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FIXED COSTS, PRODUCT VARIETY, AND WELFARE ON THE INTERNET

COSTOS FIJOS, VARIEDAD DE PRODUCTOS Y BIENESTAR DEL COMERCIO ELECTRONICO

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Abstract

The impact of product variety on welfare has received little attention in the electronic commerce literature. The problem with product variety is that more variety does not necessarily imply higher welfare. This paper finds the conditions under which more variety, if caused by lower fixed costs, implies an unambiguous welfare gain. A calibration to the market of books confirms the intuition that fixed costs –in particular, per-title fixed costs–are much lower at online stores than at conventional stores and that the welfare gains from electronic commerce are likely to be underestimated if one ignores changes in product variety.

Keywords: *Fixed Costs, Product Variety, Electronic Commerce, Welfare, Internet.*

JEL Classification: *D40, D43, D60, L00, L10, L81, D83, M20.*

Resumen

La literatura sobre el comercio electrónico no se ha enfocado en el impacto que tiene la oferta de una mayor variedad de productos sobre el bienestar general. El problema que surge al analizar la variedad de productos disponibles es que una mayor variedad no implica necesariamente más bienestar. Este artículo determina las condiciones bajo las cuales una mayor variedad de productos, si ésta es el resultado de una reducción en los costos fijos, implica inequívocamente una ganancia en términos de bienestar. Se presenta una calibración del modelo para el mercado de libros que confirma que los costos fijos –en particular, los costos fijos por producto– son significativamente menores para librerías en Internet que para librerías convencionales, y que se subestiman las ganancias de bienestar al analizar sólo cambios en precios y cantidades.

Palabras Clave: Costos Fijos, Variedad, Comercio Electrónico, Bienestar, Internet.

1. INTRODUCTION

Following the development of the Internet, many authors have concentrated on its welfare effects. They mainly analyzed price levels, costs, elasticities, and price dispersion at electronic markets and compared them with those from conventional markets. To mention some of the early work, Bailey (1998), and Brynjolfsson and Smith (2000) found evidence of lower prices and lower menu costs in electronic markets. Goolsbee (2000) found that consumers who are subject to high local taxes are more likely to purchase online, revealing high price elasticity between online and conventional stores. Degeratu, Rangaswamy, and Wu (2000) found lower price sensitivity among online grocery shoppers than among conventional shoppers. With respect to price dispersion, Bailey (1998), Clemons, Hann, and Hitt (1999), and Brynjolfsson and Smith (2000) found a wider range of prices on the Internet than in physical stores. However, the typical comparison with conventional stores implicitly put zero weight on products not offered in one of the two markets, thus ignoring the impact of product variety on consumer surplus and total welfare.[1]

One problem when analyzing product assortment is that more variety does not necessarily imply higher total welfare (consumer and producer surplus), or that markets are more efficient. Changes in price levels and costs, for example, allow for unambiguous conclusions regarding welfare. However, with free entry, markets can involve more or less products than the socially desirable. As Spence (1976b) noted, two effects operate in opposite directions. First, firms cannot capture the whole consumer surplus associated with the introduction of new goods and too few products are introduced. Second, entrants may negatively affect other firm's profits if their products are substitutes. Since the entering firms do not take this effect into account, too many products tend to be introduced. Tirole (1988) calls these two effects "non-appropriability of social surplus" and "business stealing," respectively.

This paper looks at the causes of changes in product variety to obtain unambiguous inferences regarding welfare. Additional product variety may be generated by market conditions both on the supply side and the demand side. On the supply side, fixed costs play an important role.[2] Fixed costs not only generate imperfect competition in the market, but also restrict the number of products firms wish to supply.[3]

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1 Brynjolfsson, Smith and Hu (2003) use an empirical approach to quantify the dollar value consumers place on the increased product variety available through Internet markets. This approach required, however, strong assumptions on demand cross-elasticities and short-term profit maximization.

2 Brynjolfsson, Hu, and Simester (2007) investigate how demand-side factors contribute to the Internet's "long tail," that is, increased demand for more obscure, niche products.

3 See Spence (1976b).

Electronic commerce is one example of a dramatic change in suppliers' costs that has changed the way of doing business. In particular, it has reduced entry costs for retailers. Although electronic retailers may face substantial advertising expenses, they can enter geographically distant markets and offer products that they do not stock themselves. Notably, electronic stores face lower fixed costs per product (or brand) introduced to their virtual shelves. These *per-product* fixed costs are critical in the introduction of new products.

Once an electronic store enters the market, it faces a relatively low cost of adding new products. As a result, electronic markets tend to offer a broader product line. For example, it is well known that online retailers carry a much larger selection of books and CDs than conventional retailers. Electronic stores can simply update their web sites and, as many retailers do, ship the goods directly from the manufacturer or from another retailer. They do not need to stock the goods in order to offer them. A conventional store, on the other hand, typically needs to assign a physical space and at least one unit for exhibition for every additional product introduced at every branch.[4]

This paper finds the conditions under which more product variety, if caused by lower fixed costs, implies a total welfare gain. Models of oligopolistic, monopolistic, and multiproduct competition may exhibit too many brands in the free-entry equilibrium. However, lower (per-product) fixed costs may unambiguously lead to higher welfare through further increases in product variety. In Section 2, I show this result applies under Salop's (1979) model of spatial oligopolistic competition. In Section 3, I apply the same methodology to a general model of monopolistic competition (Hart 1985a). In Section 4, I introduce a model of retailing with multiproduct firms. The three models give the same result under different, but quite general modeling environments. If variety increases because of lower fixed costs, total welfare should also increase. In Section 5, I calibrate the multiproduct retailer model from Section 4 to the market of books. The calibration confirms the intuition that per-product fixed costs are much lower at online stores than at conventional stores. Online markets tend to provide a broader product line and increase welfare even if they represent a small proportion of total sales. The calibration also confirms that welfare gains from electronic commerce are likely to be underestimated if one ignores changes in product variety.

