Revista de Análisis Económico, Vol. 21, Nº 1, pp. 3-28 (Junio 2006)

DEFAULT RATE AND PRICE OF CAPITAL IN A COSTLY EXTERNAL FINANCE MODEL*

JUAN PABLO MEDINA**
Central Bank of Chile

Abstract

Financial frictions have been used to enrich mechanisms transmission in macroeconomics. However, the predictions of real business cycle models of costly external finance imply a procyclical default rate, external premium and relative price of capital which seems at odds with the data. In this article, we include technology shocks that affect the average productivity and idiosyncratic risk of capital producers in a standard costly external finance model. These elements enhance the model to deliver a countercyclical default rate, external finance and relative price of capital premium which are more consistent with the data and contrary to the results obtained with a sector-neutral productivity shock. Intuitively, if the entrepreneurs' investment projects become more productive in average, the relative price of capital and the default rate fall while investment and output increase. Using data on the relative price of capital, we perform a calibration of this type of shocks which highlights its business-cycle relevance.

Keywords: *Financial Constraints, Costly External Finance, Default Rate.*

JEL Classification: *E32, E22, E44.*

* I thank Harold Cole, Monika Piazzesi, Martin Schneider and participants of the UCLA macro seminar for useful comments. Special thanks to my advisor, Lee Ohanian, for his help, advice and encouragement. I also thank two anonymous referees for useful comments on an earlier draft. Consuelo Edwards provides excellent help with the edition. All remaining errors are my responsibility.
** Banco Central de Chile, Santiago, Chile. E-mail:jmedina@bcentral.cl

I. Introduction

The presence of financial frictions has been used as a key element to improve the quantitative performance of economic models. In macroeconomics, the existence of financial market imperfections have enhanced the transmission mechanisms in business cycle models helping to replicate the empirical responses of aggregate variables to the shocks of the economy.[1] In finance, incompleteness in the financial markets has resulted in a higher equity premium which is more in line with the level observed in the data.[2]

In this article, we perform a quantitative analysis to some asset pricing properties of a real business cycle model with costly external finance.[3] There are several reasons to focus on a costly external finance model. First, this type of financial friction in a standard dynamic macroeconomic model offers a rationale for the amplifications and persistence of shocks observed in the macroeconomic aggregate variables. Second, models of this nature are consistent with the corporate finance literature, which has justified the imperfect substitution between internal and external funds.[4] Moreover, the idea that external funds are costly for a firm has generated empirical research testing if cash flow, leverage and other balance-sheet factors have effects on the investment decisions of firms beyond their implicit information about investment opportunities.[5] The result of this empirical literature has been to argue that financial frictions are important in investment decisions. Third, the basic costly external finance can be justified through the "costly state verification" problem first analyzed by Townsend (1979). The advantages of this model are its simplicity and descriptive realism which allow it to be embedded inside a dynamic general equilibrium framework.

Hence, if the reason to explain the amplification and propagation mechanism at the aggregate is associated with costly external finance, it seems natural to see whether the assets pricing fluctuations in this kind of model are quantitatively appealing. Gomes *et al*. (2003) have argued that using a costly external finance model can give a higher mean and volatility of the equity premium than other standard real business cycle models. However, their results show that the size of the equity premium is still very small compared with the data, and the propagation mechanism is driven by a procyclical default rate and external premium which is a property that seems at odds with the data. They argue that these findings cast doubt on the presence of financial frictions as a realistic channel for the propagation mechanism in macroeconomics models.

In this article, we show that including changes in the average productivity and idiosyncratic risk of capital producers in the basic model of costly external finance (see Carlstrom and Fuerst, 1997) can give a source of countercyclical default rate and external premium which is consistent with the data. The intuition of this result is as follows. For instance, when capital producers become more productive in average the supply of new capital expands driving down the equilibrium price of capital, the default rate and the external premium. At the same time, investment and, consequently, output expand. A similar effect has a reduction in the dispersion of the productivity of capital producers.

Using US data on the relative price of capital, we make a calibration of the stochastic process of the capital-specific productivity change. This exercise confirms the importance of the capital-specific technological change in this costly external finance model. On one hand, these types of fluctuations are required to move the supply of investment goods as a way to obtain a countercyclical relative price of capital. On the other hand, if the aggregate productivity fluctuations are eliminated, the capital-specific productivity fluctuations can explain about 30% of the volatility in the US total output. Data on Chile and Mexico confirm the importance of the capital-specific technological change in explaining business cycle fluctuations with a costly external finance model in emerging market countries.

The rest of the article is organized as follows. In section II, we describe the model with emphasis on the financial contract between entrepreneurs and financial intermediaries. After a base parameterization of the model in section III, we analyze the response of the economy to different sources of fluctuations in section IV. In section V, we make a formal calibration of the parameters governing the capital-specific technological change using some dynamic properties of the relative price of capital in the US economy. The final section VI concludes and describes directions for future research. Appendix A derives some functions related with the financial contract while appendix B describes the log-linearized system of equations used to solve and simulate the model.

II. Model

The model presented in this section is based on Carlstrom and Fuerst (1997). The framework is a standard neoclassical model with a costly external finance driven by endogenous agency costs. This element introduces financing constraints that contribute to distort the optimal capital accumulation and thus generate a model with a much richer set of dynamics. The economy consists of a continuum of consumers of unit mass. A fraction $(1 - \eta)$ are households and fraction η are entrepreneurs. The latter consumers produce capital goods and use external funds to finance this activity. A set of competitive financial intermediaries provide funds to the entrepreneurial sector. Finally, there are competitive final goods producers that face no financing constraint. We can now take a closer look at each of these agents.

2.1 Households

The households are infinitely lived with preferences given by:

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t u\left(c_t, 1-l_t\right)\right]$$

where $\mathbb{E}_0[\cdot]$ denotes the expectation operator conditional on the time 0 information, $\beta \in (0, 1)$ is the subjective discount factor, c_t and l_t are the consumption and fraction of the time that households work at t, respectively. In each period the household derives income from renting the labor services and capital holdings at competitive rates, w_t and r_t. Also, they can sell the undepreciated capital. This income is used to purchase consumption goods and capital for the next period such that the household budget constraint is:

$$c_t + q_t a_{t+1}^h = w_t l_t + \left(r_t + q_t(1-\delta)\right) a_t^h$$

where a_{t+1}^h, q_t and δ are the household capital holding for the next period, relative price of capital and the depreciation rate, respectively. The household's optimal choices can be summarized in the following first-order conditions:

$$u_1(c_t, 1-l_t) w_t = u_2(c_t, 1-l_t)$$

$$q_t u_1(c_t, 1-l_t) = \beta \mathbb{E}_t\left[u_1(c_{t+1}, 1-l_{t+1})\left(r_{t+1} + q_{t+1}(1-\delta)\right)\right]$$

The first equation is the household labor supply. The second one is the Euler equation that governs household intertemporal substitution using capital holdings to move resources across periods.

2.2 Entrepreneurs

Entrepreneurs are able to produce capital goods and live infinitely with preferences characterized by:

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty} (\beta\gamma)^t c_t^e\right]$$

The entrepreneurs are different to the household in two dimensions. They are risk neutral and more impatient than households ($\gamma < 1$). Risk neutrality implies that they only care about expected returns which in turn will ensure that they will bear all risk. That simplifies the financial contract explained below considerably. The extra discount factor prevents them from becoming wealthy enough to overcome the financing constraint in equilibrium. At the beginning of each period they have a net worth given by renting labor and capital to the final good producers at competitive rates, w_t^e and r_t, respectively.[6] They can sell the undepreciated capital such that their net worth can be written as:

$$n_t = w_t^e + \left(r_t + q_t(1-\delta)\right) a_t^e$$

where a_t^e stands for the capital holdings of the entrepreneur at the beginning of the period.

Within the period, each entrepreneur can earn additional income by investing i_t units of consumption goods in a technology that produces $\theta_t i_t$ of capital goods at the end of the period. It is assumed that θ_t is a random variable independent and identically distributed across entrepreneurs, but not over time, with cumulative distribution and probability density functions $F_t(\theta_t) = F(\theta_t,\lambda_t)$ and $f_t(\theta_t) = f(\theta_t,\lambda_t)$, respectively. λ_t is a vector of parameters that defines the distribution of θ_t. Following the costly state verification literature, it is considered that θ_t is observed only by the entrepreneur at the end of the period. It can be observed by outsiders at a cost of μi_t units of capital goods. In the beginning of the period entrepreneurs and outsiders know the distribution of θ_t but the entrepreneurs learn their types at the end of the period without any cost.

