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CHILD LABOR, HOME PRODUCTION AND THE FAMILY LABOR SUPPLY

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Abstract

This paper studies the time allocation of all family members, including the children, where it is assumed that parents distribute their time between market work and household work, and the children may work in the labor market, do the household chores or study. In this framework, I propose that wages and shadow prices play an important role in the allocation of time of household members. Since empirical papers cannot find a clear relationship between child labor, wages and exogenous income, the theoretical model presented here sheds light on the relationship between those variables.

Keywords: *Time Allocation, Child Labor, Labor Supply.*

JEL Classification: *J13, J22.*

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I. Introduction

Recent literature on child labor has developed and estimated models to find the main determinants of child labor and its effect on schooling. Many empirical papers have been written trying to find those determinants, using data from different countries around the world. Most indicate that age, gender, and household characteristics, among others variables, are some of the main determinants of child labor (for example, Ray, 2000; Ravallion and Woodon, 2000; DeGraff, Billsborrow and Herrin, 1996; DeGraff and Billsborrow, 2003; Binder and Scrogin, 1999). However, empirical papers show contradictory or counterintuitive results when it comes to the relationship between child labor and variables related to parents' earnings and family income.

If we think that child labor is closely related to poverty,[1] as basic statistics may suggest because child labor is primarily observed in poor countries, common sense tells us that child labor should decline (and schooling increase) if parents' wages increase, child wage falls or if total family income rises. This proposition has been tested several times and the empirical results in many cases do not support it. For example, Ray (2000), using data from Pakistan and Peru, found different results in these countries. When it comes to the relationship between parents' wages and child labor, these effects are not statistically significant, and a dummy variable related to "poverty" is not significant in Peru. In a different work, Levison and Moe (1998) found no effect of family income on schooling in Peru. Ravallion and Wodon (2000) observed that a transfer to households has a significant and positive effect on schooling but a small and negative effect on child labor in Bangladesh. Alarcon (1989) did not find a significant effect of head of household income on child labor in Peru. Using data from the same country, Rodriguez and Abler (1998) saw that family income has a small, negative and significant effect on participation of Peruvian children in the labor market, but no significant effect on schooling. In addition, Dar *et al*. (2002) summarized the findings of several empirical papers for many less developed countries and demonstrated that the relationship between household income and child labor is unclear.

Theories that explain child labor as the opposite of schooling fail to predict the effect of monetary variables on children's time allocation. The stylized facts tell us that something is missing in the economic analysis of child labor. In my opinion, there is an aspect that has not been studied in depth in the theoretical literature of child labor – household work. Household work is a time consuming activity that may affect the labor supply of family members and create links among them. Surveys show that individuals spend many hours at home doing chores.[2] Since this task can be performed by any of the household members, if one individual works more hours doing chores, then the others may have extra time to distribute among their activities, including work in the labor market. For example, if the children in the household perform the household work, this gives additional hours to the adults to work in the labor market, increasing the aggregate family income.

Moreover, the decision of how many hours a child should dedicate to study, work in the labor market, or work at home would depend not only on the salary the child can receive in the labor market but also on the salaries of the parents. The analysis is complex because an increase in parents' wages may mean a reduction in parents' household work and an increase in children's household work. Thus, we would observe a reduction in the children's labor supply and a reduction in hours of study. Nevertheless, this effect could be offset by an income effect if parents' wages increase enough to hire housekeeping services in the labor market, which would give the child more time to study.

The study of the determinants of household work and its effect on child labor is not new in the empirical literature on child labor.[3] Researchers have recognized that it should be studied along with the determinants of child labor and schooling. However, there is a gap in the theoretical literature on the relationship between child labor, household work, wages, and exogenous income.

This paper presents a model of family labor supply that includes child labor and production of household work. The main objective is to examine how wages and income allocate the time of family members, including the children. The question this research addresses is: How do changes in children's and parents' wages affect household work, the labor supplies and the hours spent in children's education? I propose that the wages each individual may earn working in the labor market have an important role in the intra-household allocation of time. The approach I use here is the theory of allocation of time and the family labor supply, which permits careful study of intra-family decisions that family members make when the labor supply and home production are determined. Carefully using the family labor supply approach can shed light on some of the facts that earlier models cannot, for the most part, explain. The same analysis is repeated when family income is so low that it reaches the subsistence level, a common fact in less developed countries.

The model of family labor supply that would be the base for this research is the standard model in the literature (see Blundell and MaCurdy, 1999) with the inclusion of household work and working children. Nevertheless, my model is, in some sense, an extension and application to child labor of models presented in Gronau (1977) and Rosenzweig (1980). The former shows the determination of leisure, home production and work in the market for a single individual that faces time constraints. In Gronau's model, the distribution of time depends on the wage, and changes in the exogenous income do not affect household work except when the individual does not work at all in the market. Rosenzweig's paper presents a model where a couple (husband and wife) allocates time between home production (in a small family business) and work in the market.

Birdsall (1982) and Birdsall and Cochrane (1982) develop a model of time allocation that explains the determinants of schooling. In her model, schooling and child labor are decisions taken by the family, and they depend on the wages of each individual in the household. Levison (1991) goes beyond that analysis and is concerned with the multiple activities of children in developing countries, including schooling. She analyzes the effects of changes in children's and mothers' wages on what she calls "total home production time of a specific child", which includes time spend on childcare, home maintenance and education.