2. A SPATIAL OLIGOPOLISTIC MODEL: SALOP'S CIRCLE CITY

Salop's (1979) model of spatial competition provides a simple way to analyze the impact of fixed costs on product variety and welfare. If only fixed costs are allowed to change, more variety (firms) implies higher welfare.

The product space of the industry is the unit-circumference of a circle. L consumers purchase either one unit or none of a differentiated commodity, spending the remaining income on a homogeneous commodity. There are n brands of the differentiated commodity available at prices p_i and locations l_i. Each firm can produce a unique

[4] Catalogs and telephone orders may be an exception, but they face higher administrative and customer service costs.

brand with fixed costs F and constant marginal cost m. In the constant transportation cost specification, a consumer whose most preferred brand specification (location) is l^* purchases the brand that satisfies:

$$\max_i \left[v - c\left|l_i - l^*\right| - p_i \right] \geq 0, \tag{1}$$

where $v = u - \overline{s}$ is the utility obtained from the differentiated commodity, minus the surplus obtained from the homogeneous good, not considering the transportation cost c.[5]

In the symmetric zero-profit competitive Nash equilibrium, the number of firms is given by $n = \sqrt{cL/F}$, where the number of brands under free entry decreases with fixed costs and increases with transportation costs. The equilibrium price is $p = m + c/n$, and equilibrium profits are zero. When n firms operate and serve the entire market, total welfare is:

$$W = \underbrace{\left(v - m - \frac{1}{4}\frac{c}{n} \right) L}_{\text{NET SURPLUS MINUS TRANSP. COST}} - \underbrace{nF}_{\text{TOTAL FIXED COST}}, \tag{2}$$

and replacing n by its free-entry level,

$$W = \left(v - m - \frac{1}{4}\sqrt{\frac{cF}{L}} \right) L - \sqrt{\frac{cL}{F}} F. \tag{3}$$

Differentiating with respect to F:

$$\frac{\partial W}{\partial F} = -\frac{1}{8}\sqrt{\frac{cL}{F}} + \frac{1}{2}\sqrt{\frac{cL}{F}} - \sqrt{\frac{cL}{F}}, \tag{4}$$

$$\frac{\partial W}{\partial F} = -\frac{5}{8}\sqrt{\frac{cL}{F}} < 0, \tag{5}$$

a decrease in fixed costs increases variety and total welfare. In this model, if it is known that only fixed costs have changed, more variety implies higher welfare.

The RHS of equation 4 shows the three effects of a change in fixed costs. The first term reflects the decrease in consumer surplus (i.e., an increase in transportation costs) due to the exit of firms; the second term represents the fixed costs saved by firm exit; and the last term is the direct effect of an increase in fixed costs, which equals

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5 See Salop (1979) for a complete description and derivation of the model.

the negative of the number of firms. A reduction in fixed costs attracts new firms to the market. This tends to increase total fixed costs, but not enough to outweigh the direct cost savings and the increase in consumer surplus.

It is noteworthy that the second-best situation, in which a planner can choose the optimal number of firms by equating $\partial W / \partial n$ in (2) to zero, gives $n^* = 1/2\sqrt{cL/F}$. Thus, the free-entry competitive equilibrium shows too many firms. But if a further increase in the number of firms is the consequence of a reduction in fixed costs, total welfare increases.

Note that the outcome is different if the change in variety comes from the demand side. The number of brands increases with transportation costs (i.e., less substitutability among brands) and the change in welfare is given by:

$$\frac{\partial W}{\partial c} = -\frac{5}{8}\sqrt{\frac{FL}{c}} < 0.$$

In this model, an increase in product variety may be due to a reduction in fixed costs or an increase in transportation costs. While the former increases welfare, the later reduces total welfare.[6] One can argue that degree of substitution between brands in electronic markets is likely to be higher, and consumer search costs are likely to be lower than in conventional markets. Then, if one analyzes online markets under Salop's model, an increase in product variety, given the market size, has to be the result of lower fixed costs.

3. HART'S MODEL OF MONOPOLISTIC COMPETITION

This section applies the analysis in Section 2 to a more general model that exhibits "true monopolistic competition". Hart (1985a) defines the notion of large group monopolistic competition as a situation where: (1) there are many firms producing differentiated commodities; (2) each firm is negligible in the sense that it can ignore its impact on other firms; (3) each firm faces a downward sloping demand curve and hence the equilibrium price exceeds marginal cost; and (4) free entry results in zero profits for operating firms.

According to this definition, the models in Salop (1979), Spence (1976a), and Lancaster (1979) do not capture monopolistic competition, because firms are not negligible in the sense that they can ignore their impact on other firms. Furthermore, as fixed costs are made arbitrarily small and the number of firms is arbitrarily large, the demand for each product becomes increasingly elastic and these models approach perfect competition. Although these limitations do not apply to Spence (1976b), Dixit-Stiglitz (1977), and Perloff-Salop (1985), where the number of brands may be arbitrarily large without approaching a perfectly elastic demand for each brand, these authors rely on a peculiar limiting behavior of consumers' utility functions.[7]

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[6] Product variety and welfare also increase with market size.

[7] See Hart (1985a), and Wolinsky (1986).

Hart (1985a) presents a model of monopolistic competition that does not rely on an irregular limiting behavior of the utility function. Instead, the model relies on the assumption that consumers have different tastes over a large number of brands, but each consumer is interested in only a fixed finite subset of the potential brands. Wolinsky (1986) adds to Hart's analysis by suggesting that imperfect information creates the circumstances that restrict the effective substitutability among brands.

