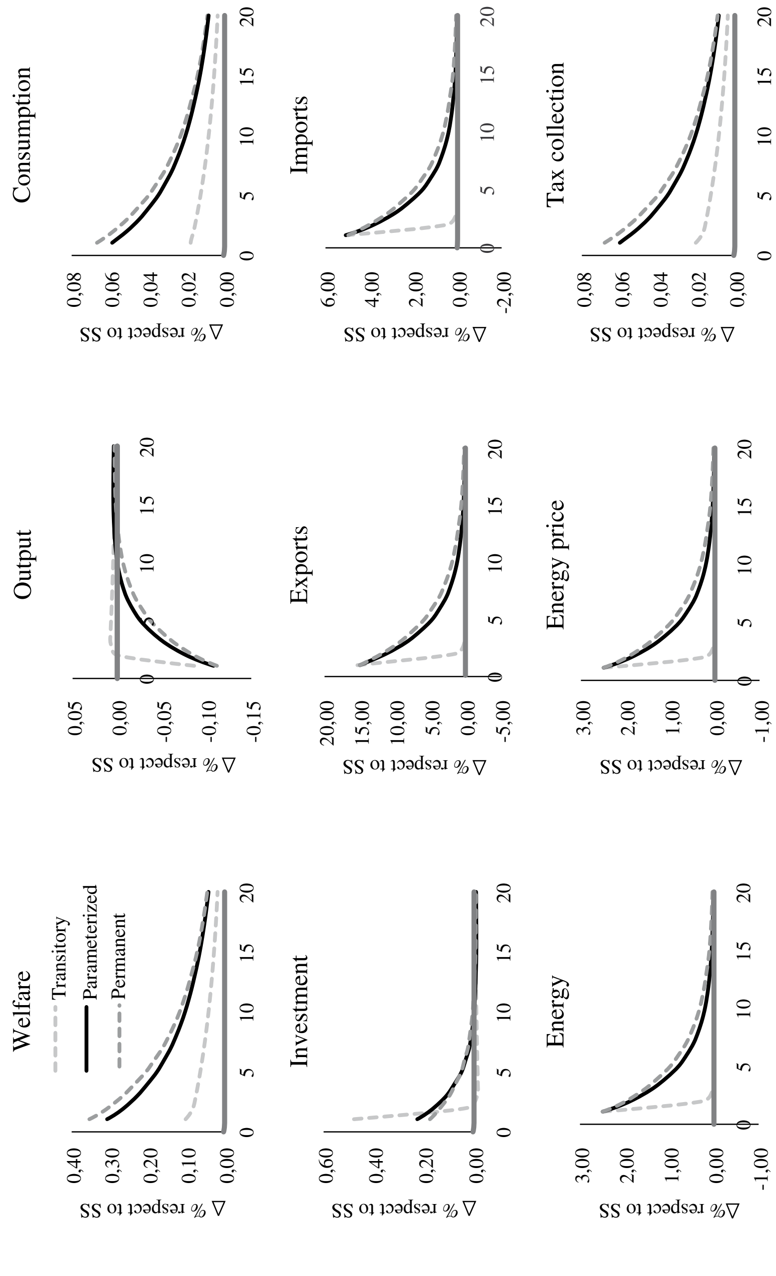


FIGURE 5

COPPER PRICE SHOCK



copper revenues and in the energy sector outweighs losses from the goods producers. This income effect explains higher consumption. The tax collection improves because the tax base improves (more energy produced with higher energy prices). The evolution of welfare shows a positive sign because the positive consumption effect offsets the negative effect based on less consumption of residential energy.

Again, the transitory and permanent shocks are above and below the baseline impulse response.

