

S&P 500 UNDER A STRUCTURAL MACRO-FINANCIAL MODEL * **S&P 500 BAJO UN MODELO MACROFINANCIERO ESTRUCTURAL**

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Abstract

In this paper, we propose a macro-financial model that combines a semi-structural, medium-term macroeconomic model with the Dynamic Gordon Model or DGM (Campbell and Shiller, 1988). The proposed framework allows us to analyze the relationship between the output gap, inflation, short-term interest rate, and stock market indicators: price, dividend, and volatility. We estimate the model for the US economy using Bayesian techniques on quarterly data from 1984 to 2020. The decomposition of the unconditional variance of the variables shows that (i) demand shocks are relevant for most macroeconomic variables and stock prices; (ii) supply shocks affect inflation mainly; (iii) shocks to the price-dividend ratio account for around 12%, 5% and 16% of the variability of the output gap, inflation, and interest rates, respectively; and (iv) the DGM mechanism helps to cushion the effects of an interest rate shock and increases the speed of convergence of all macroeconomic variables after an inflation shock, compared to a standard, semi-structural model, reflecting in this manner the importance of stock prices on the dynamics of macroeconomic variables.

* We thank the helpful comments of participants at an internal seminar at the Financial Stability Area of the Central Bank of Chile, as well as two anonymous referees. The opinions expressed herein are those of the authors and do not necessarily represent the views of the Central Bank of Chile or its Board members. All remaining errors are our own.

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Keywords: *New-Keynesian model, asset pricing, Bayesian estimation, business cycles.*

JEL Classification: *C11, E12, E17, G12.*

Resumen

En este artículo proponemos un modelo macrofinanciero que combina un modelo macroeconómico semiestructural de mediano plazo con el Modelo Dinámico de Gordon o MDG (Campbell y Shiller, 1988). El marco propuesto permite analizar la relación entre la brecha de producto, inflación, tasa de interés de corto plazo, e indicadores del mercado de valores como precio, dividendos, y volatilidad. Estimamos el modelo utilizando técnicas bayesianas con datos trimestrales de EE.UU. desde 1984 a 2020. La descomposición de la varianza incondicional de las variables revela que (i) shocks de demanda son relevantes para la mayoría de las variables macroeconómicas y el precio de las acciones; (ii) shocks de oferta afectan principalmente a la inflación; (iii) shocks al ratio precio-dividendo representan cerca de 12%, 5% y 16% de la variabilidad de la brecha de producto, inflación y tasas de interés, respectivamente; y (iv) el mecanismo MDG ayuda a amortiguar los efectos de shocks a la tasa de interés e incrementa la velocidad de convergencia de todas las variables macroeconómicas luego de un shock de inflación, respecto de un modelo semiestructural estándar, lo que refleja la importancia del precio de las acciones respecto de la dinámica de variables macroeconómicas.

Palabras clave: *Modelo neokeynesiano, valoración de activos, estimación bayesiana, ciclos económicos.*

JEL Clasificación: *C11, E12, E17, G12.*

I. INTRODUCTION

Gordon (1962) model is widely used for evaluating stock prices. The static version of this model provides a closed-form solution for the intrinsic value of a stock, which is determined by three key elements, namely future dividend payments, the dividend growth rate, and the relevant interest rate. However, the assumptions required for achieving the static version of the model are not realistic, neither useful for proper dynamic analysis. In this sense, Campbell and Shiller (1988) provide an alternative version, which is based on a first-order approximation of the log stock return. This alternative is known as Dynamic Gordon growth Model (DGM), and it

has been applied to the US stock market to decompose the impact of dividend growth and interest rates over stock returns (Campbell and Ammer, 1993), or to analyze the impact of changes in monetary policy on equity prices (Rigobon and Sack, 2003; Bernanke and Kuttner, 2005), among other empirical applications. Nevertheless, the DGM lacks key macroeconomic variables such as output growth or inflation rate. In that sense, Blanchard (1981) provides a theoretical model in which output gap, stock prices, and interest rates are included. It extends a standard IS-LM model by adding long-term interest rate and equity; however, risk premium is zero.

In this article, we propose to enrich the DGM with a rational expectations New-Keynesian macroeconomic model composed by an aggregate demand equation (or IS curve), an aggregate supply equation (or Phillips curve), and an equation that describes the dynamics of the short-term interest rate. Extending a structural macroeconomic model with asset pricing component is in line with others efforts documented in the literature. For example, Berkaert, Cho and Moreno (2010) extend a standard New Keynesian model with a non-arbitrage Affine Term Structure model; meanwhile Gray *et al.* (2010) introduce financial risk indicators into a structural macroeconomic model.