The model in Hart (1985a) assumes an economy with N potential firms and N potential differentiated commodities, where N is large and each firm produces a single brand. Firms have identical cost functions of the form $F + C(q)$.[8] There is a continuum of consumers with total mass kN (where $F > k$), each being endowed with one unit of a numeraire good. Hart's critical assumption is that each consumer likes only $m > 1$ of the potentially available brands.

In order to simplify the analysis and obtain welfare results, Hart (1985a) studies a special case in which the utility function of the typical consumer is:

$$U\left(x_{i_1},\ldots,x_{i_m},M;\mathbf{v}\right)=A\left(\sum_{b=1}^{m}v_b x_{i_b}\right)^{\alpha}+M, \tag{6}$$

where $x_{i_1},\ldots,x_{i_m},M$ are the consumption levels of the m brands and the numeraire, $\mathbf{v}$ is a finite-dimensional vector representing the consumer's valuations over the brands, with v_b, $b=1,\ldots,m$, being non-negative *iid* random variables, $A > 0$, and $0 < \alpha < 1$. This implies that each consumer has a constant marginal utility of money and that the elasticity of utility with respect to $\left(\sum_{b=1}^{m} v_b x_{i_b}\right)$ is constant.

In the symmetric monopolistically competitive equilibrium (SMCE) of Hart (1985a), in which all operating firms charge the same price p^*, it follows that when the number of firms is arbitrarily large, total demand for brand n is:

$$D\left(p,p^*,y\right)=\sum_{r=0}^{m-1}\left\{\int_0^{\bar{v}}\frac{v^{\eta-1}}{p^{\eta}}H\left(\frac{p^*v}{p}\right)^r h\left(v\right)dv\left[km^{m-1}C_r y^r\left(1-y\right)^{m-r-1}\right]\right\}, \tag{7}$$

where $0 \le y \le 1$ is the fraction of firms that operate in equilibrium, $0 \le r \le m-1$ is the number of the brands that the consumer likes, $\eta = 1/(1-\alpha)$, H is a distribution function with continuous density function $h(v)$ and support $\left[\underline{v},\bar{v}\right]$. Hart (1985b) shows

8 $F > 0$, $C(0) = 0$, $C' > 0$, and either $C'' > 0$ everywhere or $C'' \equiv 0$. See Hart (1985a) for a complete description of the model.

that $D(p, p^*, y)$ is differentiable for $p > C'(0)$, $p^*(y)$ is decreasing in *y*, and firm's profits, $\Pi(y)$, are decreasing in *y*.[9] Total welfare at the SMCE is:

$$\bar{W}(y) = \left(\int_{p^*(y)}^{\infty} D(p, p, y)\, dp + p^*(y)\, D\left[p^*(y), p^*(y), y\right] - C\left\{D\left[p^*(y), p^*(y), y\right]\right\} - F \right) y, \quad (8)$$

that is, total welfare is the sum of consumer surplus and producer surplus for one firm, multiplied by the fraction of firms operating in the market. He also shows that when the fixed costs (*F*) increase, variety (*y*) falls, since $\Pi(y)$ is decreasing in *y*. Also, p^* increases, because $p^*(y)$ is decreasing in *y*.

Differentiating with respect to *F*:

$$\frac{\partial \bar{W}\left[y(F)\right]}{\partial F} = -y + \frac{\partial y(F)}{F} \left(\int_{p^*(y)}^{\infty} \frac{\partial}{\partial y}\left[yD(p, p, y)\right] dp \right. \quad (9)$$

$$+ y\left[p^*(y) - C'\left\{D\left[p^*(y), p^*(y), y\right]\right\}\right]$$

$$\cdot\left\{D_y\left[p^*(y), p^*(y), y\right] + D_p\left[p^*(y), p^*(y), y\right] p^{*\prime}(y) + D_{p^*}\left[p^*(y), p^*(y), y\right] p^{*\prime}(y)\right\}$$

$$\left. + p^*(y)\, D\left[p^*(y), p^*(y), y\right] - C\left\{D\left[p^*(y), p^*(y), y\right]\right\} - F \right).$$

The first term of (9) is the direct welfare effect of the change in fixed costs, always negative. The second term contains $\partial y(F)/\partial F$, which is negative, multiplied by the expression in parenthesis. The expression in parenthesis equals $\partial \bar{W}(y)/\partial y$, the derivative of total welfare with respect to the fraction of firms at the SMCE. If this expression is positive (negative) it means that there are too few (too many) firms in the market relative to the second-best level.

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[9] Also, $p^* > C'(0)$, $D_p < 0$, $D_{p^*} < 0$, $D_p\,(p, p^*, y) + D_{p^*}\,(p, p^*, y) < 0$, $D_y < 0$, and $D(p^*, p^*, y) + yD_y(p^*, p^*, y) > 0$.

The first term inside the parenthesis reflects the change in consumer surplus. When y increases, aggregate demand and consumer surplus increase, because consumers have more variety to choose from. Hence, this term is positive. The second term shows that when y increases demand per firm falls ($D_y < 0$). With price above marginal cost, producer surplus tends to decrease, but the increase in y lowers prices, increasing demand ($D_p + D_{p^*} < 0$, and $p^{*\prime}(y) < 0$). Then, the second term may be positive or negative. Finally, the third term is the surplus of the new entrants, which is zero at the SMCE. Therefore, as shown by Hart (1985b), there can be too many or too few firms at the SMCE.