6.3. Welfare analysis

To perform a welfare analysis we follow the Schmitt-Grohe (2005) approach to calculate an aggregate measure of welfare that considers the whole spectrum of time. This measure is simply a recursive representation of (24):

$$W_t = \theta_c \ln c_t + \theta_l \ln(1 - l_t) + \theta_r \ln e_{rt} + \beta \mathrm{E}_t(W_{t+1})$$

This welfare measure is estimated for three different models: (i) Economy with no externality, which in other words is the first best, which assumes that there is no damage in GDP from uncontrolled emissions, (ii) externality with carbon tax, which is the second best to control emissions because the first best is not achievable, as argued in this paper, and the (iii) Economy with externality and no carbon tax, where there are no restrictions to pollute and nobody internalizes this action.

To address the question of whether there are gains or losses from the application of the tax, we calculate the welfare measure for each of the models. This allows us to compare them in relation to the model economy with externality, which will be taken as the baseline model as it explains the lowest welfare level. By computing the welfare changes with respect to the baseline, we can interpret the numbers as welfare gains. The addition of the optimal tax improves welfare, but the improvement is not the first best. The first best welfare level is reached when there is no externality.

We present the results segmented in two parts. The first reports on the welfare gains derived under the proposed scenario, while the second covers more normative issues by discussing the policy implications.

6.3.1. *Welfare in different scenarios*

This section reports the welfare results under different scenarios. Table 4 provides the welfare indices for each one of the scenarios discussed above in relation to the baseline, which assumes the externality but with no action (notice that the baseline scenario has a welfare index equal to one). The same information is described in terms of welfare (percentage) changes with respect to the baseline scenario. Moreover, to get a better understanding of these numbers, we report the consumption variation for each case in the last column. This was calculated by calculating the mean of the consumption series for each model, and reporting percent changes with respect to the baseline.

In particular, the first set of results show that welfare gains from the carbon tax are of 2.0%. In terms of consumption, it is equivalent to a variation of 0.5%, meaning that the representative agent is willing to forgive 0.5% of their consumption for a tax policy that corrects the externality. Second, in relation to the no externality model, welfare gains amount an increase of approximately 21%, which means that when the first best is achievable, agents are 21% better than in a situation where no restriction measures are taken. In terms of consumption, the representative agent of the economy is willing to pay 9.5% of their welfare to eliminate the adverse effects of climate change.

Additionally, to show the marginal welfare gains achieved by the optimal carbon tax with respect to the outcome in a world with no externalities, we calculate the "tax effectiveness rate", henceforth TER. It results from dividing the welfare gain obtained with the carbon tax by the welfare gain calculated in a scenario with no externality at all. For example, the reported TER in the table is the outcome of dividing 2.0% by 21%, which is equivalent to 10%. This means that the carbon tax policy just reach a 10% of the potential achievable welfare gain which arises from removing the externality.

TABLE 4

WELFARE COMPARISON IN LOG UTILITY MODEL

Model	Carbon tax	Welfare	Δ% W	Δ% C
No externality	No	1.21	21	9.5
Externality	Yes	1.02	2.0	0.5
	No	1.00		
		TER	**10**	

In our exercise the welfare improvement is 2.0%, which means that tax benefits outweigh the costs. Golosov *et al.* (2011) argued that this is an outcome of two contrasting effects. First, there are gradual benefits of the tax, which means that productivity increases due to the carbon tax that limits the externality, thereby consumption and welfare are higher. This effect is also present in Nordhaus and Boyer (2003), where reductions in GHGs in the carbon cycle, translates into greater GDP growth. Second, there is a cost of increasing the price of energy to reflect the higher social cost. In particular, higher energy prices increase marginal costs of final goods firms leading to lower production, consumption and therefore, decrease welfare.

One of the main reasons driving the result lies in the small share of Chile in the global CO_2, which ranges between 0.2% and 0.3%. Therefore, despite the domestic abatement's effort, net reductions have little weight in global CO_2 levels. This comes from assuming that Chile is a small open economy, instead of modeling the economy in a global manner, as in Nordhaus. Thereby, global emissions are a result of the contribution made by Chile, and the rest of the countries. This underscores a problem of coordination because the tax's power solely limits the locally generated externality; it is not possible to target the externality being produced in India or China. In other words, the local optimal tax is insufficient to deal with global climate change.