Bhalotra (2001) focuses her attention on the wage elasticities of child labor. Using a family labor supply model with child labor but without home production, she states that child labor supply depends on the child's wage, parents' wage, exogenous income and household characteristics. She finds that when the level of consumption falls to the subsistence level, the wage elasticity of child labor supply is negative because the income effect is stronger than the substitution effect, which forces the child to work more in order to provide more income for his or her family.

Other papers have analyzed different aspects of this topic. Brown, Deardorff and Stern (2003) present a model with child labor and household labor. Their model is based on the idea of comparative advantage and analyzes the specialization of family members on market work, home work and leisure. The relationship between child labor, fertility and schooling in peasant economies is analyzed in Rosenzweig and Evenson (1977). A model with labor supply and multiple activities of the family members is analyzed, applied and estimated to rural areas by Newman and Gertler (1994). Using the credit constraints approach, Rogers and Swinnerton (2004) discuss in a theoretical model if child labor decreases when parents' income increases and find that in the presence of two-sided altruism between children and parents, an increase in parents' income does not always lead to a reduction in child labor, and it could –in some cases– increase the hours worked by children.

The paper is organized as follows: Section II presents a theoretical model of family labor supply with home production. The model is solved using the Kuhn-Tucker method, paying special attention to the possible corner solutions of the variables. Section III presents the comparative static. The substitution and income effects on household work and market labor of changes in wages and unearned income are shown in the interior solution case. A complete analysis of corner solutions and the appropriate opportunity costs (shadow prices) of the activities are presented in Section IV. In Section V, the implications of the model are compared to the situation in which the family is at subsistence level (the minimum level of consumption for survival), and I find the "slope" of the child labor supply at the subsistence level, a discussion point in child labor literature. Section VI concludes the paper.

II. The Model

The model presented in this section adds home production and child labor to a family labor supply model. Suppose that the family has three members: the *husband* (head of household), the *wife* (or spouse) and a *child*. It is assumed in this family that all decisions are made in a dictatorial way (no negotiation among agents).

The family consumes three goods: the aggregated consumption good (c), a good called "household chores" (Z) and the hours of education of the child (E).

It is assumed that the family preferences are strictly quasiconcave and can be represented by a continuous twice-differentiable utility function $U(c,Z,E)$, where $U_C > 0$, $U_Z > 0$, and $U_E > 0$. Education has been included in the family utility function for altruistic reasons.

The consumption good is bought in the market. In contrast, the "household chores" can be produced at home using wife and child work (z_1 and z_2, respectively) or can be bought in the market in an amount f_0 at a price P. Then,

$$Z = f(z_1, z_2) + f_0$$

where the function $f(.)$ is the home production function. This means that the "household chores" produced at home and the chores bought in the market are perfect substitutes. It is also assumed that $f(.)$ is strictly concave and twice differentiable.[4]

The husband's labor supply is determined in an earlier step, and I assume he works a fixed number of hours a day (his labor supply is perfect inelastic).[5] Therefore, the husband's income (Y) is constant and exogenous. In this context, the family must decide how much to consume of c and Z, how many hours the mother and the child should be employed in the production of the good Z, and how many hours the wife and the child will offer to the labor market in order to maximize the family utility.

This family faces some restrictions: time and budget constraints. The total time allotted to each individual has been normalized to unity. The mother has one unit of time a day that can be employed working at home (z_1) or working in the labor market (H_1) receiving a wage w_1. The child employs their time in working at home (z_2), studying (E) or working in the labor market (H_2).

The budget constraint for this household is:

$$c + P.f_0 = Y + H_1.w_1 + H_2.w_2$$

A summary of the variables in the model is:

c	=	family aggregated consumption
Y	=	husband total income (exogenous in this model)
f_0	=	total domestic services bought in the market (measured in goods)
z_1	=	hours that wife expends on domestic work
z_2	=	hours that child expends on domestic work
H_1	=	hours of labor supply
H_2	=	hours of child labor
w_1	=	wife's wage
w_2	=	child's wage
P	=	price of home services
E	=	hours of education

Finally, let me assume that there are minimum levels of consumption of the two goods, called c^* and Z^*. This means that the family consumption of these goods cannot be below those levels in order to survive.

In order to simplify the model, I assume that there is only one period; the family spends its entire income during that period, and there is no leisure. The problem that the planner solves is the following:

$$\underset{c, f_0, z_1, z_2, E, H_1, H_2}{Max} \quad U(c, Z, E)$$

s.t.

$$c + P.f_0 = Y + H_1.w_1 + H_2.w_2$$

$$Z = f(z_1, z_2) + f_0$$

$$1 = z_1 + H_1$$

$$1 = z_2 + H_2 + E$$

$$c \geq c^*$$

$$z \quad z^*$$

$$f_0 \quad 0, \quad z_1 \quad 0, \quad z_2 \quad 0, \quad E \quad 0, \quad H_1 \quad 0, \quad H_2 \quad 0$$

Notice that the wife and the child are not symmetric since the latter has one more alternative activity: education.[6] Another point to note is that it has been assumed that education is free.

III. Solution to the Model in the Interior Case

In this section, I solve the model and find the effects of the exogenous variables Y, w_1, w_2 and P on the endogenous variables. In order to do this, I impose and discuss assumptions on the utility function and the home production function.