Based on data for the US economy, we estimate our model's parameters using quarterly data from 1984 to the third quarter of 2020. The estimation poses some challenges that we discuss in detail. In particular, the close-to-non-stationary dynamic of the price-dividend ratio suggests that results based solely on empirical approaches, such as VAR, should be taken with caution and given that we use Bayesian techniques. Overall, the estimated model provides a good representation of crucial variables' dynamics, although some sample moments related to the inflation rate are over-estimated. The decomposition of the variables' unconditional variance in the proposed model shows that demand shocks are relevant for both the output gap and interest rates, explaining 48% to 82% of the total variance, respectively. Meanwhile, supply shocks affect inflation mainly. Finally, shocks to the price-dividend ratio account for roughly 12%, 5%, and 16% of the fluctuations in the output gap, inflation, and interest rates, respectively. The impulse-response analysis shows that, relative to an otherwise standard semi-structural macroeconomic model, the DGM mechanism helps to cushion the effects of an interest rate shock and increases the speed of convergence of all macroeconomic variables after an inflation shock. Hence, these results highlight the importance of stock prices on the dynamics of macroeconomic variables.

The article is organized as follows. Section II presents the model, and Section III introduces the US aggregate variables used in the estimation of our model. Section IV shows estimation results under a Bayesian approach, and Section V concludes.

II. MODEL SETUP

The model consists of a set of four equations that we describe in detail in this section. The first three equations describe macroeconomic variables such as output

gap, inflation, and the short-term interest rate. The last equation, on its part, defines the stock price dynamics. For simplicity, we assume that all variables are expressed as linear approximations around their long-run trends. Thus, they are all demeaned or de-trended.

The first equation is aggregate demand, or IS curve, establishes that output gap (y_t) is a function of a mixed backward-forward specification, which allows for both inter-temporal optimization and some degree of inertia. The short-term real interest rate (r_t) has a negative impact on output gap through the investment channel, whereas the log P-D ratio (z_t) captures the positive impact of investment opportunities on y_t . Thus,

$$y_t = \lambda_0 y_{t-1} + (1 - \lambda_0) E_t(y_{t+1}) - \lambda_1 r_t + \lambda_2 z_t + u_{1t} \quad (1)$$

where u_{1t} is a structural aggregate demand shock. The lag and lead parameters of the output gap are forced to sum up to one, which is a standard constraint applied in practice to reduce the number of parameters.

Second equation is the aggregate supply or a hybrid Phillips curve, where the inflation rate (π_t) is a rational expectations solution with inertia given by

$$\pi_t = \delta_0 \pi_{t-1} + (1 - \delta_0) E_t(\pi_{t+1}) + \delta_1 y_t + u_{2t} \quad (2)$$

where u_{2t} is a structural aggregate supply shock. As in the case of equation (1), we consider the same constraint on the lag and lead parameters of π_t .

To complete the macroeconomic model, we include a third equation for the short-term interest rate (i_t). This equation relates the short-term interest rate with its own lag, the expectations about future inflation, and the output gap, as follows:

$$i_t = \kappa_0 i_{t-1} + \kappa_1 E_t(\pi_{t+1}) + \kappa_2 y_t + u_{3t} \quad (3)$$

where u_{3t} is a structural short-term interest rate shock. It should be noted that we do not impose any particular restriction on the parameters of this equation. Further, in our empirical application, we use the Wu and Xia (2016) shadow rate because, after the Global Financial Crisis of 2007-2008, short-term risk-free rates in the US were truncated at 0%, an episode known as the Zero Lower Bound period¹. Also, the short-term real interest rate is defined as follows: $r_t \equiv i_t - E_t(\pi_{t+1})$.

Finally, we consider a dynamic pricing equation that relates stock prices with expected future dividend payments in a similar way than Campbell and Shiller (1988). We use a log-linear approximation of the log stock return (x_{t-1}), which is the

¹ As a robustness check exercise, we conducted all our estimations using an alternative short-term: high-quality corporate bond rate. The results are similar to those presented in this paper; hence they are not reported.

following: $x_{t+1} = k + \rho p_{t+1} + (1 - \rho)d_{t+1} - p_t$, where p_t is the log of stock price and d_t is the log of dividend payments, both in real terms, and k and ρ are parameters of the Taylor approximation. In particular, the parameter ρ is related to the long run level of the price-dividend (P-D) ratio.

Ignoring constant terms ($k = 0$), we have the following ex-ante equation for the log of the stock price: $p_t = d_t + \rho E_t(p_{t+1} - d_{t+1}) + E_t(\Delta d_{t+1}) - E_t(x_{t+1})$. Further, we assume that dividends are related to the real economy as $E(\Delta d_t) = \theta_2 y_t$, meanwhile the log stock return is related to the short-term interest rate and to some risk premium as $E(x_{t+1}) = \theta_3 r_t + \theta_4 h_t$, where h_t is the log of VIX that characterizes the risk premium in the model. Taking these elements and $z_t = p_t - d_t$ as the log P-D ratio, we have the following equation: $z_t = \rho E_t(z_{t+1}) + \theta_2 E_t(y_{t+1}) - \theta_3 r_t - \theta_4 h_t$. However, various related studies have documented a high persistence of the P-D ratio in small samples (see Lettau and van Nieuwerburgh, 2008; Chevillon and Mavroeidis, 2018; Golinski, Madeira and Rambaccussing, 2018; among others). Thus, to incorporate some degree of inertia, we consider a hybrid version of the previous equation instead:

$$z_t = \theta_0 z_{t-1} + \theta_1 E_t(z_{t+1}) + \theta_2 E_t(y_{t+1}) - \theta_3 r_t - \theta_4 h_t + u_{4t} \quad (4)$$

where u_{4t} is a structural shock to the log P-D ratio. To complete the model, we assume that each structural shock follows an uncorrelated, zero-mean AR(1) process as

$$u_{it} = \rho_i u_{i,t-1} + \sigma_i v_{it} \quad (5)$$

for all $i = 1, \dots, 4$, and v_{it} is a zero-mean, unit-variance shock. Also, we assume that h_t follows an AR(1) process².