6.3.2. *Policy implications*

This section discusses policy implications of setting an optimal carbon tax. At first glance, the welfare gains from the optimal tax policy seem to be small. Accordingly, the comparatively large welfare obtained in the economy with no externality scenario with no externality in our understanding is crucial to explain the small figure of the TER indicator. We repeat that the case of no externality is a situation in which emissions do not affect the production of the goods sector. In practice, this hypothetical scenario is implemented assuming that all countries, including Chile, are successful in lowering the emissions to the level of 1990, like countries committed to the Kyoto Protocol[26]. Hence, as emissions are tied to the goal, there are no harmful effects because climate change is frozen, so welfare gains are of 21%.

The magnitude of welfare differential that can be reached suggests that there are sizeable incentives to coordinate the global action. In this sense, the Kyoto Protocol is a first important step or reference, but there are pending important additional gains from increasing participation and commitment to the goals. There are several ways to get to the goals. In this paper we stress the role of optimal taxes. There are other alternative solutions, for example, sharing cleaner technologies among nations, in order to subsidize less developed countries, etc.

A "green" tax policy was under discussion in Chile in 2012. By then, the government, in the framework of a national strategic plan, seriously analyzed the possibility of a carbon tax. The tax was not finally implemented. Again this year the current administration send a proposal of tax reform that includes a "green" tax levied on energy producers. This is under consideration by the Chamber of Deputies.

Although the previous section obtains a small effect in welfare due to the carbon tax, we believe the carbon tax should be implemented for a number of reasons:

- it corrects a market distortion. This is the main argument on economic grounds, because the tax is welfare improving in the sense of Pareto[27]. Therefore, although welfare gains are marginal, the economy is better off with the application of the carbon tax,
- it increases the tax collection, improving government revenues which opens the possibility to lower other taxes[28],

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[26] Recalling the externality function $1-D_t(S_t)=\exp\left(-\gamma\left[S_t-\bar{S}\right]\right)$, there are two ways to achieve the scenario with no externality: assuming that (i) $\lambda=0$, and (ii) $S_t=\bar{S}$.

[27] The Pareto principle of efficiency has implications for welfare. The principle says that it is impossible to make any one further better off (with a reallocation of resources) without making at least one individual worse off.

[28] The mix of different taxes are potentially important because the carbon tax is regressive, i.e., electricity, gas, and other fuels are used relatively more by the poor (see e.g. Metcalf *et al.*, 2010). To asses these

- it improves living standards by reducing CO_2 emissions (Nordhaus and Boyer),
- it allows meeting international commitments. As Chile is a member of Kyoto, a carbon tax should help to meet the goals, and
- it involves superior social awareness on the negative effects of climate change (Stern, 2007)[29].

Finally, to propel policy discussions about the need to mitigate the externality, we believe that it is important to have reasonable simulations from structural models to support decisions. Overall, the importance of a carbon tax in a potential reform seems to be small. However, to mitigate adverse reactions from the public, the extra tax revenues could be accompanied by the reduction of other types of taxes. Policymakers are asked to inquire about the balance desired by the society.

6.4. Robustness

Although our quantitative conclusions are very much model specific, this section examines how our results are affected by changes in key parameters of the benchmark model.

6.4.1. *Changes in the utility function*

To begin with, suppose a more general utility function; for example, suppose the utility function is of the type constant relative risk aversion (CRRA), which assumes constant elasticity of intertemporal substitution:

$$U_t(c_t, 1-l_t, e_{rt}) = \frac{c_t^{1-\theta_1}}{1-\theta_1} + \Omega_1 \frac{(1-l_t)^{1-\theta_2}}{1-\theta_2} + \Omega_2 \frac{e_{rt}^{1-\theta_3}}{1-\theta_3}$$

Following Riveras (2009), we set θ_1 to 1.5, θ_2 to 0.85 and θ_3 to 1.29, while Ω_1 and Ω_2 are arbitrary constants that match the energy ratio and hours worked in steady state[30]. The exercise consists of simulating the model with the adoption of the CRRA utility function. The results are reported in Table 5.

distributional effects and to arrive to a more realistic policy recommendation, a model with heterogeneous agents is needed. This extension is out of the scope of this paper.