Let us examine the case in which the restrictions of poverty are not binding ($c > c^*$, $Z > Z^*$) and assume that all the variables are strictly positive (i.e., "interior case"). From the first order conditions (see the appendix), it is easy to obtain the following expression:

$$\lambda_1 = U_C = \frac{U_Z f_1}{w_1} = \frac{U_Z f_2}{w_2} = \frac{U_E}{w_2} = \frac{U_Z}{P} \tag{3.1}$$

Equation (3.1) tells us that in equilibrium, the marginal utility of each of the components (in dollars) must be equal to the marginal utility of income.

Solving the model, I obtain five important functions: the *household work functions* $z_1 \equiv z_1(w_1, w_2, Y, P)$ and $z_2 \equiv z_2(w_1, w_2, Y, P)$, the *wife labor supply function* $H_1 \equiv H_1(w_1, w_2, Y, P)$, the *child labor supply function* $H_2 \equiv H_2(w_1, w_2, Y, P)$, and the *education function* $E \equiv E(w_1, w_2, Y, P)$.

The comparative static can be performed using Cramer's Rule. Taking differential to the first order conditions, and after a few manipulations, I obtain the expression:

$$\begin{bmatrix} U_{CC} & U_{CZ} & U_{CE} & -1 & 0 & 0 \\ U_{CZ} & U_{ZZ} & U_{ZE} & -P & 0 & 0 \\ U_{CE} & U_{ZE} & U_{EE} & -w_2 & 0 & 0 \\ -1 & -P & -w_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & U_Z f_{11} & U_Z f_{12} \\ 0 & 0 & 0 & 0 & U_Z f_{12} & U_Z f_{22} \end{bmatrix} \begin{bmatrix} dc \\ df_0 \\ dE \\ d\lambda \\ dz_1 \\ dz_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \lambda_1 & 0 & 0 & 0 \\ 0 & 0 & \lambda_1 & 0 \\ -f_0 & -H_1 & -H_2 & -1 \\ 0 & \lambda_1 & 0 & 0 \\ 0 & 0 & \lambda_1 & 0 \end{bmatrix} \begin{bmatrix} dP \\ dw_1 \\ dw_2 \\ dY \end{bmatrix} \tag{3. 2}$$

The 6x6 matrix on the left is the Bordered Hessian, which must be negative definite in order to have a maximum. Given the assumptions of strict quasiconcavity of U and strict concavity of the production function f, the determinant of the 6x6 matrix is negative and equal to $|\Theta| \cdot \Delta \cdot U_Z^2$, where $|\Theta| < 0$ is the determinant of the upper-left 4x4 matrix, and $\Delta = f_{11} \cdot f_{22} - f_{12}^2 > 0$.

The system suggests that the variables z_1 and z_2 can be solved separately from the other variables.[7] This gives the first result.

Result 1: The derivatives of the demand for household work are:

$$\frac{\partial z_1}{\partial w_1} = \frac{f_{22}}{P} \cdot \frac{1}{\Delta} < 0 \qquad \frac{\partial z_2}{\partial w_1} = -\frac{f_{21}}{P} \cdot \frac{1}{\Delta} \geq or \leq 0$$

$$\frac{\partial z_1}{\partial w_2} = -\frac{f_{12}}{P} \cdot \frac{1}{\Delta} \geq or \leq 0 \qquad \frac{\partial z_2}{\partial w_2} = \frac{f_{11}}{P} \cdot \frac{1}{\Delta} < 0$$

$$\frac{\partial z_1}{\partial Y} = 0 \qquad \frac{\partial z_2}{\partial Y} = 0$$

$$\frac{\partial z_1}{\partial P} = \frac{(f_1 f_{21} - f_2 f_{11})}{P} \cdot \frac{1}{\Delta} > 0 \qquad \frac{\partial z_2}{\partial P} = \frac{(f_2 f_{12} - f_1 f_{22})}{P} \cdot \frac{1}{\Delta} > 0$$

These derivatives depend basically on the assumptions of the production function:[8] the f function is strictly concave, $f_{11} < 0$, $f_{22} < 0$, and the cross derivatives f_{12} and f_{21} have the same sign and magnitude. In other words, the time allocation in household work is a technical matter; it does not depend on household preferences.

One of the most interesting results is that neither z_1 nor z_2 depends on the husband's exogenous income Y.[9] This result is consistent with empirical work, which usually do not find a relationship between household work and head of household income or family income.[10] The second important result is that children's household work depends negatively on their wage. The intuition behind this result says that higher wages mean a higher opportunity cost for household work.

Another result is the ambiguous sign of $\partial z_1/\partial w_2$ and $\partial z_2/\partial w_1$. Both depend on the cross derivative f_{ij}, which could be positive, negative or zero. This means that the marginal product of one factor could increase, decrease or not change when more units of the other factor are used. For example, if the cross derivative is positive (child's housework increases wife's productivity at home), an increase in w_1 (wife's wage) will cause less demand of z_1 (less wife's housework) and then less demand for child's household work z_2. On the other hand, if $f_{ij} < 0$ (the presence of child's housework reduces the wife's productivity at home), an increase in the wife's wage will cause an increase in the child's household work.

At this point, it is easy to obtain the derivatives of the *wife labor supply function* $H_1 \equiv H_1(w_1, w_2, Y, P)$.

Result 2: Since the derivatives of H_1 are opposite the derivatives of z_1, then:

$$\frac{\partial H_1}{\partial w_1} > 0 \quad \frac{\partial H_1}{\partial w_2} \geq or \leq 0 \quad \frac{\partial H_1}{\partial Y} = 0 \quad \frac{\partial H_1}{\partial P} < 0$$

The woman labor supply curve is increasing in w_1 as usual, and it does not depend on the exogenous income Y.[11] What is new is that this labor supply may depend on the child wage.

