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KNOWLEDGE CREATION AND TECHNOLOGY DIFFUSION: A FRAMEWORK TO UNDERSTAND ECONOMIC GROWTH

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Abstract

The most influential explanations of economic growth along the past five decades rely on two main items: human capital accumulation and the dissemination of knowledge/technological diffusion. These items traditionally appear as separate growth sources. In this paper an integrated perspective is adopted. We begin by building a growth model where two goals regarding technological achievements are considered; economic agents simultaneously want to expand the theoretical knowledge frontier and to reduce the gap between ready-to-use techniques and potentially available knowledge. Considering an objective function that captures the two pointed goals, one develops an intertemporal optimization setup concerning a two sector scenario. The first sector adapts existent technology to productive uses, while the second is an education sector. In this way, we can study the close relationship between technical progress and human capital generation decisions under an intertemporal perspective.

Keywords: *Technology, Human Capital, Economic Growth, Optimal Control, Transitional Dynamics.*

JEL Classification: *C61, O33, O41.*

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I. Introduction

In most economic growth models a human capital variable and a technology variable alternatively arise as the engine of growth. They acquire such quality when introduced in an aggregate production function alongside with physical capital and labor. Solow (1956) and Swan (1956) have demonstrated that under a neoclassical production function, these two last inputs are unable to produce, without anything further, long run sustained growth. Uzawa (1965), Lucas (1988), Caballé and Santos (1993), Mulligan and Sala-i-Martin (1993), Xie (1994), Bond, Wang and Yip (1996) and Ladrón-de-Guevara, Ortigueira and Santos (1999), among others, have modelled human capital formation and they have included such an input in the aggregate production function in order to support the concept of endogenous growth. Romer (1986, 1990), Aghion and Howitt (1992), Jones (1995), Evans, Honkapohja and Romer (1998) and Young (1998) have chosen to put technology in the centre of economic growth explanations, and a time dependent technological index arises in the production function as the vehicle to sustained growth.

All the referred work and all the discussion around aggregate growth models put the several inputs to production at a same analysis level. This means that in both, the physical capital-human capital approach and the physical capital-technology approach, all the inputs that are relevant to growth have to appear as arguments in the final goods production function. An early attempt on the interpretation of economic growth, Nelson and Phelps (1966), avoids such a straightforward view. For these two authors, it makes sense to assume that human capital serves as a means to generate and spread technology and that technology is then usable in the physical goods production function. Recognizing that the Solow-Swan paradigm is valid when assuming a fixed level of technology, we may continue to use such a framework but replacing the constant level of technology by a time dependent variable, where the respective time path is determined by two factors: investment in humans and technological diffusion.

Recent literature points in this direction. For example, Aghion, Howitt and Violante (2003) state that

> "skills are not just an input to the production of goods and services but also a factor in the creation and absorption of new technological knowledge" (page 444).

and Acemoglu (2003) reemphasizes that

> "Nelson and Phelps (1966) postulated that human capital is essential for the adoption of new technologies. This view has at least two important implications: First, the demand for skills will increase as new technologies are introduced, and second, economies with high human capital will effectively possess better technologies" (page 465).

Also Benhabib and Spiegel (2002) recover the Nelson-Phelps approach, presenting a generalized model, where different possibilities about technology diffusion patterns are assumed. Both, the technology frontier - current productivity level relation and the impact of human capital accumulation over technology results, are considered in this analysis.

In this paper the Nelson-Phelps approach is revisited. We study growth giving particular attention to technology and human capital and attributing a supporting role to the capital accumulation constraint of Solow and Swan, and to the consumption utility intertemporal maximization framework of Ramsey (1928), Cass (1964) and Koopmans (1964). Our main argument is that since it is the human capital and the technology variables that determine growth, we can find the basic growth results and dynamics through the analysis of the factors that influence the time evolution of these variables. The Solow-Ramsey framework becomes accessory and it may be used solely to justify that main economic aggregates (*per capita* output, physical capital per labor unit, *per capita* consumption) follow a same long run growth rate as the one that is found for the technology variable.

Modelling human capital at a different level than it is usually modelled in economic growth models will allow for two new results. First, we will be able to define a human capital production function where decreasing returns are compatible with long run constant *per capita* economic growth, what implies purging a counterfactual aspect of two sector growth models: the linearity problem raised by Solow (1994) and Jones (2003) is in this way eliminated. We do not have to assume any kind of artificial knife-edge linearity to encounter a constant long run growth rate. The second result will seem, at least at a first glance, a more awkward one. The way in which we will model human capital will lead to a long run steady state where all relevant *per capita* variables (physical capital, output, consumption, technology) grow at a same rate while human capital will exhibit a zero growth rate in the long run solution. Growth models generally present human capital as growing with the other variables. Here, we arrive to the result of convergence of the human capital variable to a constant long run value. This may be supported on the idea that we may expand techniques and physical goods indefinitely but there is a limit to the expansion of human capabilities.

In what concerns the modelling of technology we follow the Nelson-Phelps approach in distinguishing two concepts of technology. First, there is a theoretical level of technology or a technological possibilities frontier; second, we assume a level of technology in practice, which is a fraction of the first and that can be applied directly to productive uses when embodied in an aggregate final goods production function. Solow (2003) also points to a same kind of distinction:

> "Endogeneizing technological progress is a fine idea, provided that it can actually be done in a sensible way (…) the growth literature proceeds as if all improvements in total factor productivity originate in research and development activities. That is what gets modelled. But it is certain that an appreciable fraction of 'technological progress' has nothing to do with R&D. It originates on the shop (or office) floor and is 'invented' by workers and

managers who see that there are improvements to be made in the location of activities, the flow of work, the way part A is fastened to part B, and so on" (page 548).