29 This public concern could come from different sources, for instance, there could be more academic contributions. In particular, more studies and publications focusing on the net impact of the taxes can be written. Moreover, research in biotechnology may help animal species and new crops to withstand heat, droughts and floods.

30 We take a value for θ_3 such that replicates hours worked in steady state, leaving Ω_2 as close as possible to the unit.

TABLE 5

WELFARE COMPARISON IN THE CRRA UTILITY MODEL

Sensitivity to changes in:		$\theta_1 = 1.5$		$\theta_2 = 3$	
Model	Carbon tax	Δ% W	Δ% C	Δ% W	Δ% C
No externality	No	22	10	28	13
	Yes	2.1	0.6	2.7	0.7
Externality	No				
	TER	**9**		**10**	

The welfare gains derived from optimal carbon tax is 2.1%, while under no externality the potential welfare gain reaches 22%. These results are very close and consistent with those in the log form. Moreover, the representative agent is willing to pay 0.6% of their consumption to mitigate the emissions with a carbon tax and up to 10% to dissipate the effects from global warming. Similarly, the TER we get is 9%. The main reason for this small change is that $\theta_1 = 1.5$ is nearly the same as one. More intense effects can be obtained, for example, assuming $\theta_1 = 3$, which is the exercise shown in the last two columns of the table. In economic terms, a higher θ_1 means lower elasticity of intertemporal substitution $(1/\theta_1)$ or more willingness to smooth out consumption intertemporally.

As a result, the welfare gains augment from 2.1% to 2.7%, while under the no externality case, they go from 22% to 28%. The same conclusion stands when analyzing the variation in consumption. The increase in the TER is very small[31]. In brief, the main conclusions would remain under these robustness checks.

6.4.2. *Sensitivity on externality parameters*

The parameters subject to the robustness check are the damage sensitivity and the airborne fraction. These are key drivers of our results, especially for the determination of the optimal tax. We vary λ and ε 10% in both directions, which is a reasonable range for both parameters based on the literature. For the first parameter, Golosov *et al.* (2011) estimate a certain kind of certain equivalence which is quite stable due to its weights of low frequency but high impact events and high frequency but low impact events. Since this estimation is a measure of all possible scenarios of the economy, a 10% range is quite reasonable. Secondly, the airborne fraction was estimated by Knorr (2009) at 0.43, ranging from 0.4 to 0.46 and moreover, he showed that this fraction is relatively constant in time.

The robustness exercise is summarized in Table 6.

[31] Similar conclusions hold when changing the Frisch elasticity, $1/\theta_2$. Although to save space, these results are not reported, they are available on request.

TABLE 6

SENSITIVITY ON EXTERNALITY PARAMETERS

Sensitivity to changes in:			$\gamma = 1.5 \times 10^{-5}$		$\varepsilon = 0.43$	
Model	Carbon tax	Δ% W	0.9γ	1.1γ	0.9ε	1.1ε
No externality	No	21	13.8	28.2	20.7	21.3
	Yes	2.0	1.82	2.16	1.82	2.17
Externality	No					
	TER	**10**	**13**	**8**	**9**	**10**

The third column of the table shows the actual welfare estimations in each of the models, while column four to eight set the parameters for robustness exercises. First, by changing the externality damage sensitivity to 90% of the estimation cause the carbon emissions become less severe. This implies that both, the no externality and carbon tax models must reduce the gap respect to the baseline model. This is verified in the simulations as estimations decrease from 2% to 1.8% in the carbon tax model, because when analyzing the no externality case, its damaging effect is reduced from 21% to almost 14%. When damages are 10% higher, CO_2 has more severe consequences in GDP. This makes the model differences to be larger, going from 2% to almost 2.2% in the carbon tax case, because it is more necessary, and from 21% to 28% in the first best scenario. Something similar happens when varying the airborne fraction parameter. A lower level means that energy emissions are less likely to stay in the atmosphere as CO_2, meaning that overall damages are lower. In this case the tax power is reduced to 1.8% as it is less necessary, while no externality welfare decrease to 20.7% due to lower damages. When airborne fraction is 10% above the actual estimation, the opposite happens, as externality carbon tax welfare increase to almost 2.2% and no externality case to 21.3%. Despite these changes, welfare gains maintain marginal, as in all when cases the TER does not vary in a significant manner.