Now let us calculate the derivatives for the other endogenous variables. Using Cramer's Rule on (3.2) and after few manipulations, I obtain the following results.

Result 3: The derivatives of the education variable are:

$$\frac{\partial E}{\partial Y} = \frac{1}{|\Theta|} \begin{vmatrix} U_{CC} & U_{CZ} & -1 \\ U_{CZ} & U_{ZZ} & -P \\ U_{EC} & U_{EZ} & -w_2 \end{vmatrix} \tag{3.3}$$

$$\frac{\partial E}{\partial w_1} = H_1 \frac{\partial E}{\partial Y} \tag{3.4}$$

$$\frac{\partial E}{\partial w_2} = \left(\frac{\partial E}{\partial w_2} \right)_{U=\bar{U}} + H_2 \frac{\partial E}{\partial Y} \tag{3.5}$$

Equation (3.3) shows the effect on education of a change in exogenous income. The sign of that effect is ambiguous unless additional restrictions are imposed.[12] If that derivative is positive, education will be a normal good; otherwise, it will be inferior.

Equation (3.4) shows that the effect of a change in mother's wage is proportional to the hours she works, and the sign depends on whether education is a normal good or an inferior good. Equation (3.5) shows the effects of changes in child wage on the hours of education. I have decomposed those effects in the income and substitution effect. It can be shown that the substitution effect is

$$\left(\frac{\partial E}{\partial w_2}\right)_{U=\bar{U}} = \frac{\lambda_1}{|\Theta|}\cdot\left[2PU_{CZ} - U_{ZZ} - P^2 U_{CC}\right] < 0$$

This derivative is negative given the assumption of strict quasiconcavity of the utility function and the fulfillment of the second order condition. This means, holding the utility constant, an increase in child wage will reduce the hours of education.

On the other hand, the sign of the income effect depends on whether education is a normal or inferior good, and its size depends on the magnitude of H_2 (child labor). Common sense tells us that education is a normal good. However, empirical work for less developed countries does not show this as a strong result. If it is normal, the two effects go in opposite directions and then the sign of the derivative is ambiguous. An increase in child wage will increase education because the family income is higher and it is able to buy more units of this good (income effect). However, the opportunity cost of education is higher, and the child should work more hours in the labor market. Consequently, the child should study fewer hours (substitution effect).

Now, I am able to present the derivatives of the *child labor supply function* $H_2 \equiv H_2(w_1, w_2, Y, P)$.

Result 4: From the time constraints, $\partial H_2/\partial w_j = -\partial E/\partial w_j - \partial z_2/\partial w_j$, $j = 1,2$, and $\partial H_2/\partial Y = -\partial E/\partial Y$, and:

$$\frac{\partial H_2}{\partial w_1} = \underbrace{-\frac{\partial z_2}{\partial w_1}}_{HWE} + \underbrace{\frac{\partial H_2}{\partial Y}H_1}_{IE<0} \tag{3.6}$$

$$\frac{\partial H_2}{\partial w_2} = \underbrace{-\frac{\partial z_2}{\partial w_2}}_{HWE>0} \underbrace{-\left(\frac{\partial E}{\partial w_2}\right)_{U=\bar{U}}}_{SE>0} + \underbrace{\frac{\partial H_2}{\partial Y}H_2}_{IE<0} \tag{3.7}$$

The first element in the right hand side of equation (3.6) is the "household work effect" (*HWE*) because it is the effect due to a change in the demand for child's household work. Its sign depends on the sign of the cross derivative f_{21}. The second element is the standard income effect (*IE*), which is negative under the assumption of normality of education. There is no substitution effect in this equation.

Equation (3.7) states that the slope of the child labor supply curve could be positive or negative. The first element on the right is the "household work effect" (*HWE*), which is positive. A raise in w_2 will reduce child's household work and increase child market labor. The second element is the substitution effect (*SE*). The sign is positive since a higher wage represents fewer hours dedicated to study and more hours dedicated to market work. The third element is the income effect, which is negative under the assumption of normality of education. An increase in child wage also increases the family total income, and the demand for education will increase also. This reduces the supply of child's market labor.[13]

IV. Corner Solutions and Shadow Prices

The previous analysis applies to the interior case. Nevertheless, in the real world, corner solutions are frequently observed for the variables z_1, z_2, H_1, H_2 and E. In that case, they take the value zero and equation (3.1) does not hold. The decision to work or not to work is analyzed in the literature by comparing the "reservation wages" and the market wages. Indeed, equation (3.1) holds if I replace the observed wages and prices with the "shadow prices". In this model, the analysis is a bit complex since there are several possibilities for corner solutions. As before, I will do the analysis in the non-poverty case ($c > c^*$ *and* $Z > Z^*$).