Investment is financed by borrowing funds from financial intermediaries if needed. However, asymmetric information about the productivity of the entrepreneur makes the external finance costly. Gale and Hellwig (1985) in a static setting show that the optimal financial contract between a lender and an entrepreneur resembles a risky debt. Here to keep this type of contract is that it eliminates any repeated game aspects of the financial contract assuming that the contract can be only based on the current level of net worth and investment. We also assume that the financial markets are incomplete. So, they cannot write contracts that are functions of all public information.[7]

Let r_t^l denote the lending rate of this risky debt in terms of capital goods. Therefore, the contract at t states that if an entrepreneur borrows $(i_t - n_t)$ he commits to repay $(1+r_t^l)(i_t - n_t)$ in terms of capital goods. However, if the realization of θ_t is too low, the entrepreneur will be unable to repay and must default. In other words, this kind of contract determines a cutoff $\bar{\theta}_t$ such that:

- if $\theta_t < \bar{\theta}_t$, the entrepreneur defaults, the lender monitors the project outcome. It follows that the entrepreneur sets $c_t^e, a_{t+1}^e = 0$
- if $\theta_t > \bar{\theta}_t$, the entrepreneur repays $(1+r_t^l)(i_t - n_t) = \bar{\theta}_t i_t$ and his budget constraint is $q_t a_{t+1}^e + c_t^e = q_t(\theta_t i_t - (1+r_t^l)(i_t - n_t))$

With this financial contract the Euler equation for the entrepreneur can be expressed as:

$$q_t = \beta\gamma\,\mathbb{E}_t\left[\left(r_{t+1} + q_{t+1}(1-\delta)\right)R_{t+1}^n\left(n_{t+1}, i_{t+1}\right)\right]$$

where $R_t^n\left(n_t, i_t\right)$ is the expected return on the internal funds at the beginning of period t in terms of consumption goods given a net worth of n_t and an investment of i_t. Using the financing contract we can get:

$$R_t^n\left(n_t, i_t\right) = \frac{q_t}{n_t}\mathbb{E}_t\left[\left(\theta_t i_t - \left(1+r_t^l\right)\left(i_t - n_t\right)\right)1\left\{\theta_t \geq \bar{\theta}_t\right\}\right] = \frac{q_t i_t}{n_t}\mathbb{E}_t\left[\left(\theta_t - \bar{\theta}_t\right)1\left\{\theta_t \geq \bar{\theta}_t\right\}\right]$$

where $1\left\{\theta_t \geq \bar{\theta}_t\right\}$ is the indicator function that equals 1 if $\theta_t \geq \bar{\theta}_t$ and zero otherwise.

2.3 Financial intermediaries

Financial intermediaries allocate household savings by financing entrepreneur investment projects. By funding a large number of entrepreneurs, the intermediaries diversify project-specific risk and, thus, guarantee a safe return to the households since there is no aggregate risk during the life of the project.

Now we can get the expected income of intermediaries after financing a project of size i_t with a loan of $(i_t - n_t)$:

$$
\begin{aligned}
q_t i_t g_{1t}(\bar{\theta}_t) &= q_t\left[\int_0^{\bar{\theta}_t} \theta i_t f_t(\theta)d\theta - \mu F_t(\bar{\theta}_t) i_t + (1 - F_t(\bar{\theta}_t))(1 + r_t^l)(i_t - n_t)\right] \\
q_t i_t g_1(\bar{\theta}_t, \lambda_t) &= q_t i_t\left[\int_0^{\bar{\theta}_t} \theta f_t(\theta)d\theta - \mu F_t(\bar{\theta}_t) + (1 - F_t(\bar{\theta}_t))\bar{\theta}_t\right] \\
&= q_t i_t\left[\int_0^{\bar{\theta}_t} \theta f(\theta, \lambda_t)d\theta - \mu F(\bar{\theta}_t, \lambda_t) + (1 - F_t(\bar{\theta}_t, \lambda_t))\bar{\theta}_t\right]
\end{aligned}
$$

where g_1 denotes the fraction of the expected net production of capital goods received by the financial intermediary.

Similarly, the expected income received by the entrepreneur is:

$$
\begin{aligned}
q_t i_t g_{2t}(\bar{\theta}_t) &= q_t\left[\int_{\bar{\theta}_t}^{\infty} \theta i_t f_t(\theta)d\theta - (1 - F_t(\bar{\theta}_t))(1 + r_t^l)(i_t - n_t)\right] \\
q_t i_t g_2(\bar{\theta}_t, \lambda_t) &= q_t i_t\left[\int_{\bar{\theta}_t}^{\infty} \theta f_t(\theta)d\theta - (1 - F_t(\bar{\theta}_t))\bar{\theta}_t\right] \\
&= q_t i_t\left[\int_{\bar{\theta}_t}^{\infty} \theta f(\theta, \lambda_t)d\theta - (1 - F(\bar{\theta}_t, \lambda_t))\bar{\theta}_t\right]
\end{aligned}
$$

where g_2 denotes the fraction of the expected net production of capital goods received by the entrepreneur.

We can check that $g_1(\bar{\theta}_t, \lambda_t) + g_2(\bar{\theta}_t, \lambda_t) = E_t[\theta_t] - \mu F(\bar{\theta}_t, \lambda_t)$ so that an amount $\mu F(\bar{\theta}_t, \lambda_t)$ of the capital produced is lost due to monitoring cost.[8]

The optimal contract is determined by solving the following problem:

$$
\max_{\bar{\theta}_t, i_t} \; q_t i_t g_2(\bar{\theta}_t, \lambda_t)
$$

s.t.

$$
q_t i_t g_1(\bar{\theta}_t, \lambda_t) \geq (i_t - n_t) \tag{P1}
$$

$$
q_t i_t g_2(\bar{\theta}_t, \lambda_t) \geq n_t \tag{P2}
$$

where (P1) and (P2) are the participation constraint for financial intermediaries and entrepreneurs, respectively.[9] (P1) will be binding while (P2) will not be binding. This conclusion comes from the fact that there are many competitive financial intermediaries and hence they must break even at the optimal contract.

Since only (P1) is binding, the maximization above can be written as:

$$\max_{\bar{\theta}_t} \frac{q_t g_2(\bar{\theta}_t, \lambda_t) n_t}{1 - q_t g_1(\bar{\theta}_t, \lambda_t)}$$

where the FOC with respect to $\bar{\theta}_t$ is

$$\left[\frac{\partial g_2}{\partial \bar{\theta}}(\bar{\theta}_t, \lambda_t)\right]\left(1 - q_t q_1(\bar{\theta}_t, \lambda_t)\right) + q_t \left[\frac{\partial g_1}{\partial \bar{\theta}}(\bar{\theta}_t, \lambda_t)\right] g_2(\bar{\theta}_t, \lambda_t) = 0$$

Now using $g_1(\bar{\theta}_t, \lambda_t) + g_2(\bar{\theta}_t, \lambda_t) = \mathbb{E}_t\left[\theta_t\right] - \mu F(\bar{\theta}_t, \lambda_t)$ we can rewrite the FOC as:

$$1 = q_t \left[\mathbb{E}_t\left[\theta_t\right] - \mu F(\bar{\theta}_t, \lambda_t) + \mu f(\bar{\theta}_t, \lambda_t) \frac{g_2(\bar{\theta}_t, \lambda_t)}{\frac{\partial g_2}{\partial \bar{\theta}}(\bar{\theta}_t, \lambda_t)}\right]$$

This equation defines an implicit relationship among q_t, $\bar{\theta}_t$ and λ_t as $\bar{\theta}_t = \bar{\theta}(q_t, \lambda_t)$. From that last equation it can be proved that $\frac{\partial \bar{\theta}}{\partial q_t} > 0$. Also, given our parameterization of the distribution of $\bar{\theta}_t$, we can express $\mathbb{E}_t\left[\theta_t\right]$ as a function $m(\lambda_t)$.

The external premium in terms of the consumption good is $q_t\left(1 + r_t^l\right) - 1$, which can be expressed as $\bar{\theta}_t / g_1(\bar{\theta}_t, \lambda_t) - 1$. From here we can see that for a given λt the external premium is increasing in $\bar{\theta}_t$.[10] This is important because a procyclical external premium is equivalent to having a procyclical probability of default if the distribution of θ is invariant to the cyclical position of the economy. This is the case in Carlstrom and Fuerst (1997). However, if the distribution of θ changes with the cyclical position of the economy we do not know how the external premium moves with the business cycles. A critical element in this model is going to be how the set of parameters of the distribution of θ moves along the business cycles.

Other financial statistic that we can derive from this model is the default rate which is defined by $F(\bar{\theta}_t, \lambda_t)$. Using the definition of $g_2(\cdot)$ we obtain:

$$
\begin{aligned}
\mathbb{E}_t\left[(\theta_t - \bar{\theta}_t)1\left\{\theta_t \geq \bar{\theta}_t\right\}\right] &= \int_{\bar{\theta}_t}^{\infty} (\theta - \bar{\theta}_t) f(\theta, \lambda_t) d\theta \\
&= \int_{\bar{\theta}_t}^{\infty} \theta f(\theta, \lambda_t) d\theta - (1 - F(\bar{\theta}_t, \lambda_t))\bar{\theta}_t \\
&= g_2(\bar{\theta}_t, \lambda_t)
\end{aligned}
$$

which be can used to express the return of internal funds to the entrepreneur as:

$$
R_t^n(i_t, n_t) = \frac{q_t i_t g_2(\bar{\theta}_t, \lambda_t)}{n_t} = \frac{q_t g_2(\bar{\theta}_t, \lambda_t)}{1 - q_t g_1(\bar{\theta}_t, \lambda_t)}
$$

where the second equality comes from using (P1) with equality.