These two exercises support the view that our main results remain robust. However, if the rest of the word commits to Kyoto or does not makes a big difference in terms of the TER that can be reached[32].

6.4.3. *Sensitivity on the foreign commitment parameter*

The results of this section are important as Chile only issues the 0.3% of the world CO_2 emissions. Previous results depend on the damage function from Assumption 4 and on the degree of commitment of foreign countries in reducing their emissions. Regarding the calibration of the parameter $\lambda = 0.6$ –"actual" degree of commitment to

[32] A referee suggested making more explicit the effect in the TER caused by increasing the foreign commitment.

Kyoto– is estimated from the data carefully. We use the parameter's standard deviation to construct the sensitivity exercise. We redo the simulation exercise with –/+2 standard deviations of λ, that is $\lambda = 0.45$ and $\lambda = 0.75$ and we report results in Table 7.

TABLE 7

SENSITIVITY ON THE FOREIGN COMMITMENT PARAMETER

Sensitivity to changes in:				
Model	Carbon tax	Δ% W	$\lambda = 0.45$	$\lambda = 0.75$
No externality	No	21	28.7	14.3
Externality	Yes No	2.0	1.9	2.1
	TER	**10**	**7**	**15**

Briefly, we observe that the TER ranges from 7% to 15%, values that are not very different from the baseline scenario. Other simulations with further increases of λ were done and reveal that the increase of the TER is nonlinear (i.e. with positive second derivative w.r.t. λ). For instance, for values larger 0.85, increases of the TER are substantial, disregarding the utility function. In the extreme case of full commitment to Kyoto ($\lambda = 1$). The TER climbs from 10% to 80% (or 74% under the CRRA utility function). This uncovers the intuition that the "green" tax is more effective in a context in which all other countries are committed to Kyoto. This increase in the TER is explained because the denominator in the standard scenario is 21%, while in this extreme exercise is 2.7%. The gap between these TERs reflects the large shift down that the production function suffers due to the reduction of total factor productivity explained by pollution. We remark that the latter case is extreme, but not realistic since it has little support in the data. Consequently, our main results seem robust to reasonable values of λ.

7. CONCLUSIONS

In this paper we have explored the effects of human activity in global warming and climate change. We documented that it is caused by fossil fuel burning that mainly takes place in the process of energy production. This activity generates CO_2 which is released to the atmosphere and stays there causing a significant increase in temperature, damaging our environment. As a result, there are widely documented negative economic effects.

We pointed to a market failure that arises when the agents responsible of causing the negative externality do not take into account these adverse effects on others. Some studies argue that if this tendency continues, by 2100, losses may reach up to 1.1%

of annual GDP. A Pigouvian tax on carbon emissions can be designed to internalize these damages.

With the aid of a DSGE model, we quantified the magnitude of welfare improvements due to the implementation of an optimal carbon tax in Chile. The setting is a standard small open economy that trades with the rest of the world, inhabited by a representative consumer. This traditional setting is enriched with a global externality that arises from pollution which affects negatively domestic productivity.

We follow Golosov *et al.* (2011) and find the optimal "green" tax in closed-form. It equals the marginal externality damage coming from CO_2 emissions and depends on expected damages, on discounting and on the carbon cycle depreciation structure. We estimate that the optimal tax on average is \$12 per ton of carbon for 2012 (end of Kyoto), a value that is lower than those reported by Nordhaus and Golosov *et al.* The lower tax is partly explained by the fact that it is specific to the small open economy, whose pollution emissions' share is small in comparison to the world. For 2013 the tax rises to \$ 12.7 per ton, while it is estimated to reach \$ 13.3 in 2014.