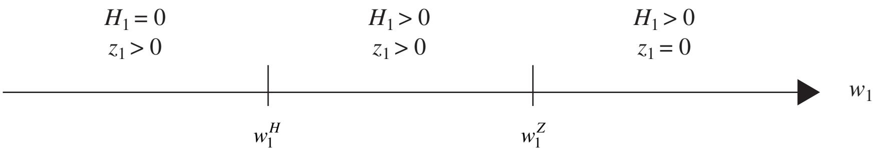
When both the wife and the child work in the job market ($H_1 > 0$, $H_2 > 0$) receiving wages w_1 and w_2 per hour and the child studies a positive number of hours, the opportunity cost of one hour employed in the production of the good Z is just the market salary. In that case, the wife works some hours at home if the marginal return of the wife's household work at $z_1 = 0$ is greater than the market wage. Let me define that marginal return or "*shadow price*" as $w_1^Z \equiv \left.\frac{U_Z \cdot f_1}{\lambda_1}\right|_{z_1=0}$ where U_Z is the marginal utility of household chores, f_1 is the marginal product of the wife's household work, and λ_1 is the marginal utility of income. Thus,

$$
\begin{aligned}
z_1 &= z_1(w_1, w_2, Y, P) && \text{if } w_1 < w_1^Z && (4.1)\\
&= 0 && \text{Otherwise}
\end{aligned}
$$

Now, let us see what determines the participation of wives in the labor market. Define the "reservation wage" as $w_1^H \equiv \left.\frac{U_Z \cdot f_1}{\lambda_1}\right|_{H_1=0}$. Following the Kuhn-Tucker conditions, the wife's labor supply function is:

$$
\begin{aligned}
H_1 &= H_1(w_1, w_2, Y, P) && \text{if } w_1 > w_1^H \\
&= 0 && \text{Otherwise}
\end{aligned}
\tag{4.2}
$$

Notice that $w_1^Z > w_1^H$ because the marginal product is decreasing and λ_1 is constant. Therefore, for any wage below w_1^H, we observe $H_1 = 0$ and $z_1 > 0$; for any wage in between w_1^H and w_1^Z, we observe $H_1 > 0$ and $z_1 > 0$; and for any wage above w_1^Z we observe $H_1 > 0$ and $z_1 = 0$.



In the case of the child, he or she can perform three activities: H_2, z_2 and E. In the interior solution, the return of each activity must be equal to the others. From equation (3.1), the marginal return of child labor (w_2) equals the marginal return of child's household work $\frac{U_Z \cdot f_2}{\lambda_1}$ and equals the marginal return of education $\frac{U_E}{\lambda_1}$.

$$
w_2 = \frac{U_Z \cdot f_2}{\lambda_1} = \frac{U_E}{\lambda_1} \tag{4.3}
$$

The second and third expressions are decreasing in z_2 and E, respectively; in these cases, an interior solution is possible.[14] However, when we have inequality signs in (4.3) and the variables (H_2, z_2, E) reach their minimum values, we observe corner solutions on one or more of these variables.

We observe that one of those variables equals zero if at least one of the other activities always has a higher marginal return.

I define the shadow price or opportunity cost of the first hour of child's household work as $w_2^Z \equiv \max\left\{ w_2|_{z_2=0}, \left.\frac{U_E}{\lambda_1}\right|_{z_2=0} \right\}$, and then:

$$z_2 = z_2(w_1, w_2, Y, P) \qquad \text{if } \left.\frac{U_Z \cdot f_2}{\lambda_1}\right|_{z_2=0} \geq w_2^Z \tag{4.4}$$
$$= 0 \qquad \text{Otherwise}$$

Likewise, in the case of the child labor supply H_2, I define the reservation wage as $w_2^H \equiv \max\left\{\left.\frac{U_Z \cdot f_2}{\lambda_1}\right|_{H_2=0}, \left.\frac{U_E}{\lambda_1}\right|_{H_2=0}\right\}$. Hence, the child works in the labor market when $w_2 \geq w_2^H$. I redefine the child labor supply function as follows:

$$H_2 = H_2(w_1, w_2, Y, P) \qquad \text{if } w_2 \geq w_2^H \tag{4.5}$$
$$= 0 \qquad \text{Otherwise}$$

In the case of hours studying, it equals zero when $\left.\frac{U_E}{\lambda}\right|_{E=0} \leq \max\left\{w_2, \frac{U_Z \cdot f_2}{\lambda_1}\right\}$.

Finally, let me present an additional interpretation for the corner solutions of z_1 and z_2. In the interior case (see footnote 6), I get the tangency condition:

$$\frac{f_1}{f_2} = \frac{w_1}{w_2} \tag{4.6}$$

In the case of corner solutions, it is not difficult to show that

$$z_1 = 0,\ z_2 > 0 \qquad \text{if } \frac{f_1}{f_2} \leq \frac{w_1}{w_2^Z}$$

$$z_1 > 0,\ z_2 = 0 \qquad \text{if } \frac{f_1}{f_2} \geq \frac{w_1}{w_2^Z}$$

where w_2^Z was defined above. As a result, the existence of positive hours of child's household work depends on the parameter w_1, the marginal productivities and the opportunity cost of household work.[15] We can use this result to interpret an empirical finding reported in the literature: girls work more hours than boys in household chores. If we assume that girls are more productive at home, so f_2 is large, and the opportunity cost is low (w_2 is low), then girls will tend to work more hours at home and we seldom observe $z_2 = 0$. In contrast, if the child were a boy, with low f_2 and high w_2, then we would expect to see $z_2 = 0$ or close to zero in most cases.

V. The Child Labor Supply at the Subsistence Level $c = c^*$ and $Z = Z^*$

It is interesting to analyze these effects when a family reaches the subsistence level restriction. In Bhalotra (2001), the slope of the child labor supply function was calculated at the subsistence level. In this section, I do the same and compare our results with hers. Given that the family is constrained to these levels, the internal allocation of resources could be different with respect to the previous case.