The distinction concerning theoretical knowledge and applied techniques has been widely discussed in the literature, from several points of view. For instance, Bresnahan and Trajtenberg (1995), Helpman and Trajtenberg (1994, 1996) and Helpman (1998) make reference to the pervasiveness of general purpose technologies (GPTs). The important argument advanced by these authors is that GPTs, like electricity or microelectronics, are generated discontinuously in time and in an exogenous way relatively to market mechanism incentives. A new GPT (a kind of scientific not directly applied discovery) generates two subsequent phases on the evolution of the economic system. The first phase, that this group of authors calls "time to sow", is characterized by a transfer of resources to the production of applied techniques and tools that make use of the new GPT. This first stage will almost certainly imply low or negative productivity growth. In the second phase, called "time to reap", there is already a generalized use of the GPT under the form of complementary inputs, making it possible, under network effects, to boost productivity growth.

This view of knowledge creation and technology adoption supports a business cycles perspective. Growth declines or accelerates as a result of the dynamics regarding the creation, development and spreading of technologies. This is also the view of several other authors, namely Andolfatto and MacDonald (1998), Boldrin and Levine (2001), Hall and Khan (2003), Mukoyama (2003) and Stiver (2003). These authors develop conceptual frameworks where aggregate fluctuations tend to arise from the discovery and diffusion of new technologies and, thus, exogenous shocks can be excluded from the analysis and from the explanation of growth dynamics. Under this perspective, both expansions and recessions are the result of good news, that is, periods of high growth mean the generalized adoption of a given new scientific / technological knowledge, while periods of low or even negative growth will correspond to periods where the technological frontier is expanding but the direct productivity gains of such expansion are not yet visible in the economic system.

Furthermore, the cited authors call the attention to the fact that inventions often occur as sudden events, while diffusion is a slow continuous process which involves a great number of individual decisions. Nevertheless, there is the clear conscience that it is diffusion and not creation that ultimately determines the pace of productivity change and economic growth. Also Keller (2002, 2004) discusses technical change and diffusion, in this case focusing the attention on international patterns of knowledge dissemination.

While the cited authors analyze in detail the nature and linkages of adopting sectors, the speed and the extent of the diffusion process, and the social and private benefits of the innovation - diffusion mechanism, these do not constitute the main concern in our present analysis. Instead, we just focus on the logical observation that without theoretical knowledge generation, there is no possible application, and hence, the absence of the use of new technologies in production implies that growth will not occur.

Another difference in approach relates to the fact that we consider the creation of basic knowledge as endogenous – the model to be developed is a social planner problem that attributes to the government a very meaningful role in the management of knowledge creation and diffusion. The government has a fundamental word to say with respect to R&D allocation of resources. If authorities choose to expand the knowledge frontier they concentrate public resources with such a purpose. If, instead, government is mainly concerned with giving to the society the possibility to diffuse knowledge in order to create wealth, fewer resources will be allocated to public R&D expenditures, and thus the knowledge frontier does not move outwards.

Therefore, our model furnishes a partial explanation of the growth process, neglecting the role of capital accumulation, savings, population growth and consumer preferences. It focus the attention on the group of features that in any society, even in the ones which have more liberal economic regimes, are under the control (at least partially) of a social entity. The social planner has the responsibility to allocate more or less resources to innovation and diffusion (the resources available for diffusion are the ones that the state does not withdraw to creation, and that simultaneously are not diverted to other private or public uses).

The goal of the decentralized economy may consist mainly or solely in having the higher level of applied knowledge it can get, in order to accelerate growth, but the authority has an allocation of resources responsibility at two levels: first, it has to have conscience that if no knowledge is created, the society cannot apply it in any way; second, the generation of scientific knowledge is also an end in itself, that is, any society, in order to progress and stand on the international arena, has to be able to display cultural and scientific potential. These points will support the introduction, along the next section, of a social utility function where knowledge adoption and knowledge generation both appear as arguments.

The treatment that we will give in the following sections to technology choices will imply an optimal control problem, where the goals (of a social planner) are simultaneously to amplify the frontier of technological knowledge and to approximate the level of ready to use technology to such frontier. This optimization problem is constrained precisely by the Nelson-Phelps motion equation that characterizes the relation between the two technology concepts.

The setup under consideration relies on a trade-off respecting technology choices: to get new inventions to be available to produce final goods one has to renounce to some of the growth of the basic science frontier and vice-versa. Economic resources are scarce in the sense that they cannot be used in a simultaneous way to give birth to new ideas and to make these ideas immediately ready to productive ends. We will end up with a two sector growth model similar to the ones in the literature already referred along this introduction, nevertheless with some important new features: physical capital and consumption are not nuclear in our analysis, the two sectors generate embodied (human capital) and disembodied (technology) knowledge and the fundamental choices are not about consumption and savings but about technology production and technology diffusion.[1]

In short, we rely on the Nelson-Phelps approach in considering a technology production function where human capital and a technology gap appear as inputs and we make a deeper analysis by considering the process through which human capital is produced and by considering as well, in alternative to an exogenous technology growth rate, a controllable technology growth rate. The model that we will be faced with is an optimal control growth model where the only variables that are considered are the fundamental growth sources: human capital and technology.

The remainder of the paper has the following contents. Section II develops the structure of the model and refers to the steady state results. Section III discusses the nature of the balanced growth path and characterizes the transitional dynamics in the neighbourhood of the steady state. Section IV concludes.