III. DATA

We consider six US aggregate variables in quarterly frequency that span the period from 1984q1 to 2020q3: output gap, annualized quarterly inflation rate, risk premium proxied by the log of the VIX, short-term shadow interest rate, stock prices, and dividends per share (Table 1). We demean all variables to be consistent with the model described previously.

² The specification of our structural model implies that its reduced-form representation follows a VAR(2) process. See the Appendix for more details.

TABLE 1
DATA DESCRIPTION AND SOURCES

Variable	Description	Source	Observations
y	Output gap	FRED	HP-filtered real GDP with $\lambda = 1,600$
π	Inflation rate	FRED	Annualized quarterly growth of CPI
h	Risk premium	FRED	Log of VIX
i	Wu and Xia (2016) shadow rate	FRBA	Annual rate
p	Price	Robert Shiller's webpage	Log of S&P 500 index divided by CPI
d	Dividend	Robert Shiller's webpage	Log of dividends per share divided by CPI

FRED stands for to the Federal Reserve Economic Data of the Federal Reserve Bank of Saint Louis; FRBA stands for the Federal Reserve Bank of Atlanta; GDP stands for Gross Domestic Product; CPI stands for Consumer Price Index; VIX stands for Chicago Board Options Exchange Market Volatility Index.

In particular, we compute the output gap (y_t) as the difference between real GDP and its HP-filtered trend component, where the smoothing parameter was set equal to 1,600. The inflation rate (π_t) is the annualized quarterly variation of the Consumer Price Index, whereas our measure of risk premium (h_t) is the log of the VIX. Since this variable is available from 1990q1, we extended the series back to 1984q1 by estimating a GARCH(1,1) model on monthly stock returns. Then, we relate the VIX with the one-month-ahead predicted standard deviation to account for the former variable being the 30-day expected volatility of the S&P 500 returns.

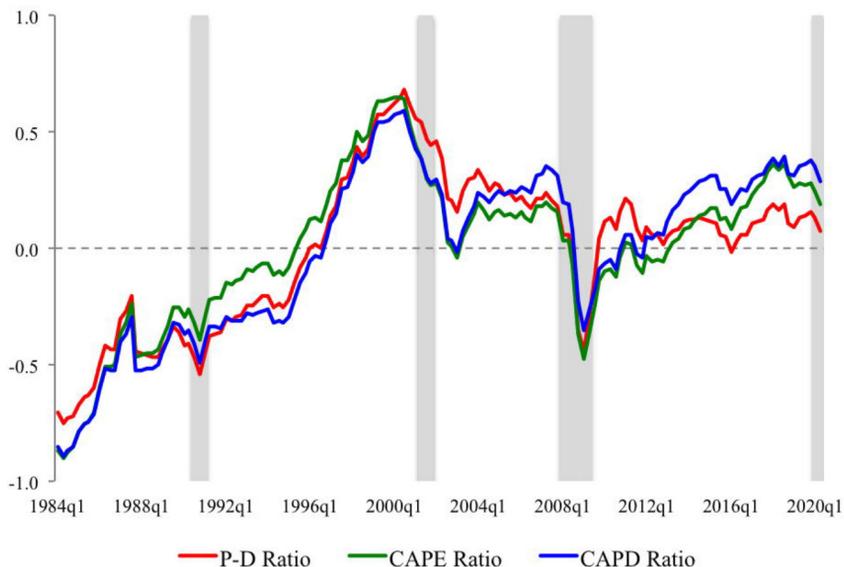
The short-term interest rate (i_t), on its part, corresponds to the shadow rate proposed by Wu and Xia (2016). Unlike the observed short-term interest rate, the shadow rate is not bounded below by 0%. In particular, it is the implicit 1-month interest rate based on a truncated Gaussian non-arbitrage affine term structure model. Moreover, when the shadow rate is positive and above 0.25%, it is equal to the Effective Federal Funds Rate.

Further, the price-dividend ratio (z_t) corresponds to the difference between the S&P 500 index and dividends per share (both in real terms and logarithms)³. However, it is common in the financial industry to use an alternative measures by considering the price-earnings ratio, where earnings per share are smoothed over a moving window of ten years. This alternative measure of price pressures is called the Cyclical-Adjusted Price to Earnings (CAPE) ratio. Figure 1 shows the P-D along with CAPE and the

³ The dataset is the same as in Shiller (1989) and is available online for download.