The mathematical problem is similar to that in Section II, except that this time the subsistence level restrictions are binding. Taking differentials to the first order conditions, and using Cramer's Rule, I show that the derivatives of the household work functions are the same as those in the non-subsistence level case.[16]

The derivatives of the education function, however, are a bit different now.

Result 5: The derivatives of the hours of education with respect to wages are:

$$\frac{\partial E}{\partial Y} = \frac{1}{w_2} > 0 \tag{5.1}$$

$$\frac{\partial E}{\partial w_1} = H_1 \frac{\partial E}{\partial Y} \tag{5.2}$$

$$\frac{\partial E}{\partial w_2} = H_2 \frac{\partial E}{\partial Y} \tag{5.3}$$

Notice that unlike equation (3.3), this time the sign of (5.1) is definitely positive. Consequently, education is a normal good at this level. This result may sound counterintuitive because we do not expect that extremely poor families spend more on education than other necessary goods if income rises. The explanation I find for this result relies on the mathematical solution of the problem. The way I introduced the "subsistence level" was fixing consumption at the level c^*, so consumption cannot increase from that quantity and the whole model works as if the family had a "target income" in order to buy c^* and Z^*. Consequently, having reached the subsistence level, an increase of Y will cause a positive change on education only *ceteris paribus* unless we allow the consumption to change also, in which case we would be in the non-subsistence case.

The following result shows the derivatives of the child labor supply with respect to wages.

Result 6: From the first order conditions, the derivatives of the child labor supply are:

$$\frac{\partial H_2}{\partial Y} = -\frac{1}{w_2} < 0 \tag{5.4}$$

$$\frac{\partial H_2}{\partial w_1} = \underbrace{-\frac{\partial z_2}{\partial w_1}}_{HWE} + \underbrace{\frac{\partial H_2}{\partial Y} H_1}_{IE<0} \tag{5.5}$$

$$\frac{\partial H_2}{\partial w_2} = \underbrace{-\frac{\partial z_2}{\partial w_2}}_{HWE>0} + \underbrace{\frac{\partial H_2}{\partial Y} H_2}_{IE<0} \tag{5.6}$$

The sign of the derivatives in equations (5.5) and (5.6) is ambiguous. In (5.5), an increase in mother's wage may reduce child market labor H_2. It depends on the sign and the magnitude of the cross derivative $\partial z_2/\partial w_1$. In equation (5.6), the net effect would depend on the magnitude of the derivatives that correspond to the household work effect (*HWE*) and the income effect (*IE*). If the child's wage increases, more time would be spent in the market and less time at home (*HWE*). In addition, since an increase in child's wage raises the family income, we will observe more education and less child labor (*IE*). Comparing (5.6) to (3.7), the only difference is the substitution effect, which does not exist at the subsistence level. This means that the family would not give up more hours of child labor in order to have more education.

In Bhalotra's paper, the derivative in equation (5.6) was negative at the subsistence level because her model did not include home production. Her explanation that the slope of the child labor supply is negative at the subsistence level because the family has a target income applies for the second term on the right hand side of equation (5.6). A drop in child wage will cause a drop in the hours of education and consequently an increase in the child labor supply in order to reach the target income. However, in this model, there is a positive effect given by the effect on child's household work. That drop in the child's wage reduces the opportunity cost of child's household work and increases the demand for child's household work z_2, reducing the supply of child labor H_2.

VI. Summary and Conclusions

In this paper, I present a model that fills the existent gap in the theoretical literature of child labor concerning the relationship between household work, market labor and wages of the household members. Empirical studies have found no clear results when it comes to the effect of parents' wages and exogenous income on child labor. In this paper, I proposed that the inclusion of household work as an activity that children perform helps to explain why we observe those results in empirical papers.

The results show that household work performed by a child and the spouse depends on the wages of the family members and the price of substitutes in the market but not on the exogenous income. The model also shows that the variables related to home production depend only on household technology and not on household preferences. These results are consistent with those of Rosenzweig (1980) or Brown, Deardorff and Stern (2003).

As I expected, the introduction of household work affects the labor supply of the "wife" and the child. In addition to the standard income and substitution effects, a third effect related to household work affects the derivatives of the labor supply functions and the education function. That effect shows how changes in wages of individual members may reallocate the time dedicated to household work. These effects usually go in opposite directions, which may cause parametric estimation of the child labor supply in different countries to show contradictory or unclear results.

Concerning the corner solutions, the model states that we can observe specialization in household work, market work or studying if the marginal return of one of these activities is high enough to overcome the returns of the other two. I can use this result to interpret why girls usually remain home doing domestic work and boys go to the labor market. If the productivity of girls at home is high, the return of education is low (measured in this model as the marginal utility of one additional hour of studying) and the wage girls could earn in the labor market is low, they would work at home. Something similar applies to boys; they work in the labor market because the return of that activity (the wage) is higher than the marginal product of domestic work and the "return" of education.

Finally, in the analysis of the slope of the child labor supply at the subsistence level, I show that it includes an additional term that does not appear in Bhalotra's paper. This occurs because the omission of home production in her model yields a negative slope. However, in this model, the slope has both a negative and a positive component.

Notes

1 Basu and Van (1998) proposed the "luxury axiom", which states that parents will send their children to work if family income is so low that it reaches the subsistence level.

2 Using Peruvian data from the 2000 Living Standards Measurement Survey, on average, a household spends 64 hours per week doing chores. This figure is the result of the sum of hours individuals spend time doing chores.