Finally, using the fact that (P1) is binding we can express investment as a function of net worth, the cutoff and the vector of parameters of the distribution or as a function of net worth, the price of the capital and the vector of parameters of the distribution:

$$
i_t = \frac{n_t}{1 - q_t g_1(\bar{\theta}_t, \lambda_t)} = \frac{n_t}{1 - q_t g_1(\bar{\theta}(q_t, \lambda_t), \lambda_t)}
$$

This equation can be interpreted as the supply of investment goods obtained from this costly external finance model. For fixed values for n_t and λ_t, this investment supply will in general be increasing in q_t.

2.4 Final goods producers

The final set of agents are the final goods producers. These are competitive firms using constant returns to scale technology:

$$
Y_t = \exp(z_t) G(K_t, H_t, H_t^e)
$$

where K_t is the aggregate level of capital in the economy in period t, H_t is the aggregate supply of household labor, H_t^e is the aggregate supply of entrepreneurial labor and z_t is the aggregate sector neutral productivity factor. The optimality conditions translate into the following equations:

$$
r_t = \exp(z_t) G_1(K_t, H_t, H_t^e)
$$

$$
w_t = \exp(z_t) G_2(K_t, H_t, H_t^e)
$$

$$
w_t^e = \exp(z_t) G_3(K_t, H_t, H_t^e)
$$

The first expression defines the rental rate of capital. The second one determines the household labor demand, while the last one specifies the entrepreneurial labor demand.

2.5 Aggregation

By the law of large numbers, aggregate investment at t is the expected value of the production of capital goods minus the monitoring cost incurred:

$$I_t = \int_0^{\infty} \theta i_t f(\theta, \lambda_t) d\theta - \int_0^{\bar{\theta}_t} \mu i_t f(\theta, \lambda_t) d\theta = i_t \left[m(\lambda_t) - \mu F(\bar{\theta}_t, \lambda_t) \right]$$

Similarly, aggregating across the entrepreneurs' budget constraint we can write:

$$q_t A_{t+1}^e + \eta c_t^e = \left[\eta w_t^e + A_t^e (r_t + q_t (1-\delta)) \frac{q_t g_2(\bar{\theta}_t, \lambda_t)}{1 - q_t g_1(\bar{\theta}_t, \lambda_t)} \right]$$

where A_t^e denotes aggregate capital holding of entrepreneurs at the beginning of t and with some abuse of notation, c_t^e is now average entrepreneurial consumption. Finally, given the linearity of investment as a function of net worth and the mapping between the price of capital and $\bar{\theta}_t$ we have:

$$n_t = w_t^e + \frac{A_t^e}{\eta} \left[r_t + q_t (1-\delta) \right]$$

$$i_t = \frac{n_t}{1 - q_t g_1(\bar{\theta}_t, \lambda_t)}$$

where now n_t and i_t are the average entrepreneurial net worth and investment.

2.6 Equilibrium conditions

A competitive equilibrium satisfies the following market-clearing conditions:

$$H_t = (1-\eta) l_t$$

$$H_t^e = \eta$$

$$(1-\eta) c_t + \eta c_t^e + \eta i_t = Y_t$$

$$K_{t+1} = (1-\delta) K_t + \eta i_t \left[m(\lambda_t) - F(\bar{\theta}_t, \lambda_t) \mu \right]$$

A recursive competitive equilibrium is defined by decision rules for K_{t+1}, A_{t+1}^e, H_t, n_t, i_t, $\bar{\theta}_t$, c_t^e, c_t, and pricing functions q_t, w_t, w_t^e, r_t, where these decision rules and pricing functions are invariant functions of (K_t, A_t^e, z_t, λ_t) and satisfy the following equations:

$$u_1(c_t, 1-l_t)w_t = u_2(c_t, 1-l_t) \tag{1}$$

$$q_t u_1(c_t, 1-l_t) = \beta \mathbb{E}_t\left[u_1(c_{t+1}, 1-l_{t+1})(r_{t+1} + q_{t+1}(1-\delta))\right] \tag{2}$$

$$K_{t+1} = (1-\delta)K_t + \eta i_t\left[m(\lambda_t) - F(\bar{\theta}_t, \lambda_t)\mu\right] \tag{3}$$

$$q_t\left[m(\lambda_t) - \mu F(\bar{\theta}_t, \lambda_t) + \mu f(\bar{\theta}_t, \lambda_t)\frac{g_2(\bar{\theta}_t, \lambda_t)}{\frac{\partial g_2}{\partial \bar{\theta}}(\bar{\theta}_t, \lambda_t)}\right] = 1 \tag{4}$$

$$(1-\eta)c_t + \eta c_t^e + \eta i_t = Y_t = \exp(z_t)G(K_t, H_t^e, H_t) \tag{5}$$

$$i_t = \frac{n_t}{1 - q_t g_1(\bar{\theta}_t, \lambda_t)} \tag{6}$$

$$n_t = w_t^e + \frac{A_t^e}{\eta}\left[r_t + q_t(1-\delta)\right] \tag{7}$$

$$A_{t+1}^e = \frac{\eta n_t g_2(\bar{\theta}_t, \lambda_t)}{1 - q_t g_1(\bar{\theta}_t, \lambda_t)} - \frac{\eta c_t^e}{q_t} \tag{8}$$

$$q_t = \beta\gamma \mathbb{E}_t\left[(r_{t+1} + q_{t+1}(1-\delta))\frac{q_{t+1}g_2(\bar{\theta}_{t+1}, \lambda_{t+1})}{1 - q_{t+1}g_1(\bar{\theta}_{t+1}, \lambda_{t+1})}\right] \tag{9}$$

$$H_t = (1-\eta)l_t \tag{10}$$

$$r_t = \exp(z_t)G_1(K_t, H_t, \eta) \tag{11}$$

$$w_t = \exp(z_t)G_2(K_t, H_t, \eta) \tag{12}$$

$$w_t^e = \exp(z_t)G_3(K_t, H_t, \eta) \tag{13}$$

III. Based Calibration

3.1 Preferences, technologies and financial parameters

The base calibration is designed to make a simple comparison among the effects on the economy of different sources of fluctuations. We follow the lines of Carlstrom and Fuerst (1997) in almost all parameters except for some financial statistics. We begin assuming that the household's utility function is:

$$u(c,1-l) = \ln(c) + \nu(1-l)$$

where ν is chosen such that in steady state households work 30% of their time and $\beta = 0.99$. This is a standard preference used in the real business cycle literature to explain quarterly US data.[11]

The final goods production is assumed to be of the Cobb-Douglas form:

$$Y_t = \exp(z_t)K_t^{\alpha_1}H_t^{\alpha_2}(H_t^e)^{(1-\alpha_1-\alpha_2)}$$

where $\alpha_1 = 0.36$, and $\alpha_2 = 0.6399$. The share of entrepreneurial labor is chosen so small such that labor income plays a very irrelevant role in determining both net worth and income distribution in this model. z_t is the final goods production technological change. The stochastic process for this exogenous variable will be explained below.

Regarding the parameters that define the financial contract as in Carlstrom and Fuerst (1997), we use a monitoring cost $\mu = 0.25$. Also, the distribution of θ_t is assumed to be log-normal. Then the set of parameters that define the distribution of θ in t are mean m_t and variance σ_t^2. The specific stochastic process for these two variables will be described in the next subsection. We assume as normalization that at the steady state $m = 1$. Hence, to match the default rate and the external premium we just need to pin down θ and σ at the steady state. Using a default premium of 200 basis points and a default rate of 0.97% we can get $\overline{\theta} = 0.14$, $\sigma = 0.66$. These values imply a steady state relative price of capital of $q = 1.09$. To avoid self-financing outcomes for the entrepreneurs, we should set γ such that $\gamma q g_2(\overline{\theta}_t, m, \sigma^2) / (1 - q g_1(\overline{\theta}, m, \sigma^2)) = 1$. That condition yields a $\gamma = 0.91$. Finally, the depreciation rate δ is set in 2%.