Although the number of firms can be above or below the optimal (second best) level, the change in welfare with respect to changes in fixed costs is unambiguously negative. This is because of the first term in (9), $-y$. The second term inside the parenthesis multiplied by $\partial y(F)/\partial F$ refers to the total change in producer surplus due to firm exit. This cannot be greater than y, the direct loss of profits, otherwise, firms would not exit. As a result,

$$\frac{\partial \bar{W}\left[y(F)\right]}{\partial F} < 0. \tag{10}$$

To conclude, as in the spatial oligopolistic model, Hart's model of monopolistic competition shows that regardless of whether the number of firms in equilibrium is optimal, there is a welfare gain when product variety increases as a result of a reduction in fixed costs.

4. A MODEL OF MULTIPRODUCT RETAILERS WITH PER-PRODUCT FIXED COSTS

As discussed above, access to electronic commerce reduced the fixed cost retailers face per product added to the product line. Once the electronic store is in place, the cost of adding a product may be negligible relative to the costs a conventional store would have. Online retailers do not need to have in stock the goods they offer, provided that they can obtain them from the manufacturer and timely deliver to the customer. Even when they have in stock the goods they offer, electronic retailers can reduce storage costs by concentrating inventories at fewer storehouses, rather than carrying every single product at each branch.[10]

[10] In some cases, however, it may be misleading to compare the number of products shown at web sites with the number of products that conventional stores have in stock. Electronic markets may not have all products immediately available. And customers at physical stores can frequently order products that are not in stock. The inclusion of time as an additional characteristic of the goods would broaden the analysis of product variety. In general, however, electronic stores have more advantages in exploiting consumers' choice for a wider spectrum of products with the characteristics that best suit their needs.

In this Section, I develop a linear demand model of multiproduct retailers. The model shows the effects of lower per-product fixed costs on product assortment and total welfare. As before, the question is whether more variety necessarily implies more efficient markets when only (per-product) fixed costs are known to have changed.

The market has N potential firms and each one can supply up to m brands. Firms have identical cost functions of the form $F + fm$, where $F \geq 0$ is the conventional fixed cost necessary to enter the market, independent of m, the number of brands made available. $f \geq 0$ is the per-product fixed cost, assumed to be constant and independent of m. For simplicity, marginal production costs are assumed to be zero.

The inverse demand function for each brand is:

$$p_i = \alpha - q_i - \delta \sum_{j \neq i} q_j \,, \tag{11}$$

where p_i and q_i are the price and quantity supplied of brand i, $0 \leq \delta \leq 1$, and $\sum_{j \neq i} q_j$ is the total quantity supplied of all other brands in all stores, including the brands supplied at the same store as brand i. The number of potentially available brands is assumed to be unlimited, and all of them are imperfect (and symmetric) substitutes for consumers.

Without loss of generality, I assume brands enter the market in a sequential manner.[11] The first of the N suppliers introduces the first brand. Then, the second retailer is able to introduce the second brand, and the process continues until all retailers supply a single brand. Then, the first retailer may introduce a second brand, and other retailers follow, until no more brands are introduced. Finally, all firms compete in either prices or quantities. Firms will enter the market if profits are sufficient to cover general fixed costs (overhead) and per-product fixed costs. They will introduce a new brand only if the additional brand increases total firm profits by at least the magnitude of the per-product fixed cost, f. Total firm profits include the revenue from the additional brand and the change in revenues from the $m - 1$ remaining brands supplied at the same store.

The model is further developed and solved for the symmetric Nash equilibrium (See Appendix). Figure 1 shows a simulation of consumer surplus, profits, and total welfare for different levels of per-product fixed costs. The simulation in Figure 1 assumes four firms compete in quantities, with at most four brands available for each firm. The general fixed cost, F, is assumed to be zero. Lower per-product fixed costs attract more firms to the market and, if all firms already entered the market, the lower the cost of adding an extra brand the more brands firms introduce at each store.

As shown in Figure 1, when the degree of substitution between products is relatively low ($\delta = 0.2$), the introduction of new brands –as a result of lower fixed costs– can only increase welfare. The "non-appropriability of social surplus" effect dominates the "business stealing" effect and brands are introduced too slowly. When new brands are introduced (from right to left in Figure 1), they create discrete "jumps" in welfare.

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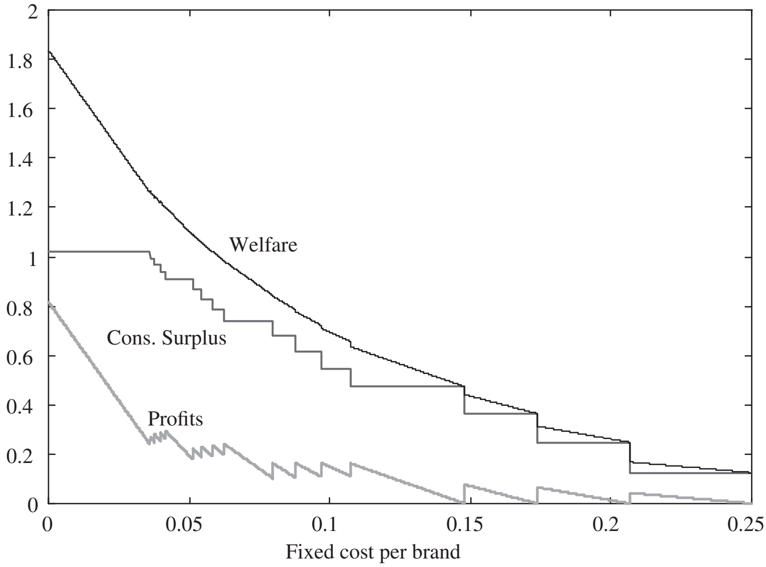
[11] This avoids dealing with multiple equilibriums, without changing the qualitative results.