II. A Two Sector Model of Optimal Technology Choices

Take technology and human capital as the engines for economic growth. The first variable, technology, may be decomposed in two. The technology possibilities frontier will be variable $T(t)$ and a ready to use in production technology variable is represented by $A(t)$. In every time moment $A(t) \leq T(t)$, representing both variables positive quantities for all $t>0$. We will be concerned with understanding the temporal evolution of a gap variable $\phi(t) \equiv A(t)/T(t)$, $\phi(t) \in [0,1]$, that has a straightforward interpretation: the higher the value of $\phi(t)$ the smaller is the gap between the values of the technology variable representing knowledge immediately available to produce and of the technology variable representing the scientific state of the art.

Other fundamental variable is the growth rate of the technology frontier: $\tau(t) \equiv \dot{T}(t)/T(t)$, $T(0)=T_0$ given. In our model this growth rate is assumed as a control variable for the representative agent (as discussed in the introduction, this is the public authority); that is, in what concerns technology decisions, the authority is able to choose the rate at which scientific progress occurs.[2] Nevertheless, obviously this must be a constrained decision process because to allocate more or less resources to research activities implies an opportunity cost respecting to the resources that remain available for other economic activities. In particular, in our framework we assume that the choice in terms of basic technology progress is constrained by the necessity of using economic resources to apply technology to the goods production process. A trade-off emerges between our first two endogenous variables: $\phi(t)$ and $\tau(t)$.

In what concerns technology choices, the basic economic goal of the social planner is to maximize the intertemporal stream of $v(\phi(t),\tau(t))$ functions, being a function v defined as follows.[3]

Definition 1. Function v. *The representative agent that makes technology choices faces a real valued objective function v:* $\Re_+^2 \mapsto \Re$ *that obeys the following properties:*

i) *Continuity, concavity and smoothness.*[4]

ii) $v_\phi \cdot \dfrac{\phi(t)}{v} = \theta \in (0,1);\ v_\tau \cdot \dfrac{\tau(t)}{v} = \mu \in (0,1)$.

The second condition in definition 1 defines constant elasticity parameters. This condition ensures that the utility of straightening the technology gap and the utility of increasing the pace of technological progress are, for the representative agent, both positive and diminishing. Note that such an assumption follows from the nature of the representative agent, as discussed in the introduction. Public policy must be oriented to the expansion of the knowledge frontier, as well as to the stimulus to apply technology to production because: (*i*) the government has the responsibility to guarantee an everlasting flow of new knowledge in order to be possible to apply it, and (*ii*) enlarging the knowledge frontier is a vital process to the survival and development of any organized society or civilization.

Therefore, there is a double concern of the representative agent: to expand the technology frontier and to guarantee that such knowledge is applied to generate wealth. Let us remark, once again, that in this process the public sector has control over the amount of knowledge that is created, through the resources it allocates to such purpose, and therefore the growth rate of technology appears as a control variable. The amount of knowledge that arrives to the direct productive activity is not under the government control; as we present bellow, a state constraint will characterize such relation.

To simplify our treatment of the model we will work with a specific functional form of the above defined *v* function. We take a Cobb-Douglas type of function (for instance, a CES function would also be a candidate functional form).

$$v(\phi(t), \tau(t)) = \phi(t)^{1/(1+\sigma)} \cdot \tau(t)^{\sigma/(1+\sigma)} \tag{1}$$

The correspondence between parameters θ and μ in definition 1 and parameter σ in equation (1) is the following: $\sigma = \mu/\theta$ and $\mu = 1 - \theta$.

The trade-off between the two endogenous technology variables, $\phi(t)$ and $\tau(t)$, becomes explicit by considering the Nelson-Phelps technology constraint:

$$\dot{A}(t) = g(h(t)) \cdot (T(t) - A(t)),\ g(0) = 0,\ g' > 0,\ g'' < 0,\ \ A(0) = A_0 \text{ given} \tag{2}$$

With (2) we state that the rate of increase of the index $A(t)$ is a function of human capital [$h(t)$ is a human capital *per* unit of labor variable or a human capital efficiency index] and of the gap that exists between the two technology variables. First and second derivatives of g indicate that there are positive but diminishing returns of human capital in the production of technology. The gap term translates the idea that the level of technology in practice will evolve faster when there is a large gap between technology possibilities and the stock of knowledge instantaneously available to produce.

Recovering variable $\phi(t)$, from equation (2) we arrive to the final form of the first resource constraint of the optimal control problem:

$$\dot{\phi}(t) = g(h(t)) \cdot (1 - \phi(t)) - \tau(t) \cdot \phi(t), \quad \phi(0) = \phi_0 \tag{3}$$

To complete the presentation of the model it is necessary to define a rule for the time evolution of the human capital variable. This can have the standard form in most growth models, i.e., human capital evolves in time through a production process that involves a production function, f, and a constant depreciation rate, δ.

$$\dot{h}(t) = f(h(t)) - \delta \cdot h(t), \quad \delta > 0, \; h(0) = h_0 \text{ given} \tag{4}$$

The human capital variable is, alongside with variable $\phi(t)$, a state variable of the intertemporal control problem. Equations (3) and (4) are the motion equations that constitute the resource constraints to which the optimization problem is subject to.