FIGURE 1

COMPARISON OF ALTERNATIVE PRICE RATIOS

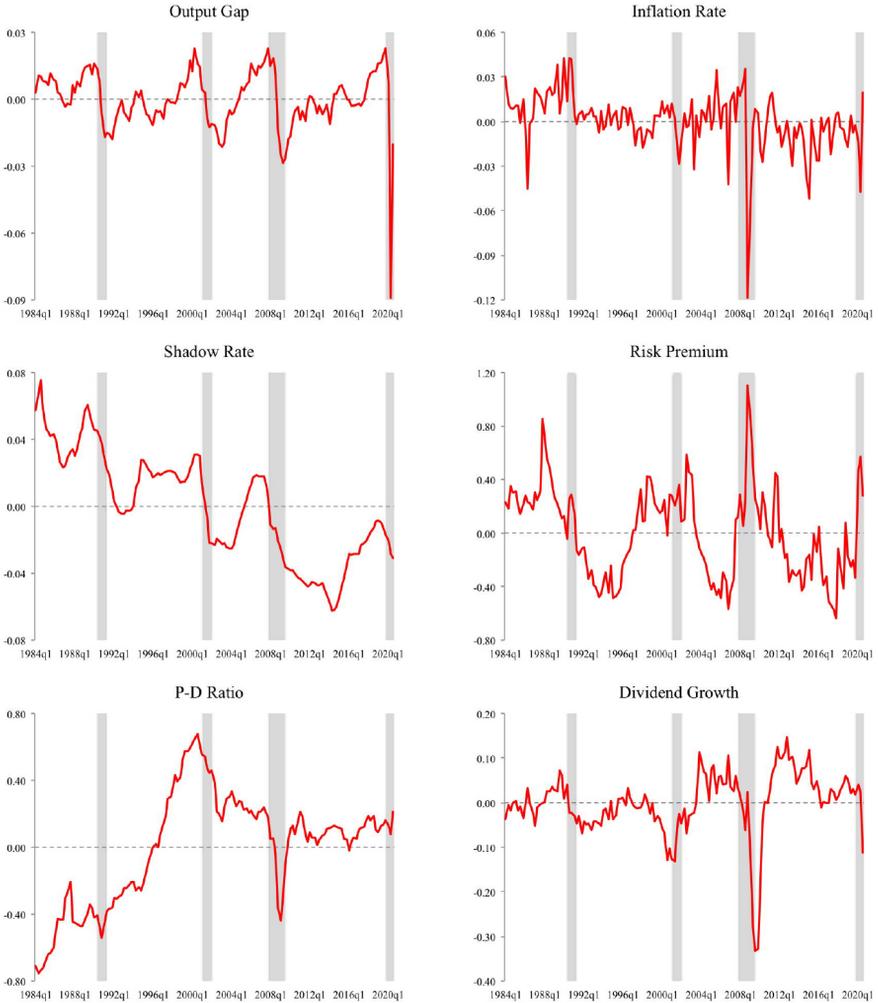


All variables are detrended. P-D, CAPE, and CAPD stand for price to dividend, cyclical-djusted price to earnings, and cyclical-adjusted price to dividend, respectively. All variables are detrended. Shaded areas correspond to recession periods, as defined by the National Bureau of Economic Research (NBER).

Cyclical-Adjusted Price to Dividend (CAPD) ratios, the latter being the traditional price-dividend ratio but with dividends per share smoothed over a moving window of ten years, all in logs. Note that, before the 2007-2008 Global Financial Crisis period, all three indicators behave similarly. However, after that event, the smoothed ratios tend to exhibit a positive trend, in sharp contrast with the P-D ratio that remains relatively stable. This result suggests that CAPE and CAPD are strongly affected by the crisis period, where both earnings and dividends per share decreased significantly.

In our sample, there is a declining trend in the short-term interest rate being below its average during the last decade (Figure 2). Although this trend reverted starting in 2014, the COVID-19 pandemic implied further decreases in the interest rate to stimulate the economy. On the other hand, the price-dividend ratio has an increasing trend before 2000, followed by some mean-reversion, and a significant drop during the Global Financial Crisis of 2007-2008. Interestingly, the P-D ratio remained relatively unchanged during the recent coronavirus pandemic, and the dividend growth rate decreased by a considerably smaller magnitude compared to its dramatic fall –around

FIGURE 2
OBSERVABLES FOR THE DYNAMIC GORDON MODEL



Shaded areas correspond to recession periods, as defined by the National Bureau of Economic Research (NBER).

30% below its historical average— during the Global Financial Crisis period. Lastly, the risk premium measure has a sharp upsurge during the Global Financial Crisis and the COVID-19 worldwide outbreak.