FIGURE 1

CONSUMER SURPLUS, PROFITS, AND WELFARE



Although lower (per-product) fixed costs tend to increase product variety and total welfare, this is not always the case when the degree of substitution between brands is high (e.g., $\delta = 0.8$). A small reduction in per-product fixed costs may actually lead to an increase in product variety and a reduction in total welfare. This is the case shown in Figure 2. Consumer surplus is always increasing in the number brands, but retailers' total profits may fall with the introduction of a new product because the firm offering the new brand does not fully internalize its impact on other firms' revenues. In Tirole's (1988) terminology, the firm introducing the last brand is not generating surplus but stealing business form its competitors.

Price competition is more appealing than competition in quantities for electronic commerce. With competition in prices, the model still shows negative changes in welfare when product variety increases, but only for very specific levels of substitutability between brands (δ). With price competition, a higher degree of substitutability between brands makes it more profitable to "steal" customers from other firms. However, as δ increases, equilibrium prices fall more under price competition than under quantity competition. This offsets the incentive to steal business by introducing new brands.

Figure 1 and Figure 2 show that welfare generally increases with product assortment, but it may fall if the reduction in per-product fixed costs is small and the degree of substitution between brands is sufficiently high. For large reductions in fixed costs, however, welfare can only increase with product variety, suggesting that

FIGURE 2

CONSUMER SURPLUS, PROFITS, AND WELFARE

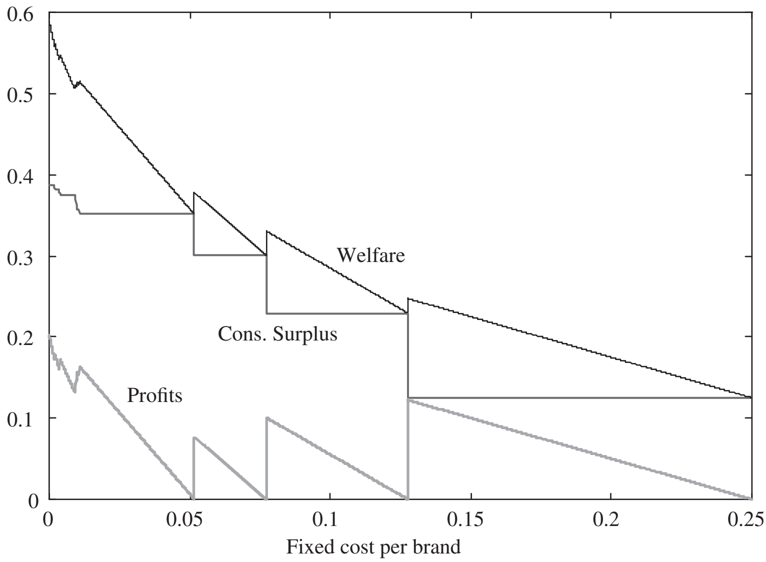


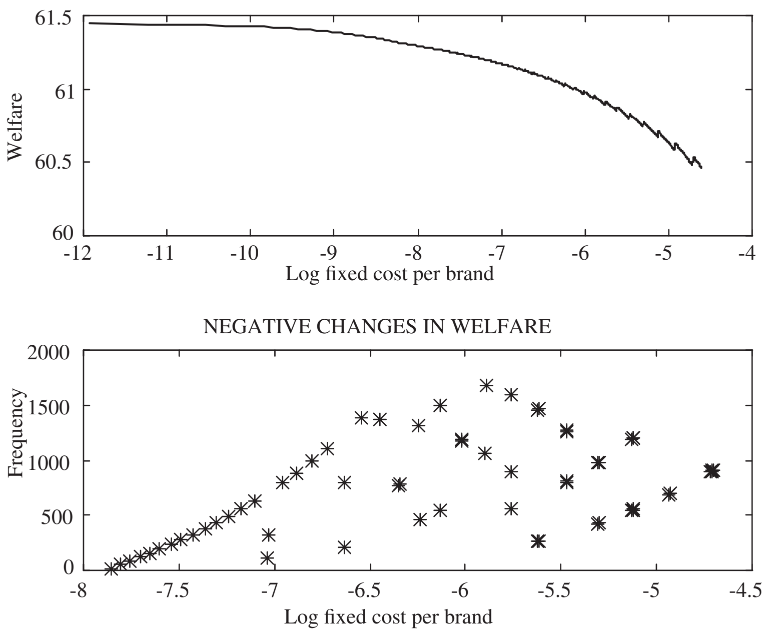
Figure 2 is just an anomaly from the integer restrictions imposed on the number of firms and brands. If this is true, a large number of firms and brands should smooth the welfare curve.

To check this, Figure 3 shows a market with 10 firms and 50 brands available to each firm. When per-product fixed costs are low, each firm introduces many brands. The top panel of Figure 3 shows total welfare, as in Figures 1 and 2, for different levels of per-product fixed costs (in logs). A small decrease in per-product fixed costs may still reduce total welfare, but these negative changes in welfare fade away as fixed costs decrease and firms introduce more brands. The same result is shown in the bottom panel of Figure 3. It shows the frequency of negative changes in welfare for small reductions in fixed costs. It shows that the frequency of these negative changes in welfare diminishes as per-product fixed costs decrease.

It is noteworthy that negative changes in welfare do not fade away as the number of firms increases. Under the inverse demand functions of this model, a large number of firms increases the incentive to steal consumers. Thus, firms are more likely to offer too many brands relative to the social optimum. Only when the number of brands in the market is large enough relative to the number of firms, an increase in product variety implies an unambiguous welfare improvement. It should also be noted that the negative changes in welfare are created by the discrete number of brands in the model, not by the introduction of multiproduct firms. In fact, the models from Sections 2 and 3 would render the same results with a small number of firms.

FIGURE 3

WELFARE



To conclude, the sections above show that the same result holds under models of oligopolistic, monopolistic, and multiproduct competition. If the increase in product variety is the result of lower (per-product) fixed costs, total welfare must also increase. The opposite may only arise for small changes in fixed costs and a discrete number of brands.