The analytical tractability of the model demands that we take explicit functional forms for functions $f(h(t))$ and $g(h(t))$. Imagining that it is possible to choose at each moment in time the shares of human capital to allocate to each of the two economic sectors (technology and education sectors), we define a new variable $u(t)$ that represents precisely the share of human capital allocated to the generation of technology. Obviously $u(t) \leq 1$, $\forall t$. The properties of function g were set forth in equation (2). Positive and diminishing returns of human capital in the production of technology imply a function with the following shape:

$$g(h(t)) = a \cdot (u(t) \cdot h(t))^{\eta}, \quad a > 0, \; \eta \in (0,1) \tag{5}$$

Function f may be defined in a similar way:

$$f(h(t)) = b \cdot [(1 - u(t)) \cdot h(t)]^{\beta}, \quad b > 0 \tag{6}$$

For equation (6) we find it essential to impose $\beta \in (0,1)$ in order to encounter a long run balanced growth path. Thus, one assumes that the education sector exhibits diminishing returns in the accumulation of human knowledge. This point was referred earlier in the introduction and should be stressed since it constitutes an innovation relatively to conventional growth models. In our model, where human capital contributes to the generation of physical goods solely in an indirect manner, we must assume diminishing returns in the accumulation of human capital in order to get a long term constant steady state growth.

As a result of this assumption, the human capital *per* unit of labor variable will have a different nature relatively to other inputs: its level will not grow in the long run in opposition to what happens to the several *per capita* variables, namely output, consumption and physical capital. Remember that $h(t)$ defines the skills of an average individual, and so it makes sense to find a result where these tend to a constant value, that is, intuitively it is hard to support the concept that human skills may be improved further and further indefinitely at a constant rate. Technology and machines can be expanded without limit, individual skills cannot - this is a fundamental argument of our analysis.

The optimal control growth problem has now all the necessary ingredients. Definition 2 states what we understand by an optimal solution.

Definition 2. Control problem optimal solution. *An optimal solution is a set of paths* $\{\phi(t), h(t), \tau(t), u(t)\}$ *that solve the maximization problem* $\underset{\tau(t),u(t)}{Max} \int_0^{+\infty} v(\phi(t), \tau(t)) \cdot e^{-\rho.t} \cdot dt$ *subject to constraints (3) and (4) and where functions v, f and g are defined respectively by (1), (5) and (6). All variables assume non negative values, initial values for state variables are given and shares* $\phi(t)$ *and u(t) remain always below unity. Furthermore, it is clear from the problem that we take an infinite horizon and that future technology accomplishments are discounted. A constant discount rate,* $\rho>0$*, is considered.*

Let $p_\phi(t)$ and $p_h(t)$ be co-state variables. We synthesize our model's information into a current value Hamiltonian function:

$$H(\phi(t), h(t), \tau(t), u(t), p_\phi(t), p_h(t)) \equiv$$
$$v(\phi(t), \tau(t)) + \left[a \cdot (u(t) \cdot h(t))^\eta \cdot (1-\phi(t)) - \tau(t) \cdot \phi(t)\right] \cdot p_\phi(t) + \tag{7}$$
$$\left\{b \cdot [(1-u(t)) \cdot h(t)]^\beta - \delta \cdot h(t)\right\} \cdot p_h(t)$$

Applying the Pontryagin's maximum principle, the first order optimality conditions can be computed:

$$H_\tau = 0 \Rightarrow \frac{\sigma}{1+\sigma} \cdot \left[\frac{\phi(t)}{\tau(t)}\right]^{1/(1+\sigma)} = p_\phi(t) \cdot \phi(t) \tag{8}$$

$$H_u = 0 \Rightarrow \eta \cdot a \cdot u(t)^{-(1-\eta)} \cdot h(t)^\eta \cdot (1-\phi(t)) \cdot p_\phi(t) = \beta \cdot b \cdot (1-u(t))^{-(1-\beta)} \cdot h(t)^\beta \cdot p_h(t) \tag{9}$$

$$H_\phi = \rho \cdot p_\phi(t) - \dot{p}_\phi(t) \Rightarrow$$

$$\dot{p}_\phi(t) = \left[\rho + a\cdot(u(t)\cdot h(t))^\eta + \tau(t)\right]\cdot p_\phi(t) - \frac{1}{1+\sigma}\cdot\left[\frac{\tau(t)}{\phi(t)}\right]^{\sigma/1+\sigma} \tag{10}$$

$$H_h = \rho \cdot p_h(t) - \dot{p}_h(t) \Rightarrow$$

$$\dot{p}_h(t) = \left[\rho + \delta - \beta\cdot b\cdot(1-u(t))^\beta \cdot h(t)^{-(1-\beta)}\right]\cdot p_h(t) - \eta\cdot a\cdot u(t)^\eta \cdot h(t)^{-(1-\eta)}\cdot(1-\phi(t))\cdot p_\phi(t) \tag{11}$$

$$H_{p_\phi} = \dot{\phi}(t) \Rightarrow \dot{\phi}(t) = a\cdot(u(t)\cdot h(t))^\eta\cdot(1-\phi(t)) - \tau(t)\cdot\phi(t) \tag{12}$$

$$H_{p_h} = \dot{h}(t) \Rightarrow \dot{h}(t) = b\cdot[(1-u(t))\cdot h(t)]^\beta - \delta\cdot h(t) \tag{13}$$

$$\lim_{t\to+\infty} p_\phi(t)\cdot e^{-\rho.t}\cdot\phi(t) = 0 \tag{14}$$

$$\lim_{t\to+\infty} p_h(t)\cdot e^{-\rho.t}\cdot h(t) = 0 \tag{15}$$

Conditions (8) to (15) are sufficient conditions of optimality given that the optimal Hamiltonian is concave in $[\phi(t),h(t)]$.

The optimality conditions are the relations necessary to prove the following proposition.