IV. BAYESIAN ESTIMATION

4.1. Posteriors

Several parameters of the model are standard in the monetary policy literature. Thus, we draw priors for this set of parameters from the Beta, Normal, and Inverse Gamma distributions. The prior mean was set equal to the related literature's calibrated values, and we choose the prior standard deviations to contemplate plausible ranges for the parameters. In particular, we consider the calibration used by Fuhrer (2010) to study inflation persistence in the US economy. More precisely, for the case of the IS equation, we set $\lambda_0 = \delta_0 = 0.5$, which provides a balance between backward- and forward-looking dynamics of the output gap and inflation. Also, we use $\lambda_1 = \delta_1 = 0.1$, although, in practice, it is possible to have lower magnitudes. In the case of λ_2 , we assume that the P-D ratio has a similar impact on the output gap than the short-term interest rate. For the short-term interest rate equation, we use $\kappa_0 = 0.8$, $\kappa_1 = (1 - 0.8) \times 1.5 = 0.3$, and $\kappa_2 = (1 - 0.8) \times 0.5 = 0.1$. For the price-dividend equation, we consider $\theta_0 = 0.05$ and $\theta_1 = 0.8$. There is no empirical counterpart for θ_2, θ_3 , and θ_4 . Thus, we use 0.5. Finally, we assume for simplicity that $\rho_i = 0.5$ and $\sigma_i = 0.005$ for all structural shocks and the dynamics of h_t .

Table 2 reports the mode, the mean, and the 90% probability interval related to the posterior distribution of the model's structural parameters obtained by the Metropolis-Hastings algorithm with 10,000 replications. Several aspects are worth highlighting from this table. First, the posterior mean of the IS curve parameters is somewhat similar to the standard calibrations in semi-structural DSGE models. The mean sensitivity of the output gap to variations of the short-term interest rate is estimated to be 0.17, close to the calibrated value. In contrast, the lag parameter of this curve is about half of its prior mean, while the output gap is almost insensitive to variations of the P-D ratio. Second, the estimated parameters of the hybrid Phillips curve reveal a forward-looking behavior of inflation. Further, our results also show that the output gap has relevant effects on inflation, which is roughly 60% larger than its calibrated value. Third, the posterior mean of the parameters associated with the equation for the short-term interest rate shows an important backward-looking component consistent with the downward trend exhibited by the shadow rate during the whole sample (Figure 2). Also, the point estimate of the parameter k_1 is around 1.6 times larger than its calibrated value, implying that the short-term interest rate is very sensitive to expected inflation fluctuations. Lastly, the estimates related to the pricing equation support the idea that there exists an interaction between the output gap and the stock market. In particular, the posterior mean of θ_2 is close to 0.38, suggesting that the P-D ratio reacts to changes in the expected value of future output growth.

TABLE 2
POSTERIOR ESTIMATE OF MODEL PARAMETERS

Parameter	Prior Distribution			Posterior Distribution		
	Distrib.	Mean	Std. Dev.	Mode	Mean	90% PI
λ_0	Beta	0.500	0.100	0.214	0.229	[0.214, 0.253]
λ_1	Beta	0.100	0.050	0.170	0.156	[0.140, 0.176]
λ_2	Beta	0.100	0.050	0.017	0.018	[0.012, 0.023]
δ_0	Beta	0.500	0.100	0.130	0.111	[0.091, 0.130]
δ_1	Beta	0.100	0.050	0.164	0.162	[0.148, 0.171]
k_0	Beta	0.800	0.150	0.912	0.920	[0.903, 0.943]
k_1	Beta	0.300	0.100	0.465	0.465	[0.424, 0.499]
k_2	Beta	0.100	0.050	0.092	0.098	[0.072, 0.118]
θ_0	Beta	0.050	0.010	0.045	0.049	[0.044, 0.054]
θ_1	Beta	0.800	0.050	0.883	0.879	[0.867, 0.890]
θ_2	Normal	0.500	0.150	0.383	0.364	[0.299, 0.414]
θ_3	Normal	0.500	0.150	0.388	0.383	[0.347, 0.428]
θ_4	Normal	0.500	0.150	0.042	0.044	[0.032, 0.055]
ρ_1	Beta	0.500	0.200	0.965	0.962	[0.945, 0.978]
ρ_2	Beta	0.500	0.200	0.046	0.084	[0.056, 0.113]
ρ_3	Beta	0.500	0.200	0.448	0.461	[0.377, 0.551]
ρ_4	Beta	0.500	0.200	0.964	0.958	[0.939, 0.979]
ρ_h	Beta	0.500	0.200	0.801	0.796	[0.731, 0.855]
σ_1	Inv. Gamma	0.005	∞	0.002	0.002	[0.002, 0.003]
σ_2	Inv. Gamma	0.005	∞	0.015	0.015	[0.013, 0.018]
σ_3	Inv. Gamma	0.005	∞	0.005	0.005	[0.005, 0.006]
σ_4	Inv. Gamma	0.005	∞	0.006	0.006	[0.005, 0.008]
σ_h	Inv. Gamma	0.005	∞	0.190	0.192	[0.176, 0.214]

The posterior distribution was obtained using the Metropolis-Hastings algorithm with 10,000 replications. PI stands for probability interval.

4.2. Moment Comparison

To evaluate our proposed model's performance, we compare some key sample moment conditions of the output gap, inflation, the P-D ratio, the short-term interest rate, and the measure of risk premium, with their corresponding moments derived from the estimated model. In particular, we focus our attention on the volatility, the first-order autocorrelation, and the cross-correlation between these variables.