5. CALIBRATION TO THE MARKET OF BOOKS

In this Section, I calibrate the multiproduct retailer model from Section 4 to the introduction of online booksellers in the U.S. The data covers the period 1998-2000. The calibration shows that per-product fixed costs were quite low for online retailers. It also suggests a substantial increase in welfare from the expansion in product variety that took place with the introduction of online booksellers.

I calculate per-product fixed costs from data on annual sales and the number of titles available at conventional and online bookstores. This requires strong assumptions regarding demand functional forms and the nature of competition. After calibrating for per-product fixed costs and the parameters of the demand function for each market, it is possible to calculate the number of titles that conventional bookstores would offer if they had the same per-product fixed costs as their online counterparts. The associated increase in welfare provides a measure of the efficiency improvements brought by online stores. The calibration process is further described in the Appendix.

It is unrealistic to assume that all books are substitutes to some degree. Thus, following Hart (1985a), I impose in the model a limit to the number of brands that consumers may consider substitutes. The inverse demand function for product i at store k is:

$$p_{ik} = A - q_{ik} - \frac{\delta h}{H}\left(\sum_{j \neq i} q_{jk} + \sum_{l=1}^{n}\sum_{j=1}^{m_l} q_{jl}\right), \qquad \text{for } i, j = 1, \ldots m_k, \tag{12}$$

where m_k is the number of titles at store k, n is the number of bookstores, h is the maximum number of substitutes for each title, H is the total number of titles potentially available, and $0 < \delta < 1$. The degree of substitutability between brands is given by $\delta h / H$, and assumed to be known.[12] This parameter sets the slope of the demand function. With data on store sales, I can then estimate the origin of the inverse demand function.

I assume a symmetric equilibrium in which retailers sell the same number of units of each book title. This assumption seems strong, because retailers would first add the bestseller titles and then the less popular ones, which do not create much consumer surplus. However, I (partially) address this problem by adjusting down the number of titles from stores that supply a wide selection of titles. A more realistic approach would require data from individual titles. As a first step, I consider conventional and online markets to be separate.[13] Each one has an inverse demand function of the form in (12) with a different origin, A.

Section 4 considered both competition in prices and quantities. Among these two, it seems more realistic to consider competition in prices for electronic commerce. It is hard to argue that online stores first choose quantities (or capacity) and the market determines prices. As mentioned before, electronic retailers do not necessarily have in stock the goods they offer. It is more realistic to assume that booksellers choose prices and consumer demand determines quantities.

Other forms of competition may be considered. Brand reputation, customer service, delivery time, location, advertisement, information about the product, transaction's security, etc., are all variables found relevant for both electronic and physical markets.[14] For simplicity, however, this model assumes price competition for both markets, holding other variables constant. Additional results are provided for quantity competition.

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[12] I analyze below the sensitivity of the results to this parameter.

[13] The term market is used loosely in this paper to reflect the different sales channels (conventional and online). As such, it may differ from the products market as defined by the antitrust literature.

[14] For example, Brynjolfsson and Smith (2000b) found that, even among shopbot consumers, branded retailers hold significant price advantages. Also, customers are very sensitive to how the total price is allocated among the book price, shipping cost, and tax. See also Clay *et al.* (2000).

Given the number of stores and titles available, each store maximizes profits by solving:

$$\max_{\{p_{ik}\}} \sum_{i=1}^{m_k} (p_{ik} - \overline{c})(q_{ik}(p_{ik})) - fm_k - F, \tag{13}$$

for each p_{ik}, where $\overline{c}$ is the marginal cost of the book, f is the per-product fixed cost, and F is the store fixed cost, all common to all firms and titles. Then, the symmetric Nash equilibrium provides equilibrium prices and quantities.

The nation's largest retail bookstore chains and online bookstores report annual sales and the approximate number of titles they carry.[15] Thus, the average pq can be estimated by dividing total annual sales by the number of titles. $\overline{c}$ equals cp where c (a proxy for marginal cost) is estimated as the ratio of the reported cost of sales to total sales. The origin of the inverse demand function, A, is obtained as a function of pq, m , n, and $\delta h / H$, as shown in the Appendix.

The key to estimate f, the per-product fixed cost, is that when the title $m_k + 1$ is added to store k at the symmetric Nash equilibrium, the marginal revenue from the title should equal the marginal cost f.[16] In the profit function in (13), the increased revenues from the new title minus the lost revenues from other titles offered by the same store should equal the per-product fixed cost. Only F, the total fixed cost, is unknown in (13). However, in a static model, F is bounded from above by firms' net revenues. Otherwise, they would not stay in business. F is also bounded from below by the revenues of each firm at equilibrium when an additional firm enters the symmetric competition. I use the simple average of these values as the general fixed cost.[17]

The calibration is first performed independently for the conventional market and the electronic market, to obtain A_c, A_o, f_c, f_o, W_c, and W_o, where the subscripts c and o refer to conventional stores and online stores, respectively. Table 1 shows the results of the calibration for $\delta = 0.5$, h = 10 (the number of titles considered by consumers), and where H, the total number of titles potentially available, is assumed to be 25 millions. Although these parameter assumptions are entirely arbitrary, Table 3 shows a sensitivity analysis and finds an upper limit to the degree of substitution.

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15 See Publishers Weekly at www.publishersweekly.com, and 10-K reports to the Securities and Exchange Commission.

16 With a large number of titles, the marginal revenue minus marginal cost of adding one title should be arbitrarily close to zero.

17 An alternative method to measure fixed costs would consider the opportunity cost of the assets reported by the firms on their balance sheets, but this may not be the appropriate approach for an industry in its initial phase.