Proposition 1. Existence and uniqueness of a balanced growth equilibrium. *Under the condition $\beta+\rho/\delta<1$ there exists a unique balanced growth path or unique steady state four dimensional point that satisfies (8) to (15).*

Proof: To prove the existence of a unique steady state point one has to solve the system $\left[\dot{\phi}(t)\ \dot{h}(t)\ \dot{\tau}(t)\ \dot{\mu}(t)\right] = \overline{0}$. The solution for this system consists on a set $\left\{\overline{\phi}, \overline{h}, \overline{\tau}, \overline{\mu}\right\}$ of constant values. To solve the system one is compelled to find equations of motion for the two control variables. Let us start with $\tau(t)$.

Differentiating (8) in order to time, the following growth rates relation is obtained:[5]

$$\gamma_\tau = -\sigma\cdot\gamma_\phi - (1+\sigma)\cdot\gamma_{p_\phi} \tag{16}$$

Replacing γ_{p_ϕ} and γ_ϕ in (16) by the corresponding expressions attainable from (10) and (12), the motion equation for $\tau(t)$ comes

$$\dot{\tau}(t) = \left[\frac{1}{\sigma}\cdot\tau(t) - (1+\sigma)\cdot\rho - a\cdot(u(t)\cdot h(t))^{\eta}\cdot(1+\sigma/\phi(t))\right]\cdot\tau(t) \qquad (17)$$

The time evolution of $u(t)$ is derived from the following growth rates relation that is true under (9):

$$\gamma_u = \frac{1-u(t)}{(1-\eta)-(\beta-\eta)\cdot u(t)}\cdot\left[\gamma_{p_\phi} - \gamma_{p_h} - \frac{\phi(t)}{1-\phi(t)}\cdot\gamma_\phi + (\eta-\beta)\cdot\gamma_h\right] \qquad (18)$$

We arrive to

$$\dot{u}(t) = \frac{1-u(t)}{(1-\eta)-(\beta-\eta)\cdot u(t)}\cdot\left\{\left[\frac{\phi(t)}{1-\phi(t)} - \frac{1-\sigma}{\sigma}\right]\cdot\tau(t) + \right.$$
$$\left.\left[\eta+(\beta-\eta)\cdot u(t)\right]\cdot b\cdot\left[(1-u(t))\cdot h(t)\right]^{-(1-\beta)} - (1-\beta+\eta)\cdot\delta\right\}\cdot u(t) \qquad (19)$$

Equations (17) and (19), alongside with the two resource constraints, constitute the system from which we derive the steady state solution. The system has in fact a unique solution, which is

$$\begin{bmatrix}\bar{\phi}\\ \bar{h}\\ \bar{\tau}\\ \bar{u}\end{bmatrix} = \begin{bmatrix}\dfrac{(1-\sigma)\cdot\bar{g}}{\sigma\cdot\rho+\bar{g}}\\ \left[\dfrac{b}{\delta}\cdot(\beta+\rho/\delta)^{\beta}\right]^{1/(1-\beta)}\\ \dfrac{\sigma}{1-\sigma}\cdot(\rho+\bar{g})\\ 1-\beta-\rho/\delta\end{bmatrix} \qquad (20)$$

with $\bar{g} = a\cdot\left(\bar{u}\cdot\bar{h}\right)^{\eta}$. If $\beta+\rho/\delta<1$ then we guarantee that $\bar{u}\in(0,1)$ and this is the only boundary condition that must be imposed in order to (20) to be a feasible four dimensional steady state point■

The steady state results deserve some comments. First, we notice that the share $\bar{u}$ depends upon parameters β, ρ and δ. The higher the elasticity parameter

β and the discount rate the lower the value of the share of human capital allocated to technological development. The faster the depreciation of human capital the more this form of capital is allocated to its own production relatively to a technological use. Second, the human capital efficiency index is indeed a constant amount on the balanced growth path. The better the sector technical conditions (the higher the value of *b*), the higher the elasticity parameter β, the higher the discount rate and the lower the depreciation rate, the larger will be the accumulated human capital stock under a long run perspective. Third, we observe that $\bar{\phi}$ obeys the boundary condition $\bar{\phi} \leq 1$, because $\rho > 0$ and $\bar{g} > 0$. Fourth, the technology growth rate arises in the steady state depending on a multiplicity of factors, namely: (i) the objective function parameter, (ii) the intertemporal discount rate, (iii) both sectors education functions parameters (*a* and *b*), (iv) the elasticity parameters (β and η) and (v) the depreciation rate.

For the steady state growth rate result,

$$\bar{\tau} = \frac{\sigma}{1-\sigma} \cdot \left\{ \rho + a \cdot \left[((1-\beta) - \rho/\delta) \cdot \left((b/\delta) \cdot (\beta + \rho/\delta)^{\beta} \right)^{1/(1-\beta)} \right]^{\eta} \right\} \tag{21}$$

we highlight that the partial derivatives $\partial\bar{\tau}/\partial\sigma > 0$, $\partial\bar{\tau}/\partial a > 0$, $\partial\bar{\tau}/\partial b > 0$ and $\partial\bar{\tau}/\partial\eta > 0$ have unquestionable signs. The same is not true for β, ρ and δ because when changes in these factors benefit the accumulation of human knowledge (make *h* to rise) they injure the transference of human capital to innovation purposes (make *u* to fall).

At this stage, found the long term technology growth rate, we could transform the model by adding to it a physical capital accumulation constraint that should include a labor augmenting aggregate production function; then, we would get an economic growth framework. This framework will not be developed, but the results that its introduction would bring can be stated in some detail. Under such a scenario the growth rate in (21) would be the steady state economic growth rate for the various *per capita* aggregates: physical capital, consumption and output. In such a model we have endogenous growth through technology choices, that is, long run constant *per capita* growth is obtained and this is a function of several parameters of our analysis. These parameters concern the way in which the economy is able to create and diffuse technology and to invest in human capabilities. The accomplished growth rate is precisely the result of the way the economy handles the two true engines of growth: human capital and technology.