Table 3 shows the results of our estimations for the baseline model and a specification without the DGM mechanism⁴. Overall, we note that the model can capture several

⁴ The specification without the DGM mechanism considers equations (1) to (3) plus the AR(1) process for the structural shocks only and shuts-down the z_t process in the IS curve. In other words, this specification is simply a standard, semi-structural macroeconomic model.

TABLE 3
MOMENT COMPARISON

Moment	Point Estimate	Std. Err.	Baseline		No DGM	
			Model Moment	z -stat	Model Moment	z -stat
σ_y	0.013	0.001	0.017	3.23	0.021	5.59
σ_π	0.020	0.001	0.022	4.49	0.021	3.32
σ_z	0.332	0.024	0.286	1.94		
σ_i	0.032	0.003	0.038	2.15	0.010	8.87
σ_h	0.336	0.022	0.323	0.58		
$\varphi_y(1)$	0.648	0.064	0.774	1.97	0.809	2.51
$\varphi_\pi(1)$	0.314	0.078	0.383	0.88	0.462	1.89
$\varphi_z(1)$	0.968	0.016	0.943	1.61		
$\varphi_i(1)$	0.980	0.013	0.989	0.69	0.937	3.24
$\varphi_h(1)$	0.823	0.047	0.802	0.45		
$\varphi_{y,\pi}$	0.321	0.079	0.532	2.69	0.414	1.18
$\varphi_{y,z}$	0.037	0.083	0.125	1.07		
$\varphi_{y,i}$	0.377	0.077	0.259	1.54	-0.482	11.2
$\varphi_{y,h}$	-0.155	0.082	-0.179	0.30		

σ denotes volatility, $\varphi(1)$ denotes first-order serial correlation, and $\varphi_{a,b}$ denotes the cross-correlation between variables a and b . Model moments based on the posterior mean of the estimated parameters.

univariate and multivariate dynamic behavior of the endogenous variables mentioned before. For instance, in our dataset, the price-dividend ratio is about 25 times more volatile than the output gap. In comparison, the estimated model delivers a P-D ratio that is roughly 17 times more volatile, although this difference is not statistically relevant at standard significance levels. Analogously, the sample autocorrelation of the output gap and the P-D ratio and the cross-correlation between them is close to 0.65, 0.97, and 0.04, respectively, whereas the estimated model predicts values for these statistics of around 0.77, 0.94, and 0.13. However, the model fails to replicate some moments related to the inflation rate. More concretely, the volatility of π and the correlation between y and π predicted by the estimated model are significantly above their sample counterparts.

From the previous table, we also note that incorporating the DGM mechanism into an otherwise standard, semi-structural macroeconomic model improves the overall fit to data. In particular, when this mechanism is not present, then several moments of the short-term interest rate and inflation move away from their sample counterparts. For example, the volatility of i predicted by the model without the DGM channel is about one-third of that in the data, its first-order autocorrelation becomes smaller compared to the baseline model, and the cross-correlation between this variable and output gap turns negative. In the case of inflation, the model without DGM generates

a significantly higher persistence of π . However, its volatility and cross-correlation with the output gap are closer to their sample counterparts than the proposed model.

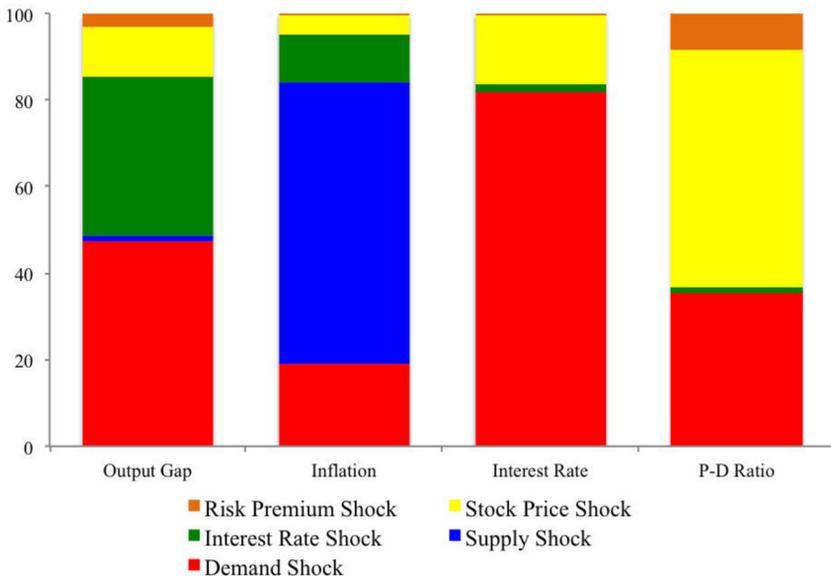
4.3. Variance Decomposition

Figure 3 shows the decomposition of the unconditional variance of the output gap, inflation, the short-term interest rate, and the price-dividend ratio based on the mean of the model posterior distribution shown in Table 2.