TABLE 1

CALIBRATION TO THE INTRODUCTION OF ONLINE BOOKSELLERS

	Conventional Stores (1)	Online Stores (2)
Sales (million $ per store)	1,378	624
Titles per store	150,000	1,000,000
Number of stores	8	3
Average sales in $, (pq)	9,192	624
Mark-up, (p – c) / p	29%	19%
Demand origin in $, A	243	78
Per-product fixed cost in $, f	2,106	68
	Welfare (in million $)	
Consumer surplus	1,935	249
Total revenues minus costs	672	150
General fixed cost, F	76	45
Welfare	2,002	264

Notes: $\delta = 0.5$, h = 10, H = 25,000,000.
Source: Publishers Weekly and annual reports to the Securities and Exchange Commission.

For the brick-and-mortar market, annual sales were obtained as the average of the four largest retail chains (Barnes & Noble, Borders Group, Books-A-Million, and Crown Books), excluding sideline sales and international sales. The average number of titles reported in the late 1990's was around 150,000. Since these chains represented approximately 50% of the market, eight firms were included in the calibration. For the electronic market, Amazon.com sales were considered, excluding international sales. The model assumes one million titles and three stores in a symmetric competition. As discussed below, these assumptions are not critical for the results obtained.

Under the assumptions of this model and the parameters in Table 1, the estimated per-product fixed cost for online stores is about 3.3% of the corresponding cost for physical stores. Although per-product fixed costs are lower and variety is higher in electronic bookstores, welfare was still higher in conventional markets. This is simply because demand was much larger, as reflected in total sales.

To measure changes in efficiency, I impose per-product fixed costs from online stores to physical stores. This provides a measure of the welfare improvement as the market moves towards electronic sales. When per-product fixed costs decrease, product variety should increase until the marginal brand revenue equals the marginal cost per brand, *f*. Defining m^* as the optimal number of titles given the demand for conventional stores and the per-product fixed costs from online stores, prices and quantities are given by (18) and (19) in the Appendix, respectively. Table 2 shows the hypothetical physical market with per-product fixed costs from online stores, and the relationship with the actual physical market in Table 1.

TABLE 2

WELFARE CHANGE WITH ONLINE PER-PRODUCT FIXED COSTS

	Conventional Stores (3)	Ratio (3/1)
Sales (million $ per store)	5,427	3.94
Titles per store	1,158,860	7.73
Average sales in $, (pq)	4,683	0.51
Mark-up, (p – c) / p	20%	0.69
Per-product fixed cost in $, f	68	0.033
	Welfare (in million $)	
Consumer surplus	11,357	5.87
Total Revenues	8,027	11.95
Welfare	18,778	9.38

Notes: $\delta = 0.5$, h = 10, H = 25,000,000.
Source: Publishers Weekly and annual reports to the Securities and Exchange Commission.

The number of titles in physical stores would increase sharply if they had per-product fixed costs similar to those of electronic retailers. Sales per title would decrease, however, because both quantities and prices would be lower. In this example, welfare increases by a factor of nine because of the increased consumer surplus (more titles and total sales) and increased profits (more sales and lower per-product fixed costs).

These results are quite robust to changes in the number of stores and the relevant titles supplied by online stores. As mentioned above, the popularity of each title is expected to decrease as stores add more titles, and marginal titles may become irrelevant in terms of welfare. But suppose online stores only offered the titles available at conventional stores (with lower sales). Even with lower sales at online stores, the implied welfare is higher and the implied per-product fixed costs are lower than those at brick-and-mortar stores. Welfare calculations are sensitive, however, to the degree of substitution among brands, $\delta \frac{h}{H}$, assumed so far as known. If consumers consider a large number of titles as a set of close substitutes (high $\delta \frac{h}{H}$), additional variety only contributes marginally to total welfare. Table 3 shows the sensitivity of the results to different assumptions on this parameter.

TABLE 3

SENSITIVITY TO DIFFERENT DEGREES OF SUBSTITUTION

Choice of h (fixed δ and H)	2	5	10	15	30	42
Ratio of per-product fixed costs (fo/fc)	4.2%	3.8%	3.3%	2.8%	1.4%	0%
Welfare ratio (Wo/Wc)	45.0	18.4	9.4	6.3	3.1	2.2

As discussed in Section 4, if the degree of substitution between brands is high, lower fixed costs are less likely to generate a significant increase in welfare. But with price competition, however, it is possible to find an upper bound to the degree of substitution and, therefore, a lower bound for the change in total welfare. In this example, given δ and H , h cannot be greater than 42. Otherwise, the per-product fixed costs for online stores would have to be negative. Intuitively, under price competition and a high degree of substitution, online bookstores would not offer so many titles. Nevertheless, at this extreme level of substitutability among brands, total welfare increases by a factor of 2.2 as the market moves towards the electronic marketplace.

Similar arguments apply to changes on the demand side. So far the model assumes that only fixed costs change and that substitutability among brands is the same online as off-line. However, it is reasonable to expect lower search costs in electronic markets and, consequently, higher substitutability among brands. If this is true, the results above would underestimate the welfare increase from online stores. Furthermore, higher substitutability online would imply a lower ratio f_o/f_c in Table 3, that is, online per-product fixed costs would be lower than those obtained in this calibration.

The results are also robust to competition between the two markets and competition in quantities. It is reasonable to assume that online and physical stores compete in the same market. The same model with competition between all stores results in similar estimations of per-product fixed costs and welfare. If competition in quantities is assumed, f_o/f_c and W_o/W_c are similar, although there would be no upper bound for the degree of substitution among brands.[18]

Assumptions in this model with respect to functional demand forms and nature of competition may seem restrictive. Nevertheless, the estimated per-product fixed cost at online bookstores is always below 5% of the corresponding cost for conventional bookstores. This confirms the intuition that "virtual shelving" reduces the cost of adding extra brands at electronic stores. Changes on the demand side are unlikely to explain such differences in product assortment between markets. If online stores had the same per-product fixed costs as off-line stores, they would not supply so many brands. The first order condition for the choice of variety (see (17) in the Appendix) does not hold for any value of $\delta \frac{h}{H}$.