III. Stability and Dynamics

To make a meaningful analysis of the stability and dynamics of the model we have to take some simplifying assumptions over the problem proposed in the previous section. Under the two following assumptions we prove that the model is saddle-path stable and we proceed to the characterization of the dynamic behavior of the various endogenous variables.

Assumption 1*. Let* $\phi(t) = \phi \in (0,1)$*, for all time moments.*

Assumption 2. *Let the following constraint over parameter values hold:* $\sigma = \beta /(\eta - \beta)$.

Assumption 1 eliminates the variable $\phi(t)$ of the model, and from (3) we realize that under this assumption the other three variables are not independent from each other any more, that is, for every moment of time is now true that

$$\tau(t) = a \cdot \frac{1-\phi(t)}{\phi(t)} \cdot (u(t) \cdot h(t))^{\eta} \tag{22}$$

Assumption 2 implies the condition $\eta > 2 \cdot \beta$ if we want $\sigma \in (0,1)$ to continue to hold. The optimization problem is hereafter the one in definition 3.

Definition 3. Optimal control problem with a fixed technology gap. *Let assumptions 1 and 2 hold. The goal of the representative agent is now to maximize* $\int_0^{+\infty} \phi^{(\eta-2\cdot\beta)/\eta} \cdot [a\cdot(1-\phi)]^{\beta/\eta} \cdot [u(t)\cdot h(t)]^{\beta} \cdot e^{-\rho . t} \cdot dt$ *subject to* $\dot{h}(t) = b \cdot [(1-u(t))\cdot h(t)]^{\beta} - \delta \cdot h(t)$, $h(0)=h_0$.

To prove the existence of a unique steady state one has to proceed as in the previous section. Necessary optimality conditions are:

$$\phi^{(\eta-2\cdot\beta)/\eta} \cdot [a\cdot(1-\phi)]^{\beta/\eta} \cdot u(t)^{-(1-\beta)} = b \cdot [1-u(t)]^{-(1-\beta)} \cdot p(t) \tag{23}$$

with $p(t)$ a shadow-price variable. And

$$\dot{p}(t) = \left\{\rho + \delta - \beta \cdot b \cdot [(1-u(t))\cdot h(t)]^{-(1-\beta)}\right\} \cdot p(t) \tag{24}$$

$$\lim_{t\to+\infty} p(t)\cdot e^{-\rho . t} \cdot h(t) = 0 \tag{25}$$

The constraint (4) is also an optimality condition.

From the optimality conditions we arrive to the expression for the time movement of $u(t)$,

$$\dot{u}(t)=\frac{1}{1-\beta}\cdot\left\{\beta\cdot b\cdot[(1-u(t))\cdot h(t)]^{-(1-\beta)}-(\rho+\delta)\right\}\cdot u(t)\cdot[1-u(t)] \tag{26}$$

Equations (4) and (26) turn possible the presentation of the steady state under the new assumptions,

$$\begin{bmatrix}\bar{h}\\ \bar{u}\\ \bar{\tau}\end{bmatrix}=\begin{bmatrix}\left(\frac{\beta}{\rho+\delta}\right)^{\beta/(1-\beta)}\cdot\frac{b^{1/(1-\beta)}}{\delta}\\ \frac{\rho+(1-\beta)\cdot\delta}{\rho+\delta}\\ a\cdot\frac{1-\phi}{\phi}\cdot\left[\frac{\rho+(1-\beta)\cdot\delta}{\rho+\delta}\cdot\left(\frac{\beta}{\rho+\delta}\right)^{\beta/(1-\beta)}\cdot\frac{b^{1/(1-\beta)}}{\delta}\right]^{\eta}\end{bmatrix} \tag{27}$$

All the equilibrium values in (27) obey the necessary boundary conditions.

In the vicinity of the balanced growth path a two-dimensional linearized system may be computed concerning the problem in definition 3:

$$\begin{bmatrix}\dot{h}(t)\\ \dot{u}(t)\end{bmatrix}=\begin{bmatrix}-(1-\beta)\cdot\delta & \beta\cdot\delta\cdot\frac{\bar{h}}{1-\bar{u}}\\ -\beta\cdot\delta\cdot\frac{\bar{u}}{\bar{h}} & \beta\cdot\delta\cdot\frac{\bar{u}}{1-\bar{u}}\end{bmatrix}\cdot\begin{bmatrix}h(t)-\bar{h}\\ u(t)-\bar{u}\end{bmatrix} \tag{28}$$

System (28) allows to prove proposition 2.

Proposition 2. Saddle-path stability. *Under assumptions 1 and 2 the model that relates the quantity and the allocation of human capital exhibits saddle-path stability.*

Proof: For a two-dimensional Jacobian matrix as the one in system (28), saddle-path stability means that one of its eigenvalues is positive (respecting to the unstable component of the system) while the other is negative (respecting this to the stable path). Noticing the algebra result $Tr(J)=\lambda_1+\lambda_2$ and $Det(J)=\lambda_1.\lambda_2$, one has just to obtain the signs $Tr(J)>0$ and $Det(J)<0$ to prove the expected stability result. Computing the trace we realize that $Tr(J)=\rho$; the determinant is a negative amount given that $\beta<\eta/2$ and $\eta<1$: $Det(J)=-(1-2\cdot\beta)\cdot\delta\cdot[\rho+1(1-\beta)\cdot\delta]$. We confirm in this way the existence of a saddle-path stable equilibrium■

To understand how the endogenous variables jointly evolve towards the steady state we make use of a graphical analysis through the construction of a phase diagram. This analysis allows to prove proposition 3.