3 For example, see DeGraff, Bilsborrow and Herrin (1996), DeGraff and Bilsborrow (2003) and Binder and Scrogin (1999).

4 It is important to note that f_0 and the output of the production function $f(z_1, z_2)$ are measured in goods and z_1 and z_2 are measured in units of time.

5 This assumption relies on what we observe in less developed countries: due to cultural reasons, the great majority of men work, but only a low proportion of women participate in the labor market. According to statistics of the Peruvian Living Standard Measurement Survey, 97.5% of male heads of household participate in the labor market.

6 The role of education has been introduced in this model in a very simple way. In fact, it is just an activity performed by the child that affects positively the family utility (as leisure does in standard models). Since leisure has not been included in the model, there is no confusion between those variables.

7 This is true because from the first order conditions, I get $w_1 = P \cdot f_1$ and $w_2 = P \cdot f_2$. These equations constitute a subsystem of two equations and two unknowns, z_1 and z_2. This result relies on the assumption of perfect substitution between chores produced at home and bought in the market.

8 These assumptions mean that the marginal product is decreasing for both inputs and the production function exhibits decreasing returns to scale.

9 A similar result was found in Rosenzweig (1980). If I change the assumption of perfect substitution between chores produced at home and bought in the market, then that derivative could be different from zero.

10 For example, Binder and Scrogin (1999) use the occupation of household head as a proxy of head of household's wage; their estimates show no clear effect on household work of these variables. Levison and Moe (1998) find that the effect of family income on housework is not significant.

11 This result is a shortcoming of the model because standard theory and empirical work show that wife's labor supply depends on husband's wage. This occurs in this model because leisure has not been included.

12 If the utility function were a strictly concave function, the derivative would be positive.

13 These results are consistent with Rosenzweig (1980) who found three effects for the husband and wife labor supply in a model with home production and two symmetric agents. In our model, they are not symmetric because of education and because we do not have leisure here.

14 See Newman and Gertler (1994), pp. 993-994.

15 This result is similar to that in Brown, Deardorff and Stern (2003). The difference is that they assume constant marginal products and, consequently, they have specialization.

16 From the first order condition (see the appendix), it is easy to get $w_1 = P \cdot f_1$ and $w_2 = P \cdot f_2$, which are the same equations I obtained in the non-subsistence case. Thus, the discussion in Section II regarding household work and the wife's labor supply does not change.

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APPENDIX

The optimization problem presented in Section II can be rewritten as

$$\underset{c,\ f_0,\ z_1,\ z_2,\ E}{Max} \quad U(\mathrm{c},\ f(z_1,z_2)+f_0,\ E)$$

s.t.

$$c+P_0=Y+(1-z_1)w_1+(1-z_2-E)w_2$$

$$1-z_1\geq 0$$

$$1-z_2-E\geq 0$$

$$c \quad c^*$$

$$f(z_1,z_2)+f_0 \quad z^*$$

$$f_0 \quad 0,\ z_1 \quad 0,\ z_2 \quad 0,\ ,\ E \quad 0$$

The lagrangean to this problem is:

$$L=U(\cdot)+\lambda_1(Y+(1-z_1)w_1+(1-z_2-E)w_2-c-Pf_0)+\lambda_2(c-c^*)+\lambda_3(f(z_1,z_2)+f_0$$
$$-z^*)+\lambda_4(1-z_1)+\lambda_5(1-z_2-E)$$

The first order necessary conditions are:

$$\frac{\partial L}{\partial c}=U_C-\lambda_1+\lambda_2\leq 0, \qquad c\geq 0$$

$$\frac{\partial L}{\partial f_0}=U_Z-\lambda_1\cdot P+\lambda_3\leq 0, \qquad f_0 \quad 0$$

$$\frac{\partial L}{\partial z_1}=U_Zf_1-\lambda_1\cdot w_1+\lambda_3 f_1-\lambda_4\leq 0, \qquad \mathrm{z}_1 \quad 0$$

$$\frac{\partial L}{\partial z_2}=U_Zf_2-\lambda_1\cdot w_2+\lambda_3 f_2-\lambda_5\leq 0, \qquad \mathrm{z}_2 \quad 0$$

$$\frac{\partial L}{\partial E}=U_E-\lambda_1\cdot w_2-\lambda_5\leq 0, \qquad \mathrm{E} \quad 0,$$

and the complementary slackness conditions:

$$c \cdot (U_C - \lambda_1 + \lambda_2) = 0$$

$$f_0 \cdot (U_Z - \lambda_1 \cdot P + \lambda_3) = 0$$

$$z_1 \cdot (U_Z f_1 - \lambda_1 \cdot w_1 + \lambda_3 f_1 - \lambda_4) = 0$$

$$z_2 \cdot (U_Z f_2 - \lambda_1 \cdot w_2 + \lambda_3 f_2 - \lambda_5) = 0$$

$$E \cdot (U_E - \lambda_1 \cdot w_2 - \lambda_5) = 0$$

When consumption levels are above the subsistence level, λ_2 and λ_3 equal zero.