3.2 Stochastic process for shocks

Technological shocks follow a joint autoregressive process:

$$\begin{pmatrix} z_t \\ \ln(m_t) \end{pmatrix} = \begin{pmatrix} \rho_z & \rho_{zm} \\ \rho_{mz} & \rho_m \end{pmatrix} \begin{pmatrix} z_{t-1} \\ \ln(m_{t-1}) \end{pmatrix} + \begin{pmatrix} \varepsilon_{z,t} \\ \varepsilon_{m,t} \end{pmatrix} \tag{14}$$

where we consider that $\varepsilon_{z,t}$ and $\varepsilon_{m,t}$ have a joint normal distribution independently and identically distributed over time. Also, the mean of these perturbations are zero and are independent of each other with variance denoted by υ_z^2 and υ_m^2, respectively. It is assumed that the variance of the distribution of θ in t is constant.[12]

In the real business cycle literature when the final goods technological change is the only source of exogenous fluctuations, are typically used $\rho_z = 0.95$ and $\upsilon_z = 0.71\%$. Unfortunately, we do not have unambiguous data to obtain values for other parameters governing the joint stochastic process of these exogenous fluctuations. For example, being consistent with this model, data on the fluctuations of the relative price of capital or the default premium contain information of the two shocks considered here, and there are multiple ways to decompose the fluctuations of the endogenous variables as coming from these shocks.

Since the parameters in the stochastic process of the exogenous variables will imply a particular dynamic of the endogenous variables, one method to get these parameters is to choose them such that the model matches some moments of the endogenous variables observed in the data. However, there is a large number of potential moments that can be used and an equally large set of parameters to be calibrated (8 in the autoregressive matrix and 2 variances). Hence, it seems important to understand the quantitative dynamics in a simpler context of the stochastic process. On these lines, in the next section we will analyze the impulse-response functions in a constrained case which fixes $\rho_{zm} = \rho_{mz} = 0$. In other words, there are no spillover effects among exogenous fluctuations.

IV. Impulse Responses for a Simple Case

After having the base calibration of the model, we can make numerical analysis using the well known method of taking a log-linear expansion of the equations of the model around the deterministic steady state. Then the log-linear decision rules are computed using the method of undetermined coefficients. Having this we are ready to compute the impulse-response functions to the three sources of fluctuations: (i) aggregate-sector-neutral productivity changes (z), and (ii) changes in the average productivity of capital producer (m).

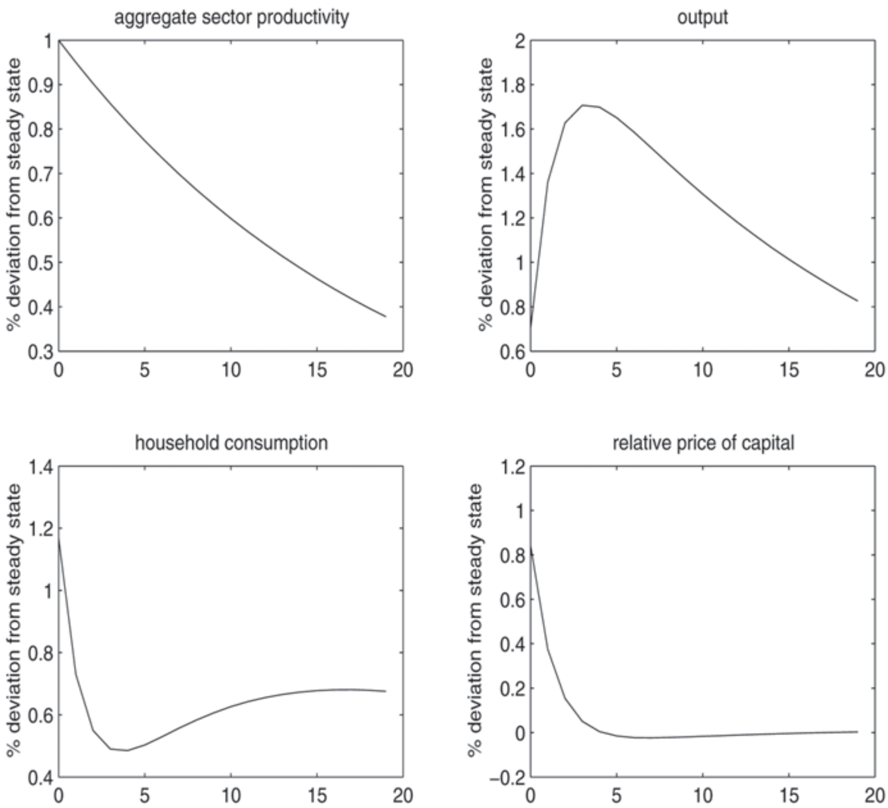
The impulse responses are computed with an initial shock that deviates each one of the exogenous variables 1% from the steady state. Since we did not choose a particular value for ρ_m and ρ_σ we consider two cases: (a) $\rho_m = 0.9$; and (b) $\rho_m = 0$. The first case describes a highly persistent evolution of this exogenous variable and the second considers a path that is independent over time.

4.1 Shock in the aggregate sector neutral productivity

The responses of the main economic variables to an aggregate productivity shock of 1% are displayed in Figures 1 and 2. The results are equivalent to

FIGURE 1

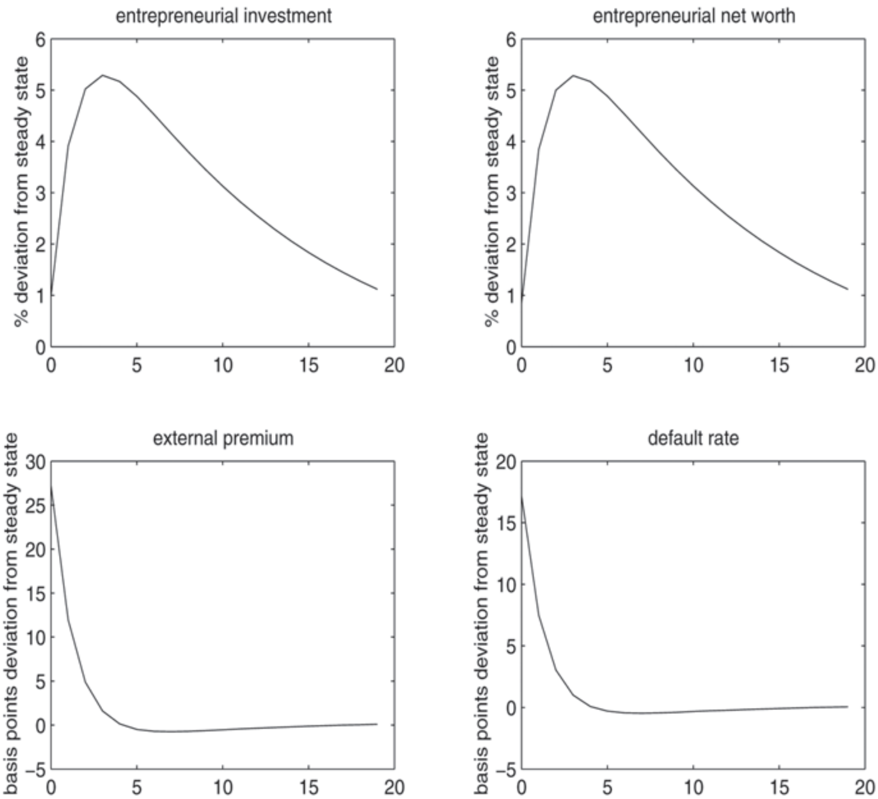
RESPONSES TO A SHOCK IN THE AGGREGATE SECTOR PRODUCTIVITY (PART 1)



Carlstrom and Fuerst (1997)'s. Output, investment and consumption exhibit a hump-shaped pattern that reflects a delayed response to this shock that is not present in the standard neoclassical model. The agency cost and the persistency of the aggregate productivity shock generate a higher autocorrelation in output and investment growth. Output increases slowly until it reaches a positive deviation from its steady state of almost 1.8% at the third quarter after the shock. Investment also displays the same path profile as output with a peak deviation of more than 5%. The relative price of capital increases by 0.8% instantaneously to die out monotonically afterwards. The external premium and the default rate increase about 25 and 15 basis points at the time of the shock, with a path similar to that of the relative price of capital.

FIGURE 2

RESPONSES TO A SHOCK IN THE AGGREGATE SECTOR PRODUCTIVITY (PART 2)



The intuition behind this result is related with the increase in the marginal cost of investment due to the agency problem. An increase in the productivity of the final goods producers shifts out the demand for new capital, and then entrepreneurs want to increase their production of capital goods. However, the increase in investment calls for external funds which are costly and the net worth of entrepreneurs does not rise too much initially since they cannot adjust their capital holdings until the next period. Hence, the supply of investment does not shift out too much compared to the increase in the demand for new capital which delivers an increase in the price of capital. This rise in the price of capital drives up the cutoff $\overline{\theta}_t$ and therefore it pushes up the default rate and the external premium. Although we obtain a richer propagation dynamics of the aggregate productivity shock in this costly external finance model, the impulse responses characterize a procyclical default rate, external premium and relative price of capital.

4.2 Shock in average entrepreneurial productivity

Figures 3 and 4 depict the responses of the economy to an increase of 1% in the mean of the entrepreneurial productivity when this shock is persistent (ρ_m = 0.9). This can be interpreted as an exogenous force that drives up the average productivity of entrepreneurs in the production of capital goods. This change shifts out the supply curve of investment goods without affecting simultaneously the demand for investment. Hence, the investment and the equilibrium price of capital go down, which lowers the default rate and the external premium. Thus, we get a countercyclical relative price of capital, default rate and external premium, which is more consistent with the data.