It is also interesting to calibrate the model for changes in prices, not considering differences in product variety, as done in much of the research on electronic markets. As mentioned above, this may largely underestimate the welfare improvements from the Internet. While changes in product variety increase total welfare by at least 120%, when prices are assumed to fall due to a drastic reduction in marginal costs, the same

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[18] A limitation of this model is that the number of firms is considered exogenous. A reduction in per-product fixed costs would induce new entry. Nevertheless, the results of the calibration are quite robust to the number of firms and, as noted above, whether one considers three online stores or one store (Amazon.com), the results are the same. As noted in Section IV, when the number of brands is large relative to the number of firms, it is unlikely for a reduction in per-product fixed costs to be welfare decreasing.

model results in a 95% welfare increase. The difference is larger if one assumes that prices fall because of changes in demand and the nature of competition. In this case, welfare would increase by approximately 20%. The larger is the parameter that measures the degree of substitutability among brands, the larger these differences are.

6. CONCLUSION

Following the development of online commerce, many authors have concentrated on its welfare effects, analyzing mainly price levels, costs, elasticities, and price dispersion. The impact of product variety is more problematic, however, since more variety does not necessarily imply that markets are more efficient.

This paper concentrates on one main driver of the difference in product assortment between markets to obtain unambiguous inferences on market efficiency. It analyzes the effect on product variety and welfare from changes in general and per-product fixed costs. Then, the increase in the number of products in a market can be used as a measure of market efficiency and to obtain accurate welfare estimates.

Two general models of product differentiation show that a reduction in fixed costs unambiguously increases the number of firms in the market and total welfare, regardless of whether the free-entry market equilibrium has too many or too few firms relative to the social optimum. A model of retailing with multiproduct firms shows that small reductions in per-product fixed costs can lead to an increase in variety and a decrease in welfare. In general, however, total welfare increases as fixed costs go down and more brands are introduced.

Finally, a calibration of this model to the market of books confirms the intuition that per-product fixed costs are much lower at online stores than at conventional stores, and consequently, welfare is substantially higher. Alternative explanations for changes in prices (e.g. lower search costs or lower marginal costs) fail to fully explain the large supply of titles at electronic retailers.

APPENDIX

From the inverse demand functions in (12), direct demand functions are:

$$q_{ik} = Ax - yp_{ik} + z\left(\sum_{j\neq i} p_{jk} + \sum_{l=1}^{n}\sum_{j=1}^{m_k} p_{jl}\right), \tag{14}$$

where $x = [1 + \frac{\delta h}{H}(2\Sigma_{k=1}^{n} m_k - 3)]/w,\quad y = [1 + \frac{\delta h}{H}(\Sigma_{k=1}^{n} m_k - 2)]/w,\quad z = \frac{\delta h}{H}/w,$

$w = 1 + (\Sigma_{k=1}^{n} m_k - 2)\frac{\delta h}{H} - (\Sigma_{k=1}^{n} m_k - 1)(\frac{\delta h}{H})^2$; $\dfrac{\delta h}{H}$ is assumed to be known.

Given the number of stores and titles in the market, each store maximizes profits by solving:

$$\max_{\{p_{ik}\}} \sum_{i=1}^{m_k} (p_{ik} - \bar{c})\left\{Ax - yp_{ik} + z\left(\sum_{j\neq i} p_{jk} + \sum_{l=1}^{n}\sum_{j=1}^{m_k} p_{jl}\right)\right\} - fm_k - F, \tag{15}$$

where $\bar{c}$ is the marginal cost of the book, f is the per-product fixed cost, and F is the store fixed cost, all common to all firms. In a symmetric Nash equilibrium, prices and quantities are:

$$p = \frac{Ax - y\bar{c} - z(m-1)\bar{c}}{2y - z(m(n+1) - 2)}, \tag{16}$$

$$q = Ax - yp + z(nm-1)p. \tag{17}$$

$\bar{c}$ equals cp, where c is estimated as the fraction of the reported cost of sales to total sales (a proxy for marginal cost). Replacing for $\bar{c}$:

$$p = \frac{Ax}{\Omega}, \tag{18}$$

$$q = \frac{Ax(\Omega - y + z(nm-1))}{\Omega}, \tag{19}$$

where $\Omega = y(2-c) - z(m(n+1) - 2 - (m-1)c)\cdot$

Multiplying (18) by (19):

$$pq = \frac{A^2 x^2 (\Omega - y + z(m-1))}{\Omega^2}. \tag{20}$$

pq is estimated by dividing total annual sales at retail bookstore chains by the number of titles. Under the above assumptions, only the origin of the inverse demand function, A, is unknown, and is given by:

$$A = \sqrt{\frac{pq}{\Omega - y + z(m-1)}} \frac{\Omega}{x}. \tag{21}$$

Once A is estimated, (18) and (19) provide p and q. The next step is to estimate f, the per-product fixed cost. From the profit function in (15), the revenues from a marginal new title, minus the lost revenues from titles sold at the same store, should equal the per-product fixed cost. With a large number of brands, the increase in total earnings when adding one more title should be arbitrarily close to the cost of adding it, f.

$$f = (m+1)(p_{(m+1)} - \overline{c})q_{(m+1)} - m(p_{(m)} - \overline{c})q_{(m)}. \tag{22}$$

Finally, given the demand, prices, quantities, and costs, welfare is:

$$W = nm\frac{(A-p)q}{2} + nm(p - \overline{c})q - nmf - nF, \tag{23}$$

where F is estimated as described in Section 5.

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