From those first order conditions, in the interior case we have a system of nine equations and nine unknowns. The equations are the following.

$$U_C = \lambda_1 \tag{a.1}$$

$$U_Z = \lambda_1 \cdot P \tag{a.2}$$

$$U_Z f_1 = \lambda_1 w_1 \tag{a.3}$$

$$U_Z f_2 = \lambda_1 w_2 \tag{a.4}$$

$$U_E = \lambda_1 w_2 \tag{a.5}$$

$$c + P \cdot f_0 = Y + H_1.w_1 + H_2.w_2 \tag{a.6}$$

$$Z = f(z_1, z_2) + z_0 \tag{a.7}$$

$$1 = z_1 + H_1 \tag{a.8}$$

$$1 = z_2 + H_2 + E \tag{a.9}$$

The unknowns are: f_0, z_1, z_2, Z, H_1, H_2, c, E and λ_1. The lagrange multiplier λ_1 is the marginal utility of income. Taking differentials to equations (a.1) – (a.5), and rearranging in matrix form I have

$$\begin{bmatrix} U_{CC} & U_{CZ} & U_{CE} & -1 & U_{CZ}f_1 & U_{CZ}f_2 \\ U_{CZ} & U_{ZZ} & U_{ZE} & -P & U_{ZZ}f_1 & U_{ZZ}f_2 \\ U_{CE} & U_{ZE} & U_{EE} & -w_2 & U_{EZ}f_1 & U_{EZ}f_2 \\ -1 & -P & -w_2 & 0 & -w_1 & -w_2 \\ U_{CZ}f_1 & U_{ZZ}f_1 & U_{EZ}f_1 & -w_1 & U_{ZZ}f_1^2+U_Zf_{11} & U_{ZZ}f_1f_2+U_Zf_{12} \\ U_{CZ}f_2 & U_{ZZ}f_2 & U_{EZ}f_2 & -w_2 & U_{ZZ}f_1f_2+U_Zf_{12} & U_{ZZ}f_2^2+U_Zf_{22} \end{bmatrix} \begin{bmatrix} dc \\ df_0 \\ dE \\ d\lambda \\ dz_1 \\ dz_2 \end{bmatrix} =$$

$$\begin{bmatrix} 0 \\ \lambda_1 dP \\ \lambda_1 dw_2 \\ -dY-(1-z_1)dw_1-(1-z_2-E)dw_2-f_0dP \\ \lambda_1 dw_1 \\ \lambda_1 dw_2 \end{bmatrix} \qquad \text{(a.10)}$$

After row and column operations, I obtain the expression in equation (3.2).

The subsistence level case

The mathematical problem is similar to that presented in Section II, but in this case $c = c^*$ and $Z = Z^*$. The Kuhn-Tucker conditions in the interior case are:

$$U_C + \lambda_2 = \lambda_1 \qquad \text{(a.11)}$$

$$U_Z + \lambda_3 = \lambda_1 \cdot P \qquad \text{(a.12)}$$

$$(U_Z + \lambda_3)f_1 = \lambda_1 w_1 \qquad \text{(a.13)}$$

$$(U_Z + \lambda_3)f_2 = \lambda_1 w_2 \qquad \text{(a.14)}$$

$$U_E = \lambda_1 w_2 \qquad \text{(a.15)}$$

And the constraints:

$$c + P \cdot f_0 = Y + H_1.w_1 + H_2.w_2 \qquad \text{(a.16)}$$

$$c = c^* \qquad \text{(a.17)}$$

$$f(z_1, z_2) + f_0 = Z^* \qquad \text{(1.18)}$$

$$1 = z_1 + H_1 \tag{a.19}$$

$$1 = z_2 + H_2 + E \tag{a.20}$$

Taking differentials to equations (a.11) to (a.20),

$$\begin{bmatrix} U_{CC} & U_{CZ} & U_{CE} & -1 & 1 & 0 & U_{CZ}f_1 & U_{CZ}f_2 \\ U_{CZ} & U_{ZZ} & U_{ZE} & -P & 0 & 1 & U_{ZZ}f_1 & U_{ZZ}f_2 \\ U_{CE} & U_{ZE} & U_{EE} & -w_2 & 0 & 0 & U_{EZ}f_1 & U_{EZ}f_2 \\ -1 & -P & -w_2 & 0 & 0 & 0 & -w_1 & -w_2 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & f_1 & f_2 \\ U_{CZ}f_1 & U_{ZZ}f_1 & U_{EZ}f_1 & -w_1 & 0 & f_1 & \begin{matrix} U_{ZZ}f_1^2 + U_Z f_{11} \\ +\lambda_3 f_{11} \end{matrix} & \begin{matrix} U_{ZZ}f_1 f_2 + U_Z f_{12} \\ +\lambda_3 f_{12} \end{matrix} \\ U_{CZ}f_2 & U_{ZZ}f_2 & U_{EZ}f_2 & -w_2 & 0 & f_2 & \begin{matrix} U_{ZZ}f_1 f_2 + U_Z f_{12} \\ +\lambda_3 f_{12} \end{matrix} & \begin{matrix} U_{ZZ}f_2^2 + U_Z f_{22} \\ +\lambda_3 f_{22} \end{matrix} \end{bmatrix} \begin{bmatrix} dc \\ df_0 \\ dE \\ d\lambda_1 \\ d\lambda_2 \\ d\lambda_3 \\ dz_1 \\ dz_2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ \lambda_1 dP \\ \lambda_1 dw_2 \\ -dY - (1 - z_1)dw_1 - (1 - z_2 - E)dw_2 - f_0 dP \\ dc* \\ dz* \\ \lambda_1 dw_1 \\ \lambda_1 dw_2 \end{bmatrix}$$

The derivatives presented in section V have been computed using his matrices.