The TER that results is 10% and it is robust to changes in the specification in the utility function (a CRRA functional form delivers similar results) and in the parameters relating to the externality. Although this tax seems to have low impact from the viewpoint of the world, our recommendation is to implement the tax because it corrects a market failure and raises public concern on climate change in the domestic country. Consequently, given the global scope of the problem, policy recommendations should also focus on increasing the number of participants in the Kyoto Protocol. A final robustness exercise reveals that the carbon tax remains similarly effective for reasonable ranges of foreign commitment to cut CO_2 emissions (λ in the range between -/+2 standard deviations).

The contribution of this paper to the literature lies in the formulation of a general equilibrium micro-founded model to analyze the effects of climate change on welfare in Chile. In these terms, there are no similar studies applied to Chile. With this methodology we quantify the welfare gains from applying optimal carbon taxes. A further interesting extension for this paper could be the explicit modelling of the coordination problem among social planners in each country. Finally, if quantitative suggestions with the model are requested, a full estimation of the model would be mandatory (to take the model to the data it should be more interesting: at least has to include financial assets, nominal price rigidities, fiscal and monetary policy).

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A. APPENDIX

A.1. Competitive equilibrium definition

A Competitive Equilibrium in an RBC economy is a "stochastic process" for the collection

$$\{c_t, i_t, k_t, l_t, A_t, E_t, e_{rt}, e_{mt}, e_{et}, M_t, T_t\}_{t=0}^{\infty}$$

and prices

$$\{p_t, \tau_{et}, p_t^x, q_t, r_t, w_t\}_{t=0}^{\infty}$$

such that:

1. Taking the stochastic process $\{p_t, r_t, w_t, T_t\}_{t=0}^{\infty}$ as given, the representative consumer solves:

$$\max_{c_t, e_{rt}, l_t, i_t, k_{t+1}} \mathsf{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t, 1-l_t, e_{rt})$$
$$st\ c_t(1+\tau_{ct}) + p_t e_{rt} + i_t \le r_t k_t + w_t l_t + \Pi_{gt} + \Pi_{mt} + \Pi_{et} + T_t$$
$$k_{t+1} = i_t + (1-\delta)k_t$$
$$k_0 > 0$$

This problem gives optimal paths $\{c_t, e_{rt}, l_t, k_t, i_t\}_{t=0}^{\infty}$.

2. Taking the stochastic process $\{p_t, \tau_{et}\}_{t=0}^{\infty}$ as given, the energy firms solves:

$$\max_{m_t} \Pi_{et} = (p_t - \tau_{et})E_t - q_t M_t$$
$$st\ E_t = F_{et}(M_t)$$
$$\ln q_t = \left(1-\rho_q\right)\ln \overline{q} + \rho_q \ln q_{t-1} + \varepsilon_{qt}, \quad \varepsilon_{qt} \sim N(0, \sigma_q^2)$$

This problem gives optimal sequences $\{E_t, M_t\}_{t=0}^{\infty}$.

3. Taking the stochastic process $\{p_t\}_{t=0}^{\infty}$ as given, the mining firms solves:

$$\max_{e_{mt}} \Pi_{mt} = p_t^x F_{mt}(e_{mt}) - p_t e_{mt}$$
$$st\ y_{mt} = F_{mt}(e_{mt})$$
$$\ln p_t^x = \left(1-\rho_{px}\right)\ln \overline{p^x} + \rho_{px} \ln p_{t-1}^x + \varepsilon_{px,t},\ \ \varepsilon_{px,t} \sim N(0,\sigma_{px}^2)$$

This problem gives the optimal $\{e_{mt}\}_{t=0}^{\infty}$.