Proposition 3. Dynamic adjustment. *Along the stable trajectory, the adjustment to the steady state implies that a growing stock of human capital is accompanied by a shift of human capital from the education sector to the technology sector. On the other hand, if the stock of human capital per labor unit grows negatively then this form of capital tends to be reallocated from the technology generation to the education activities.*

Proof: The proof of proposition 3 is made through the construction of a phase diagram. To illustrate graphically the dynamics of the model we need to represent two auxiliary lines: $h(t)=0$ and $\dot{u}(t)=0$; these can be taken from the linear system (28):

$$\dot{h}(t)=0 \Rightarrow u(t)=\frac{\rho}{\rho+\delta}+\frac{(1-\beta)\cdot(1-\bar{u})}{\beta\cdot\bar{h}}\cdot h(t) \tag{29}$$

$$\dot{u}(t)=0 \Rightarrow u(t)=\frac{\rho+(1-2\cdot\beta)\cdot\delta}{\rho+\delta}+\frac{(1-\bar{u})}{\bar{h}}\cdot h(t) \tag{30}$$

Under the assumptions of the model, line (29) has a larger slope than (30), and the intercept of (30) is a higher value than the intercept of (29). Given that $\partial\dot{h}(t)/\partial h(t)<0$ and $\partial\dot{u}(t)/\partial h(t)<0$, the following vector fields apply:

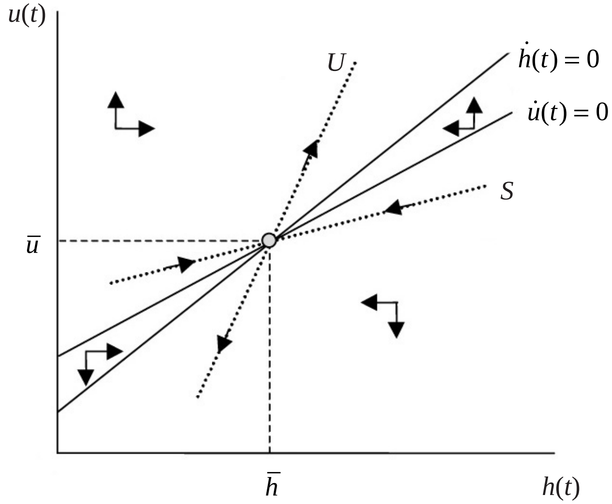
- to the right of line (29) the value of $h(t)$ decreases and to the right of the same line that value increases;
- to the right of line (30) the value of $u(t)$ decreases and to the right of (30) it increases.

The corresponding directional arrows are drawn below in the phase diagram of Figure 1.

The directions to which the referred arrows point allow for an exact notion about the stable trajectory location. The stable trajectory, as the unstable arm, is upward sloped, being the slope of the first less accentuated than the one concerning the unstable path. Drawing the phase diagram we confirm the saddle-path stable nature of the relation between $h(t)$ and $u(t)$ and we find that the variables evolve in the same direction towards the equilibrium point. If $h_0<\bar{h}$, then $h(t)$ and $u(t)$ grow positively until the point $(\bar{h},\bar{u})$ is attained. The phase diagram is sketched in Figure 1.

FIGURE 1

PHASE DIAGRAM



To conclude the dynamic analysis we have interest in knowing how the technology growth rate evolves to the steady state given the stable path found for the relation between $h(t)$ and $u(t)$. The growth rate is not an independent variable; it relates to $h(t)$ and $u(t)$ through (22). Our dynamic analysis becomes complete with proposition 4.

Proposition 4. Growth rate dynamics. *In a technology choices model where the gap variable is a given constant value, the dynamic adjustment to the steady state where none of the variables $h(t)$, $u(t)$ and $\tau(t)$ grows is of the following kind: if $h_0 < \bar{h}$ in the neighbourhood of the steady state then $h(t)$, $u(t)$ and $\tau(t)$ all grow at positive rates to the equilibrium position; if $h_0 < \bar{h}$ then $\tau(t)$ follows the same negative growth behavior of $h(t)$ and $u(t)$.*

Proof: Assume several possible $\tau(t)$ values, $\tau_0 < \tau_1 < ... < \bar{\tau} < ... < \tau_n$. From equation (22) follows that for any τ_i, $i=1, \ldots, n$,

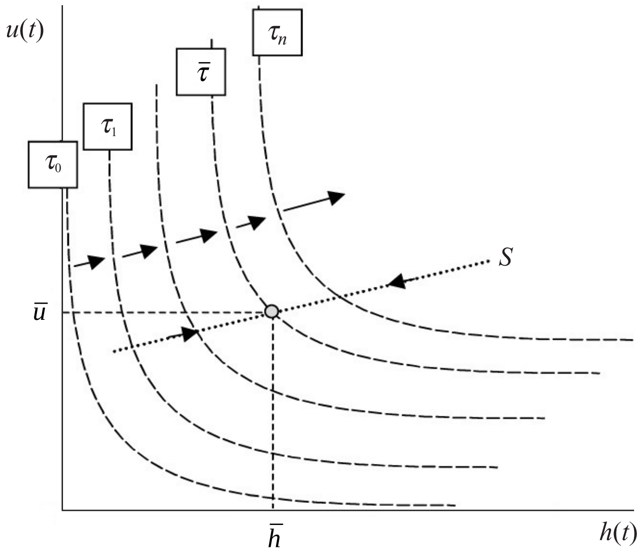
$$u(t) = \left(\tau_i \cdot \frac{1}{a} \cdot \frac{\phi}{1-\phi} \right)^{1/\eta} \cdot \frac{1}{h(t)} \tag{31}$$

Equation (31) represents a concave function in the referential $[h(t),u(t)]$ that is as much far from the origin as the higher is the value of τ_i. Thus, we may draw in such referential a set of functions (31) for the various levels of τ_i. We regard that as we approach the steady state, the value of $\tau(t)$ will be higher if we start from the left of $\bar{h}$, or lower if we start from the right of $\bar{h}$. In this way, variable $\tau(t)$ has a same qualitative behavior in the adjustment process as the control variable $u(t)$■

Figure 2 elucidates the argument in the proof of proposition 4.