From this figure, we note that demand, interest rate, and stock price shocks drive the output fluctuations, thus reflecting the importance of this variable's dynamics and the importance of stock prices and interest rates on aggregate demand through consumption and investment. A similar situation occurs when decomposing movements in the shadow rate, which are propelled mainly by demand and stock price shocks and, to a lesser extent, by interest rate shocks. In the case of supply shocks, they are more relevant to inflation in general. Their incidence is about 65% of the total variance due to the instead forward-looking nature of this variable. Finally, in the case of the P-D ratio, roughly 55% of all variations are explained by stock price shocks, while demand shocks account for about 35% of the total variance. To understand this

FIGURE 3

UNCONDITIONAL VARIANCE DECOMPOSITION



result, we should look at the prior and posterior mean of θ_2 . From our estimations, the sensitivity of the price-dividend ratio to changes in the expected value of future output gap is smaller than its prior value, which is in line with the fact that, previous to the Global Financial Crisis of 2007-2008, episodes of positive output gaps tend to coincide with stock market expansions.

4.4. Impulse Response Functions

Figure 4 shows the results for a set of key Impulse-Response Functions (IRFs) under our baseline model (blue lines) and under the model without the DGM mechanism (green lines).

The first column of this figure shows the impact of an unanticipated increase in the short-term interest rate, reducing the output gap, inflation, and the price-dividend ratio in the short run. The impact on the stock market is driven by both the output gap effect and the discounting effect. In the former case, it is clear that the higher interest rate will deteriorate the output gap for several periods through the investment channel, which in turn will narrow the growth rate of dividends and hence the P-D ratio. In the second case, the higher short-term interest rate and lower expectations about future inflation imply a higher rate at which future dividend payments are discounted. Therefore, the price-dividend ratio will fall below its steady-state value for several periods. Further, note that the DGM mechanism tends to cushion the interest rate shock because, in the absence of this mechanism, the investment channel's magnitude (λ_1) is closer to its traditional calibration value. Hence the output gap and, ultimately, inflation reacts less to this shock.

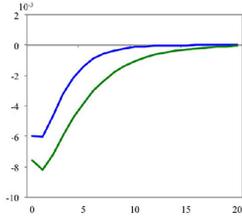
The second column of Figure 4 shows that a positive inflation shock will worsen both the output gap and the P-D ratio. This shock can be related, for instance, to an increase in oil prices, thus implying an inverse relationship with stock prices given the predominance of the real activity channel in the short-run. However, in the medium-run, the discounting effect becomes more relevant. Thus, the price to dividend ratio will increase because the short-term real interest rate is still below its steady-state value. In this context, stock prices return to their long-run value in about ten quarters after the shock occurred. We also note that when the DGM channel is not present, the short-term interest rate's sensitivity to expected inflation is about one-third of its estimated value under the baseline model. In this way, inflation rises more than proportionally. Consequently, the short-term real interest rate falls below its steady-state value and positively stimulates the output gap in the short-run. However, the speed of convergence to the long-run equilibrium values is slower than in the baseline model because of the absence of the discounting effect that positively affects the P-D ratio and the output gap in the medium-run.

Lastly, the third column of Figure 4 reports the impact of a positive, one-standard-deviation shock in the price-dividend ratio equation. This shock can be viewed, for example, as an increase in the risk appetite of investors. The positive shock in the

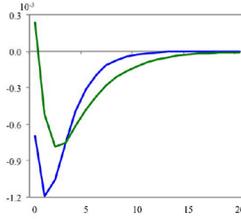
FIGURE 4

IMPULSE-RESPONSE FUNCTIONS

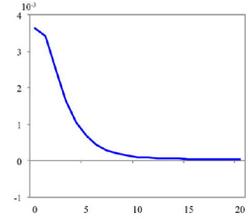
Output Gap to ST Interest Rate



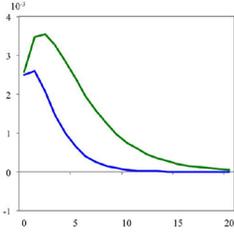
Output Gap to Inflation



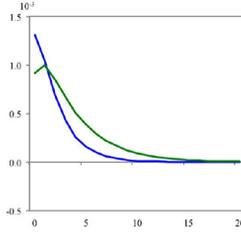
Output Gap to P-D Ratio



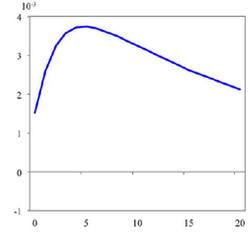
ST Interest Rate to ST Interest Rate



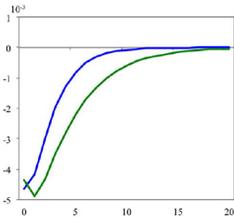
ST Interest Rate to Inflation



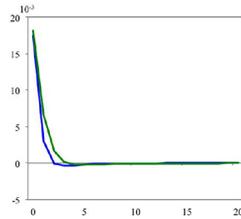
ST Interest Rate to P-D Ratio



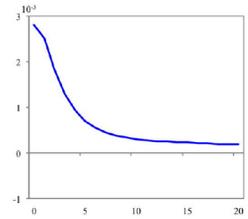
Inflation to ST Interest Rate



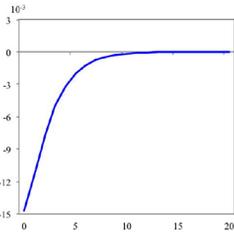
Inflation to Inflation



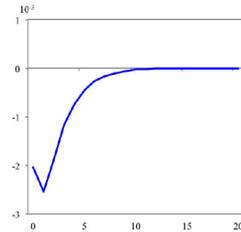
Inflation to P-D Ratio



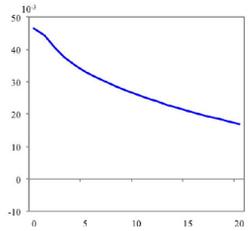
P-D Ratio to ST Interest Rate



P-D Ratio to Inflation



P-D Ratio to P-D Ratio



Response of each variable to a one-standard-deviation shock under the baseline model (blue lines) and no DGM model (green lines).