4. Taking the stochastic process $\{p_t, r_t, w_t\}_{t=0}^{\infty}$ as given, the final goods firms solves:

$$\max_{e_{gt},k_t,l_t} \Pi_{gt} = y_{1t} - p_t e_{gt} - r_t k_t - w_t l_t$$
$$st\ y_{gt} = F_{gt}(A_t, S_t, e_{gt}, k_t, l_t)$$
$$S_t = L(E_t)$$
$$\ln A_t = \rho_a \ln A_{t-1} + \varepsilon_{at},\ \ \varepsilon_{at} \sim N(0,\sigma_a^2)$$

This problem gives the optimal $\{e_{gt}, k_t, l_t\}_{t=0}^{\infty}$.

5. The government budget constraint is balanced for all t:

$$T_t = \tau_{ct} c_t + \tau_{et} E_t.$$

6. Other conditions hold for all t:

$$\ln S_t^{rw} = (1-\rho_{rw})\ln[\lambda \overline{S} + (1-\lambda) S_{t^*}^{rw}] + \rho_{rw} \ln S_{t-1}^{rw} + \varepsilon_{rwt},\ \ \varepsilon_{rwt} \sim N(0,\sigma_{rw}^2).$$

7. Markets clear for all t:

   a. r_t clears the capital market: $k_t^s = k_t^d$
   b. w_t clears the labor market: $l_t^s = l_t^d$
   c. Energy sector clears: $E_t = e_{gt} + e_{mt} + e_{rt}$
   d. Goods market clears: $Y_t = c_t + i_t + q_t M_t - p_t^x F_{mt}(e_{mt})$

This completes the definition of competitive equilibrium.

A.2. Decentralized optimal conditions

The first order conditions to the decentralized model are:

$$F_{gt}(A_t, S_t, e_{gt}, k_t, l_t) = c_t + i_t + q_t M_t - p_t^x F_{mt}(e_{mt}) \quad (A.1)$$

$$k_{t+1} = i_t - (1-\delta)k_t \quad (A.2)$$

$$U'_{c_t} w_t = U'_{1-l_t}(1+\tau_{ct}) \quad (A.3)$$

$$U'_{c_t} p_t = U'_{e_{rt}}(1+\tau_{ct}) \quad (A.4)$$

$$U'_{c_t} = \beta \mathrm{E}_t \left[U'_{c_{t+1}} \frac{1+\tau_{ct}}{1+\tau_{c,t+1}} \left(1 + r_{t+1} - \delta\right) \right] \quad (A.5)$$

$$p_t = F'_{gt,e_{gt}} \quad (A.6)$$

$$r_t = F'_{gt,k_t} \quad (A.7)$$

$$w_t = F'_{gt,l_t} \quad (A.8)$$

$$p_t = p_t^x F'_{mt,e_{mt}} \quad (A.9)$$

$$q_t = F'_{e,M_t}(p_t - \tau_{et}) \quad (A.10)$$

$$E_t = F_{et}(M_t) \quad (A.11)$$

$$E_t = e_{gt} + e_{m_t} + e_{rt} \quad (A.12)$$

$$\ln A_t = \rho_a \ln A_t + \varepsilon_{at} \quad (A.13)$$

$$\ln S_t^{rw} = (1-\rho_{rw}) \ln \overline{S}^{rw} + \rho_{rw} \ln S_{t-1}^{rw} + \varepsilon_{rwt} \quad (A.14)$$

$$\ln p_t^x = (1-\rho_{px}) \ln \overline{p}^x + \rho_{px} \ln p_{t-1}^x + \varepsilon_{pxt} \quad (A.15)$$

$$\ln q_t = (1-\rho_q) \ln \overline{q} + \rho_q \ln q_{t-1} + \varepsilon_{qt} \quad (A.16)$$

This gives a solution to $\left\{ c_t, i_t, k_t, l_t, A_t, S_t^{rw}, E_t, e_{rt}, e_{mt}, e_{et}, M_t, p_t, w_t, r_t, q_t, p_t^x \right\}_{t=0}^{\infty}$.

A.3. Other figures

FIGURE 6

WORLD GREENHOUSE GAS EMISSION IN 2005