FIGURE 3

RESPONSES TO A SHOCK IN THE AVERAGE ENTREPRENEURIAL PRODUCTIVITY
(ρ_m = 0.9, PART 1)

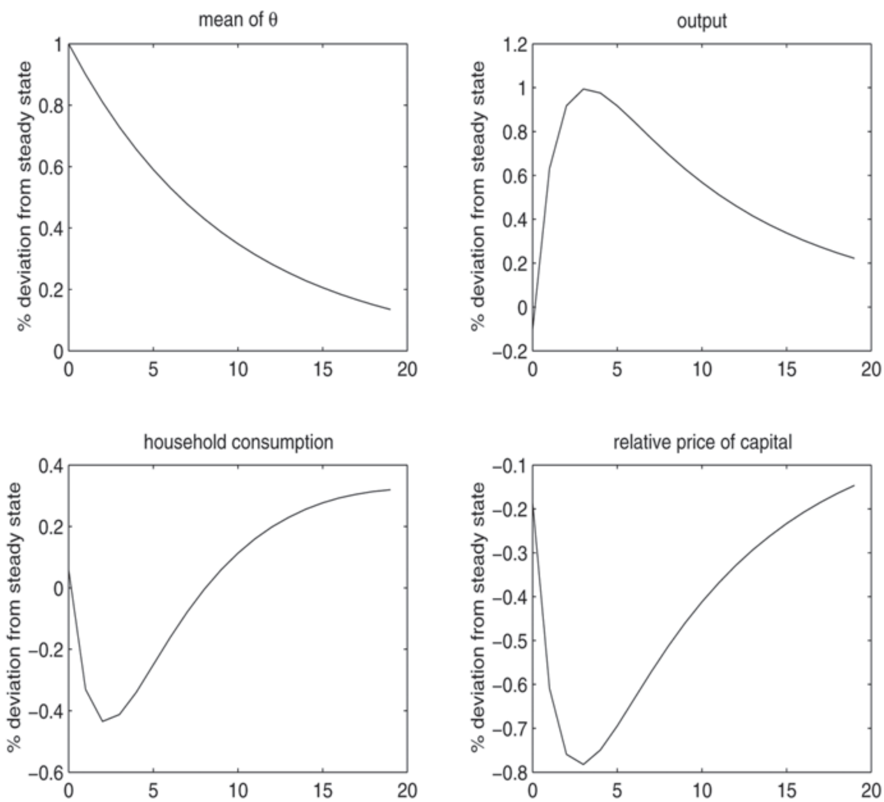
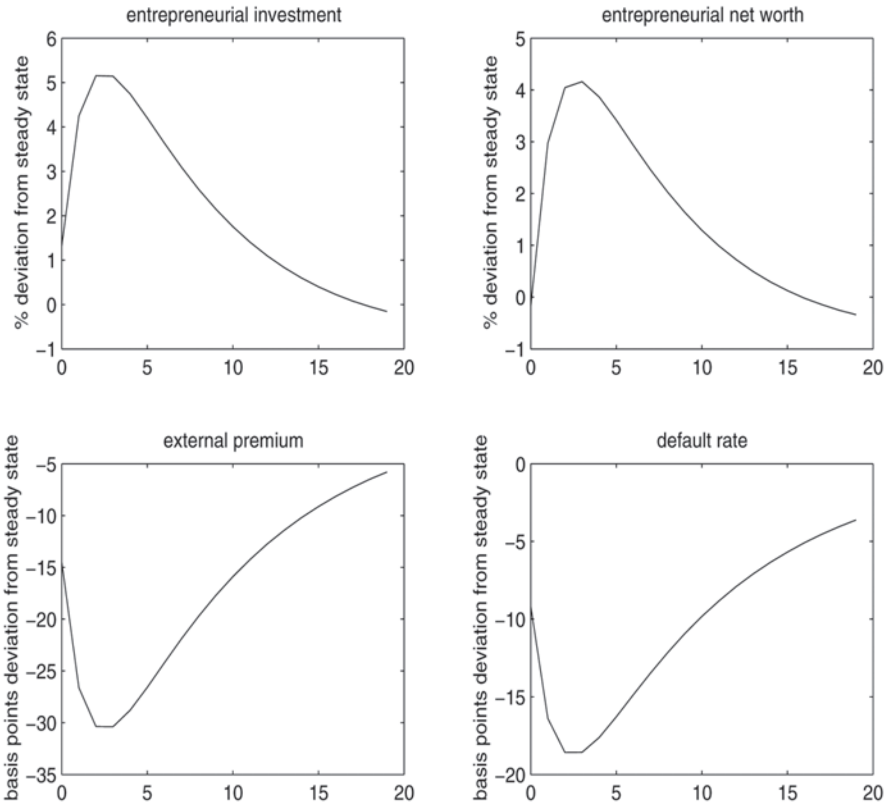


FIGURE 4

RESPONSES TO A SHOCK IN THE AVERAGE ENTREPRENEURIAL PRODUCTIVITY
(ρ_m = 0.9, PART 2)



It is worth noting that the biggest deviation in the premium and default rate is observed around the second or third quarter after the initial shock in the average productivity. In those periods the premium is 30 basis points below its steady-state value, while the default rate displays almost 20 basis points of reduction with respect to its steady-state level. This hump-shaped path of these two variables is not present under aggregate sector productivity shock (see Figure 2). Although, the magnitude in output response is somewhat smaller after the shock in the average entrepreneurial productivity than after the aggregate sector productivity shock, the response in the investment has the same quantitative reach, which coincides with a decrease in the external finance premium.

The responses of the economy to this same shock but for the no persistent case (ρ_m = 0) are displayed in Figures 5 and 6. Although the responses of the economic variables die out very fast in this case, the results still share the basic property of having a countercyclical relative price of capital. That pattern is driven by a short period shift out of the supply of investment which in turn implies a contraction of the default rate and the external premium.

V. Calibration of the Stochastic Processes

In this section we will focus on calibrating formally the stochastic processes assuming the presence of only two shocks described in (14): aggregate sector productivity and average entrepreneurial productivity. We will consider no spillover

FIGURE 5

RESPONSES TO A SHOCK IN THE AVERAGE ENTREPRENEURIAL PRODUCTIVITY (ρ_m = 0, PART 1)

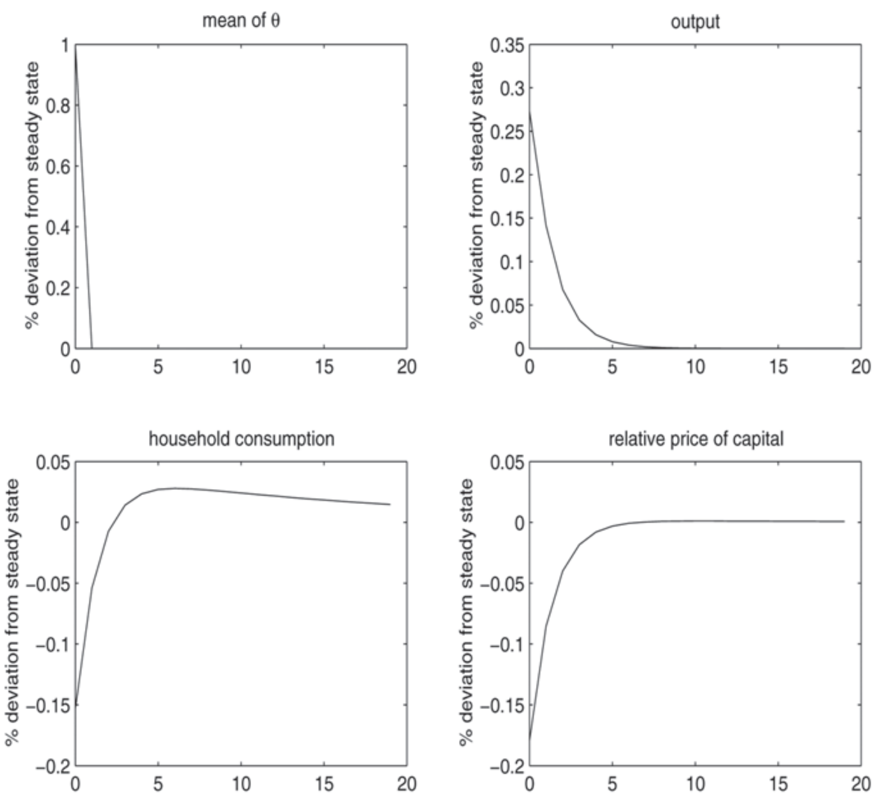
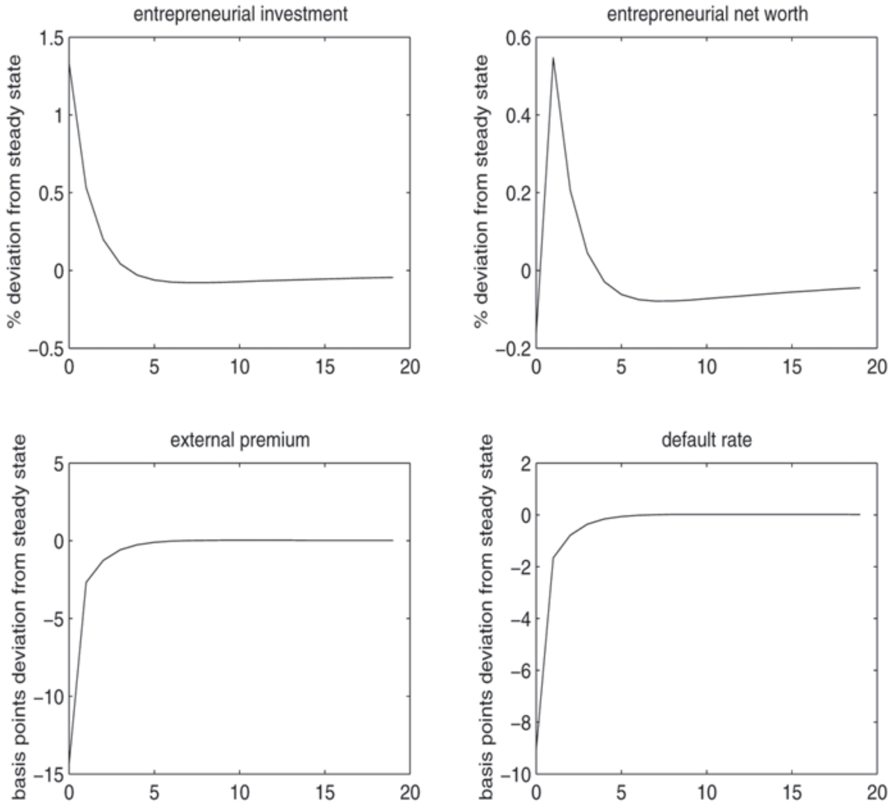