P-D ratio will stimulate aggregate spending and the output gap through consumption and investment (Blanchard, 1981). Specifically, in the former case, since stock shares are part of the wealth, a higher valuation of the stock market will positively affect consumption. In the latter case, a larger price-dividend ratio will boost the value of capital in place relative to its replacement cost, hence expanding investment.

V. CONCLUSIONS

This article extends the Dynamic Gordon growth Model (DGM) with a medium-term, semi-structural macroeconomic model. Thus, the proposed four-equations framework allows us to analyze the relationship between the output gap, inflation, stock price, and interest rate. The parameters of the model are estimated using Bayesian techniques. The decomposition of the unconditional variance of the model variables, evaluated in the posterior means, shows that demand shocks are relevant for both macroeconomic variables and stock prices. Moreover, shocks to the price-dividend ratio account for around 12%, 5%, and 16% of the variability of the output gap, inflation, and interest rates, respectively. The impulse-response analysis shows that, relative to an otherwise standard semi-structural macroeconomic model, the DGM mechanism helps to cushion the effects of an interest rate shock and increases the speed of convergence of all macroeconomic variables after an inflation shock. Hence, these results highlight the importance of stock prices on the dynamics of macroeconomic variables.

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APPENDIX

A.1. Reduced-form Representation

Let $X_t = (y_t, \pi_t, i_t, z_t, h_t)'$ be the vector of all endogenous variables of the model, and $U_t = (u_{1t}, u_{2t}, u_{3t}, u_{4t}, u_{ht})'$ the vector of structural shocks. With these definitions, the model can be written in matrix form as follows:

$$AX_t = BE_t(X_{t+1}) + CX_{t-1} + U_t \quad (\text{A.1})$$

where the matrices A , B , and C contain structural parameters. We can take a structural VAR (SVAR) of order one as a candidate solution for expression (A.1):

$$X_t = PX_{t-1} + QU_t \quad (\text{A.2})$$

where P is assumed to have all its eigenvalues inside the unit circle. The solution for P and Q depends on the structure of U_t . In our case, we assumed that each structural shock follows an AR(1) process. Hence

$$U_t = RU_{t-1} + SV_t \quad (\text{A.3})$$

where R accounts for the persistence of the shocks (ρ_i), and S for their variances (σ_i). Taking all all previous expressions together, we have that $E_t(X_{t+1}) = PX_t + QRU_t$, and replacing into the structural model (A.1) yields $AX_t = B(PX_t + QRU_t) + CX_{t-1} + U_t$, which is equivalent to

$$X_t = (A - BP)^{-1}CX_{t-1} + (A - BP)^{-1}(BQR + I)U_t$$

By the undetermined coefficient approach (Uhlig, 1999), matrix P solves $P = (A - BP)^{-1}C$, meanwhile matrix Q satisfies $Q = H(BQR + I)$. In the case of P , it requires to solve a quadratic matrix equation:

$$BP^2 - AP + C = 0 \quad (\text{A.4})$$

Several procedures can be used to solve equation (A.4), such as a generalized eigenvalue problem, and Schur decomposition. In particular, we consider an iteration process that involves a matrix $H(P) = (A - BP)^{-1}C$. Thus, the process initiates with P_0 , then $H_0 = H(P_0)$, and updated with $P_{i+1} = H_i C$.

After getting P , matrix Q can be obtained using the $vec(\cdot)$ operator. By using this operator, we have that $vec(Q) = vec[(HB)QR] + vec(H)$. Noting also that $vec(ABC) = (C' \otimes A)vec(B)$, then we have the following expression for Q :

$$vec(Q) = [I - R' \otimes (HB)]^{-1} vec(H) \quad (A.5)$$

Given equation (A.2), we also have $X_{t-1} = PX_{t-2} + QU_{t-1}$, which can be multiplied by QRQ^{-1} as follows: $QRQ^{-1}X_{t-1} = QRQ^{-1}PX_{t-2} + QRU_{t-1}$. Subtracting the latter from (7), and replacing (8), yields the following VAR(2) model (Cho and Moreno, 2006):

$$X_t = (P + QRQ^{-1})X_{t-1} - QRQ^{-1}PX_{t-2} + QSV_t \quad (A.6)$$

In conclusion, the reduced form of our proposed structural model corresponds to a VAR(2) process. More importantly, note that the solution relies crucially on solving expressions (A.4) and then (A.5), for getting matrices P and Q , respectively.