FIGURE 2

GROWTH RATE DYNAMICS



IV. Final Remarks

In this paper we have argued that physical capital accumulation plays just a subsidiary role in terms of long run growth. The analysis focused on the two central engines of growth, which are human capital accumulation and technological diffusion. The way in which technological diffusion is modelled relies on the Nelson-Phelps approach: the technology index to include in an aggregate production function evolves in time according to a technological gap that relates to a reference value that may be understood as the scientific frontier or state of the art knowledge capabilities of the economy.

The major new modelling assumption is that the economy has the ability to control the growth of the knowledge frontier, in the extent that public authorities choose the level of basic R&D that is pursued in universities and state laboratories. This assumption makes sense if one thinks about a trade-off that is clear from the technology resource constraint: to create knowledge less resources will be available to apply knowledge, to apply knowledge it is necessary to devote relatively less resources to its creation. In this way, the control of decisions about technology creation is conditioned by the economic non controllable rules of technology adoption. The way we have chosen to study simultaneously the behavior of human capital and technology variables leads to a two-sector optimal control problem, from which we have withdrawn some meaningful results:

(i) In the steady state, state variables "technology gap" and "human capital efficiency index" and control variables "growth rate" and "human capital share", they all display constant values.
(ii) The constant steady state result is possible only if one assumes human capital decreasing returns in both the technology and the education sectors. In this way decreasing returns become compatible with sustained growth.
(iii) Assuming a constant gap in terms of technology adoption, the model gives a saddle-path stable result that implies that variables human capital index, human capital share and technology growth rate will evolve qualitatively in the same way towards the steady state.

In what concerns welfare implications, the model makes a clear statement about the role of the public sector in research and more generally in economic growth. Without the incentive of the authorities in promoting scientific knowledge, there would be no knowledge to apply, and therefore the economy would stagnate. From a welfare point of view, our framework indicates the absence of long run growth under a fully decentralized setup (without government intervention) and positive growth, according to (21), when there is a management of resources between public basic knowledge creation and private innovation and diffusion processes.

One might argue that basic research is not a monopoly of the public sector; many general purpose inventions are just the fruit of brilliant minds that took the time to arrive to new results, without important financial incentives. In fact, this is not questionable; however, it is also true that from a macroeconomic point of view sustained economic growth became possible only when some important public institutions were created. Today's economic structures are not conceivable without a basic R&D sector controlled and financed publicly; and therefore the knowledge frontier does not move at a pace determined by a few altruistic geniuses. Science is financed by public resources and governments decide in what extent the science frontiers are stimulated.

A last word to the endogenous nature of the growth process. As we have stated, to translate our analysis to an economic growth setup it is necessary just to include a capital accumulation constraint and, eventually, an intertemporal consumption utility optimization framework. In this way, the rate that describes the technology variables evolution in time would simultaneously be the rate of economic growth, under a steady state perspective. Thus, it is possible to talk about endogenous economic growth because the growth rate is determined endogenously (in fact, it is controllable under some resource constraints) and because the several parameters that appear in its long run expression may be influenced by the economic agents decisions, including the policy measures that the government undertakes.

Nevertheless, the endogenous nature of our growth model is different from the nature of the conventional growth models. First, only technology and education decisions influence long run growth; factors as the eagerness to consume, savings decisions or the rate of population growth are absent from the steady state growth rate expression. Second, despite output, the physical capital stock and technology all grow in the steady state, the human capital average efficiency does not - under our assumptions the model is pessimistic about human capabilities, that is, the way in which we may acquire more skills is bounded.

Notes

[1] To highlight the innovation - diffusion process, we do not consider capital accumulation and private consumption utility maximization. The introduction of these features would not constitute any analytical obstacle; combining a Ramsey framework with the Nelson-Phelps setup, we would have all of the most common ingredients on growth analysis. The point is that under a Solow perspective of decreasing marginal returns in capital accumulation, this variable does not add important information to long run growth analysis, which Solow indeed associated with technical progress.

[2] More accurately, the social planner is able to choose the amount of resources allocated to basic research; it is expected that there will be a direct macro relation between such effort and the obtained results, despite the uncertainty associated with the creative activity.

[3] The social planner objective function reflects micro-level economic and social goals that individual agents pursue. On one hand, bridging the gap between available and attainable techniques is an instrumental objective of each individual firm. On the other hand, an ever-expanding scientific frontier has not as single purpose to provide the conditions for a wider ready-to-use technology development; it is also a way through which households acquire human capital that provides individual satisfaction (scientific knowledge can be thought as an argument of the representative agent utility function) and unique capabilities that offer to them a stronger position when facing labour markets.

[4] By smoothness we mean that function v is differentiable and that the correspondent first order derivative is a continuous function.

[5] The symbol γ represents the growth rate of the variable referred in index.

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