FIGURE 6

RESPONSES TO A SHOCK IN THE AVERAGE ENTREPRENEURIAL PRODUCTIVITY
(ρ_m = 0, PART 2)



effects: $\rho_{zm} = \rho_{mz} = 0$. For simplicity we will keep the same parameter values known for the aggregate sector productivity fluctuation, i.e., $\rho_z = 0.95$ and $\upsilon_z = 0.71\%$. This implies that we need to calibrate two parameters: ρ_m and υ_m. Hence, we would require to match at least two moments observed in the data to obtain these parameters.

In Table 1 we show some moments computed on a quarterly basis from the US economy during the period 1954:Q1 - 2006:Q1. The variables shown are total real GDP (*Y*), real fixed investment (*I*), and the relative price of fixed investment defined as the investment deflator divided by the GDP deflator (*q*). The statistics are computed using the log-deviation of each series with respect to their Hodrick-Prescott filter.

From the table we can conclude that the relative price of capital is countercyclical and that a reduction in the last quarter can predict an increase in total output in the current one. Recalling the impulse responses graphed in the last section, we can see that the aggregate sector productivity shock cannot produce this negative correlation with output. However, the presence of fluctuations in the average productivity of capital goods producers can induce a negative correlation between output and the relative price of capital.

TABLE 1

US DATA 1954.1-2006.1. STANDARD DEVIATIONS AND AUTOCORRELATIONS WITH GDP

	st. dev.	x(–1)	x(0)	x(+1)
Y	1.60	0.85	1.00	0.85
I	5.1	0.81	0.90	0.82
q	0.72	–0.16	–0.07	0.04

Also, since in this model the cyclical properties of the default premium resemble those of the relative price of capital, this result will imply a countercyclical default premium. This last property has been well documented in finance (see, for example, Gomes *et al.*, 2006), but given that this model contains a highly simplified corporate sector, it is harder to see what premium in the data is actually described by this model. Hence, we prefer to focus on the relative price of capital as the variable containing information about the shift in the supply and demand for investment good which in turn will convey something about the default rate and external finance premium in this model.

Thus, we consider the following moments from the data to be matched with this model: (i) the contemporaneous correlation between GDP and the relative price of capital (–0.07) and (ii) the correlation of GDP with one lag of the relative price of capital (–0.16). These values imply $\rho_m = 0.98$ and $\upsilon_m = 1.21\%$. The moments inferred from the model are shown in Table 2.

This final calibration highlights the relevance of the capital-specific technological change to explain business cycles and, at the same time, gets plausible cyclical properties for the default rate and the external premium in a model with financing constraints. If we shut down the average entrepreneurial productivity shocks, we can get an estimate of the relative importance of the capital specific technological change in terms of output volatility. Removing this specific technological shock the standard deviation of GDP falls from 0.63% to 0.43%. Hence, the simulation of this model concludes that a 30% of the fluctuations in GDP could be attributed to a capital-specific technological change. Surprisingly, Greenwood *et al.* (2000) obtain the same result for the contribution of the capital-specific technological change using a different model and methodology.

TABLE 2

MODEL: STANDARD DEVIATION AND AUTOCORRELATIONS WITH GDP

	st. dev.	x(–1)	x(0)	x(+1)
	Both Shocks			
Y	0.63	0.41	1.00	0.41
I	2.42	0.53	0.85	0.37
q	0.62	–0.16	–0.07	–0.22
	Only aggregate productivity shocks			
Y	0.43	0.54	1.00	0.54
I	1.49	0.26	0.89	0.82
q	0.45	0.62	0.21	–0.63

TABLE 3

EMERGING COUNTRIES DATA. STANDARD DEVIATION AND AUTOCORRELATIONS WITH GDP

	st. dev.	x(–1)	x(0)	x(+1)
	Chile: 1989.1-2005.4			
Y	1.93	0.71	1.00	0.71
I	7.74	0.53	0.79	0.79
q	3.12	– 0.15	–0.04	0.13
	Mexico: 1993.1-2005.4			
Y	2.51	0.83	1.00	0.83
I	9.52	0.87	0.96	0.75
q	1.91	–0.69	–0.72	–0.54

Despite this model has been constructed to analyze the implications of the financial factors in the behavior of the US economy, the relevance of financial frictions in explaining salient features of emerging market countries has had an increased interest. Céspedes *et al*. (2004) and Gertler *et al*. (2003) are notable contributions. Building on a costly external finance model, these authors analyze the role of balance sheet and imperfect market access in investment decisions in general equilibrium models of small open economies. This literature has been focussed in the choice of the exchange rate regime (e.g. flexible vis-à-vis fixed) using calibrated models. However, less attention has had analyzing whether there is evidence that favors a costly external finance model in an open economy setting.[13] The results presented in this work deliver critical elements for analyzing the relevance of costly external finance elements to explain business cycle behavior of aggregate variables. A preliminar discussion can be done with Table 3, which shows standard deviations and autocorrelations with GDP of the same variables shown in Table 1 for Chile and Mexico.[14] In both countries, the relative price of capital also displays a negative correlation with GDP. Hence, a costly external finance model can describe the business cycle behavior of these two countries as long as the average entrepreneurial productivity is present as a source of fluctuations.

VI. Final Thoughts

Financial frictions have been used to explain persistence in macroeconomics and asset pricing anomalies. Also, the empirical research on determinants of aggregate demand has assigned an important role to credit markets' imperfections.

In this paper, we analyze simultaneously the quantitative implications of financial frictions on macroeconomics and their implications on the behavior of the default rate, the external finance premium and the relative price of capital. We extend a costly external finance model to allow for more plausible cyclical properties of the default rate, external premium and relative price of capital.

The basic ingredient is the inclusion of changes in the average entrepreneurial productivity or idiosyncratic entrepreneurial risk. These elements deliver a countercyclical pattern for the default rate, external premium and relative price of capital that is more consistent with the data. This is a very simple case that makes the supply of investment goods shift more than the demand for investment, which implies that the relative price of capital drops when investment rises. The increasing relationship between the price of capital and the default rate in the model gives a source to reconcile the default rate's behavior along the cyclical position of the economy. In other words, in good times when investment and output go up, the default rate, the external finance premium and the relative price of capital fall.

The result of countercyclical relative price of capital is very important since US postwar aggregate fluctuations show this feature. Greenwood *et al.* (2000) show that capital-specific technology shocks can explain such behavior of the relative price of capital and about 30% of output fluctuations. This article obtains the same quantitative result and offers another dimension for their observations because the shocks that induce a countercyclical price of capital are specific to the capital producers. In other words, the modification to the costly external finance model suggested here shows another way to rationalize the sector-specific technological changes as an important source of economic fluctuations and be consistent with the countercyclical features of the relative price of capital. It is also worth noting that this cyclical behavior of these variables is obtained without affecting the propagation mechanism emphasized by the costly external finance model.

Interestingly, the countercyclical behavior of the relative price of capital is also a feature of two developing countries: Chile and Mexico. Hence, this preliminar analysis shed light on the presence of a costly external finance mechanism and the relevance of capital-specific technological change in emerging market countries. A formal empirical assessment of the relevance of the costly external finance model to explain emerging countries fluctuations is left for future research.

It is worth stating several directions for future research. First, the modifications to the basic model of costly external finance were done with the purpose of getting the right directions for the cyclical behavior of the default rate. It is still the case that technological changes in capital production are not completely understood from an economic point of view. For that reason it is interesting to investigate other ways to induce a shift in the supply of investment that comes from the economic environment and not just from new exogenous state variables. Second, this model does not offer a complete theoretical counterpart for asset pricing. In particular, it is hard to interpret what can be called equity in this model. Gomes *et al*. (2003) use the return on capital holdings as the return on equity, but that definition is not very satisfactory. Hence, other extension would be how to introduce clearly the presence of two sources of financing: risky debt and equity in the model. Third, having defined clearly equity in the model, it is straightforward to ask whether the costly external finance model gives sensitive results in other dimension of asset pricing. For example, it would be interesting to find out what other elements should be added to have a high equity premium.

Notes

1 Some prominent examples of this propagation features are Bernanke and Gertler (1989), Bernanke *et al*. (1996, 1999), Carlstrom and Fuerst (1997) and Kiyotaki and Moore (1997).

2 For instance, Mankiw (1986), Constantinides and Duffie (1996) and Lustig (2005) use incompleteness in the financial markets to obtain a high equity premium in line with the postwar data.

3 The costly external finance feature has been called *financial accelerator* extensively in the literature after the work of Bernanke *et al*. (1999).

4 Examples where asymmetric information has been the main reason for this imperfect substitution are Gale and Hellwig (1985), Jensen and Meckling (1976) and Myers and Majluf (1984).

5 In the empirical research about investment and financial constraints, we can find examples of this idea in Fazzari *et al*. (1988) and Hoshi *et al*. (1991).

6 Since leisure does not enter in the entrepreneur preferences, they work all their available time.

7 In a partial equilibrium framework, Albuquerque and Hopenhayn (2004) and Clementi and Hopenhayn (2006) characterize the full dynamic optimal contract between a financially constrained entrepreneur and a lender when the financial frictions come from enforceability problems and asymmetric information, respectively. In these works, the contracts are intertemporally optimal and based on all public information.

8 Appendix A describes g_1, g_2, F and f for the case that θ_t has a log-normal distribution function which is the case used in the calibration.

9 Recall that the financial contract is within the period so that opportunity cost for the funds of entrepreneurs and financial intermediaries is zero.

10 From the definition of $g_1(\bar{\theta}_t, \lambda_t)$ it follows that $\frac{\partial g_1(\bar{\theta}_t,\lambda_t)}{\partial \bar{\theta}_t}$ is negative which guarantees that $\frac{\partial\left[\bar{\theta}_t / g_1(\bar{\theta}_t,\lambda_t)\right]}{\partial \bar{\theta}_t} > 0$.

11 See Hansen (1985).

[12] Alternatively, one could have assumed that σ_t^2 fluctuates as well. This exogenous variable would also be able to generate a countercyclical relative price of capital, default rate and external premium. However, the quantitative responses of variables to shocks to the variance of θ are not very significant.

[13] One exception is the work of Elekdag *et al.* (2005), who estimate a costly external finance model for the Korean economy.

[14] Chilean and Mexican data are from http://si2.bcentral.cl/Basededatoseconomicos/951 portada.asp?idioma=I and http://dgcnesyp.inegi.gob.mx/cgi-win/bdieintsi.exe, respectively.

References

ALBUQUERQUE, R. and H. HOPENHAYN (2004). “Optimal Lending Contracts and Firm Dynamics”, *Review of Economic Studies,* 71 (2), pp. 285-315.

BERNANKE, B. and M. GERTLER (1989). “Agency Costs, Net Worth, and Business Fluctuations”, *American Economic Review,* 79 (1), pp. 14-31.

BERNANKE, B., M. GERTLER and S. GILCHRIST (1996). “The Financial Accelerator and the Flight to Quality”, *Review of Economics and Statistics,* 78 (1), pp. 1-15.

BERNANKE, B., M. GERTLER and S. GILCHRIST (1999). “The Financial Accelerator in a Quantitative Business Cycle Framework”, in M. Woodford and J. Taylor (eds.), *Handbook of Macroeconomics*, North Holland.

CARLSTROM, C. and T. FUERST (1997). “Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis”, *American Economic Review.* 87 (5), pp. 893-910.

CESPEDES, L.F., R. CHANG and A. VELASCO (2004). “Balance Sheets and Exchange Rate Policy”, *American Economic Review*, 94 (4), pp. 1183-1193.

CLEMENTI, G.L. and H. HOPENHAYN (2006). “A Theory of Financing Constraints and Firm Dynamics”, *Quarterly Journal of Economics,* 121 (1), pp. 229-265.

CONSTANTINIDES, G. and D. DUFFIE (1996). “Asset Pricing with Heterogeneous Consumers”, *Journal of Political Economy* 104 (2), pp. 219-240.

ELEKDAG, S., A. JUSTINIANO and I. TCHAKAROV (2005). “An Estimated Small Open Economy Model of the Financial Accelerator”, *IMF Working Paper* 05/44.

FAZZARI, S., R. HUBBARD and B. PETERSEN (1988). “Financing Constraints and Corporate Investment”, *Brookings Papers on Economic Activity,* 0 (1), pp. 141-195.

GALE, D. and M. HELLWIG (1985). “Incentive Compatible Debt Contracts I: The One-period Problem”, *Review of Economic Studies,* 52 (4), pp. 647-663.

GERTLER, M., S. GILCHRIST and F. NATALUCCI (2003). “External Constraints on Monetary Policy and the Financial Accelerator”, *BIS Working Paper* 139.

GOMES, J., A. YARON and L. ZHANG (2003). “Asset Prices and Business Cycles with Costly External Finance”, *Review of Economic Dynamics,* 6 (4), pp. 767-788.

GOMES, J., A. YARON and L. ZHANG (2006). “Asset Pricing Implications of Firms’ Financing Constraints”, *Review of Financial Studies*, *forthcoming*.

GREENWOOD, J., Z. HERCOWITZ and P. KRUSELL (2000). “The Role of Investment- Specific Technological Changes in the Business Cycle”, *European Economic Review,* 44 (1), pp. 91-115.

HANSEN, G. (1985). “Indivisible Labor and the Business Cycle”, *Journal of Monetary Economics,* 16 (3), pp. 309-27.

HOSHI, T. A. KASHYAP and D. SCHARFSTEIN (1991). “Corporate Structure, Liquidity, and Investment: Evidence from Japanese Industrial Groups”, *Quarterly Journal of Economics,* 106 (1), pp. 33-60.

JENSEN, M. and W. MECKLING (1976). “Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure”, *Journal of Financial Economics,* 3 (4), pp. 305-60.

KIYOTAKI, N. and J. MOORE (1997). “Credit Cycles”, *Journal of Political Economy,* 105: pp. 211-48.

LUSTIG, H. (2005). “The Market Price of Aggregate Risk and the Wealth Distribution”, *NBER Working Paper* 11132.

MANKIW, N. (1986). "The Equity Premium and the Concentration of Aggregate Shocks", *Journal of Financial Economics,* 17 (1), pp. 211-219.

MYERS, S. and N. MAJLUF (1984). "Corporate Financing and Investment Decisions when Firms have Information that Investors do not have", *Journal of Financial Economics,* 13 (2), pp. 187-221.

TOWNSEND, R. (1979). "Optimal Contracts and Competitive Markets with Costly State Verification", *Journal of Economic Theory,* 21 (2), pp. 265-93.

APPENDIX A

DISTRIBUTION OF θ AND FINANCIAL CONTRACT FUNCTIONS

As described in the article, we assume that θ follows a log-normal distribution function. In this case the set of parameters that determine the distribution can be reduced to the mean (m_t) and variance (σ_t^2) of θ_t. Let $\Phi(\cdot)$ and $\phi(\cdot)$ denote the cumulative and density functions of a normal standard distribution. Then the default rate is given by:

$$F(\bar{\theta}_t, m_t, \sigma_t^2) = \Phi\left(\frac{\ln(\bar{\theta}_t) - \ln(m_t) + \frac{1}{2}\ln(\sigma_t^2 / m_t^2 + 1)}{(\ln(\sigma_t^2 / m_t^2 + 1))^{1/2}}\right) \tag{15}$$

The density function of θ_t can be written as:

$$f(\bar{\theta}_t, m_t, \sigma_t^2) = \Phi\left(\frac{\ln(\bar{\theta}_t) - \ln(m_t) + \frac{1}{2}\ln(\sigma_t^2 / m_t^2 + 1)}{(\ln(\sigma_t^2 / m_t^2 + 1))^{1/2}}\right) \frac{1}{\bar{\theta}_t(\ln(\sigma_t^2 / m_t^2 + 1))^{1/2}} \tag{16}$$

The fraction of the expected net production of capital goods received by the financial intermediaries is:

$$g_1(\bar{\theta}_t, m_t, \sigma_t^2) = m_t\Phi\left(\frac{\ln(\bar{\theta}_t) - \ln(m_t) - \frac{1}{2}\ln(\sigma_t^2 / m_t^2 + 1)}{(\ln(\sigma_t^2 / m_t^2 + 1))^{1/2}}\right) \tag{17}$$

The fraction received by the entrepreneurs is:

$$\begin{aligned} g_2(\bar{\theta}_t, m_t, \sigma_t^2) = m_t&\left[1 - \Phi\left(\frac{\ln(\bar{\theta}_t) - \ln(m_t) - \frac{1}{2}\ln(\sigma_t^2 / m_t^2 + 1)}{(\ln(\sigma_t^2 / m_t^2 + 1))^{1/2}}\right)\right] \\ &-\bar{\theta}_t\left[1 - \Phi\left(\frac{\ln(\bar{\theta}_t) - \ln(m_t) + \frac{1}{2}\ln(\sigma_t^2 / m_t^2 + 1)}{(\ln(\sigma_t^2 / m_t^2 + 1))^{1/2}}\right)\right]. \end{aligned} \tag{18}$$

APPENDIX B

LOG-LINEARIZED EQUATIONS

This appendix lists the set of equations that characterizes the equilibrium and their log-linearized versions used to solve and simulate the model. The variables without a subscript t are the steady-state values and the symbol '$\sim$' denotes the log deviation of the variable with respect to its steady-state value.

Household-leisure decision

$$\tilde{w}_t - \sigma\tilde{c}_t = 0 \tag{log. 1}$$

Household-Euler Equation

$$\mathbb{E}_t\left\{\beta q(1-\delta)\tilde{q}_{t+1} + \beta r^k \tilde{r}^k_{t+1} - q\tilde{c}_{t+1} - q\tilde{q}_{t+1} + q\tilde{c}_t\right\} = 0 \tag{log. 2}$$

Aggregate Capital Evolution

$$\begin{aligned}&(1-\delta)K\tilde{K}_t + \eta i\Big[m_t - F(\bar{\theta},m,\sigma^2)\mu\Big]\tilde{i}_t + \eta i\left[1 - \frac{\partial F}{\partial m}(\bar{\theta},m,\sigma^2)\mu\right]m\tilde{m}_t \\ &-\eta i\frac{\partial F}{\partial \sigma^2}(\bar{\theta},m,\sigma^2)\mu\sigma^2\tilde{\sigma}^2_t - \eta i\frac{\partial F}{\partial \theta}(\bar{\theta},m,\sigma^2)\mu\bar{\theta}\tilde{\bar{\theta}}_t - K\tilde{K}_{t+1} = 0\end{aligned} \tag{log. 3}$$

Aggregate Expenditure

$$(1-\eta)c\tilde{c}_t + \eta c^e\tilde{c}^e_t + \eta i\tilde{i}_t - Y\tilde{Y}_t = 0 \tag{log. 4}$$

Price of Capital

$$\begin{aligned}&q\Big[m - \mu F(\bar{\theta},m,\sigma^2) + \mu f(\bar{\theta},m,\sigma^2)g_2(\bar{\theta},m,\sigma^2)/\tfrac{\partial g_2}{\partial\bar{\theta}}(\bar{\theta},m,\sigma^2)\Big]\tilde{q}_t \\ &+q\Big[1 - \mu\tfrac{\partial F}{\partial m}(\bar{\theta},m,\sigma^2) + \mu\tfrac{\partial}{\partial m}\Big(f(\bar{\theta},m,\sigma^2)g_2(\bar{\theta},m,\sigma^2)/\tfrac{\partial g_2}{\partial\bar{\theta}}(\bar{\theta},m,\sigma^2)\Big)\Big]m\tilde{m}_t \\ &+q\Big[-\mu\tfrac{\partial F}{\partial\sigma^2}(\bar{\theta},m,\sigma^2) + \mu\tfrac{\partial}{\partial\sigma^2}\Big(f(\bar{\theta},m,\sigma^2)g_2(\bar{\theta},m,\sigma^2)/\tfrac{\partial g_2}{\partial\bar{\theta}}(\bar{\theta},m,\sigma^2)\Big)\Big]\sigma^2\tilde{\sigma}^2_t \\ &+q\Big[-\mu\tfrac{\partial F}{\partial\bar{\theta}}(\bar{\theta},m,\sigma^2) + \mu\tfrac{\partial}{\partial\bar{\theta}}\Big(f(\bar{\theta},m,\sigma^2)g_2(\bar{\theta},m,\sigma^2)/\tfrac{\partial g_2}{\partial\bar{\theta}}(\bar{\theta},m,\sigma^2)\Big)\Big]bar\theta\tilde{\bar{\theta}}_t = 0\end{aligned} \tag{log. 5}$$

Investment Supply

$$i(1-qg_1(\bar{\theta},m,\sigma^2))i\tilde{i}_t - n\tilde{n}_t - iqg_1(\bar{\theta},m,\sigma^2)\tilde{q}_t - iq\tfrac{\partial g_1}{\partial\theta}(\bar{\theta},m,\sigma^2)\bar{\theta}\,\tilde{\bar{\theta}}_t$$
$$-iq\tfrac{\partial g_1}{\partial m}(\bar{\theta},m,\sigma^2)m\tilde{m}_t - iq\tfrac{\partial g_1}{\partial\sigma^2}(\bar{\theta},m,\sigma^2)\sigma^2\tilde{\sigma}_t^2 = 0 \qquad \text{(log. 6)}$$

Net Worth

$$w^e\tilde{w}_t^e + A^e q(1-\delta)/\eta\tilde{q}_t + A^e r^k/\eta\tilde{r}_t^k + A^e q(1-\delta)+r^k)/\eta\tilde{A}_t^e - n\tilde{n}_t = 0 \qquad \text{(log. 7)}$$

Entrepreneurs' Capital Holdings

$$\eta g_2(\bar{\theta},m,\sigma^2)i\tilde{i}_t + \eta i\tfrac{\partial g_2}{\partial\bar{\theta}}(\bar{\theta},m,\sigma^2)\bar{\theta}\,\tilde{\bar{\theta}}_t + \eta i\tfrac{\partial g_2}{\partial m}(\bar{\theta},m,\sigma^2)m\tilde{m}_t$$
$$+\eta i\tfrac{\partial g_2}{\partial\sigma}(\bar{\theta},m,\sigma^2)\sigma^2\tilde{\sigma}_t^2 - \eta c^e/q\tilde{c}_t^e + \eta c^e/q\tilde{q}_t - A^e\tilde{A}_{t+1}^e = 0 \qquad \text{(log. 8)}$$

Entrepreneurs' Euler Equation

$$\mathbb{E}_t\Big\{\beta\gamma\Big[2q(1-\delta)+r^k\Big]\frac{qg_2(\bar{\theta},m,\sigma^2)i}{n}q\tilde{q}_{t+1} + \beta\gamma r^k\,\frac{qg_2(\bar{\theta},m,\sigma^2)i}{n}\tilde{r}_{t+1}^k$$
$$+\beta\gamma(q(1-\delta)+r^k)q\tfrac{\partial g_2}{\partial\bar{\theta}}(\bar{\theta},m,\sigma^2)\tfrac{i}{n}\bar{\theta}\,\tilde{\bar{\theta}}_{t+1} + \beta\gamma(q(1-\delta)+r^k)q\tfrac{\partial g_2}{\partial m}(\bar{\theta},m,\sigma^2)\tfrac{i}{n}m\tilde{m}_{t+1}$$
$$+\beta\gamma(q(1-\delta)+r^k)q\tfrac{\partial g_2}{\partial\sigma^2}(\bar{\theta},m,\sigma^2)\tfrac{i}{n}\sigma^2\tilde{\sigma}_{t+1}^2$$
$$+\beta\gamma(q(1-\delta)+r^k)qg_2(\bar{\theta},m,\sigma^2)\tfrac{i}{n}(\tilde{i}_{t+1}-\tilde{n}_{t+1}-q\tilde{q}_t)\Big\} = 0$$
(log. 9)

Final-Good Production

$$z_t + \alpha_1\tilde{K}_t + \alpha_2\tilde{H}_t - \tilde{Y}_t = 0 \qquad \text{(log. 10)}$$

Rental Rate of Capital

$$z_t + \alpha_2\tilde{H}_t - (1-\alpha_1)\tilde{K}_t - \tilde{r}_t^k = 0 \qquad \text{(log. 11)}$$

Household wage

$$z_t + \alpha_1\tilde{K}_t - (1-\alpha_2)\tilde{H}_t - \tilde{w}_t = 0 \qquad \text{(log. 12)}$$

Entrepreneurs wage

$$z_t + \alpha_1\tilde{K}_t + \alpha_2\tilde{H}_t - \tilde{w}_t^e = 0 \qquad \text{(log. 13)}$$