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FIXED COSTS AND ASSET MARKET PARTICIPATION*

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Abstract

This paper investigates the effects of fixed costs on investor's decision of asset market participation. The model features a continuum of agents with heterogeneous initial wealth and attitude toward risk. We show that under certain conditions there exists a unique competitive equilibrium in which investors optimally choose to stay in autarky, participate just in the riskless asset market or in both the riskless and the risky asset markets. The model is calibrated based on earnings profile from the U.S. We find that using fixed costs that are comparable to the current commission charged by brokers the model can generate participation patterns similar to observed ones. Further, we find participation rates to be very sensitive to the cost differentials associated with entering the risky asset market while relatively less sensitive to the overall levels of fixed costs. Finally, we find that fixed costs make it even harder for dynamic models to replicate the risk free rate and in that sense deepen that puzzle.

I. Introduction

This paper investigates the effects of fixed costs[1] on agents' asset market participation decisions. These costs can represent information acquisition, up-front trading commissions, time and psychological costs associated with asset trading. The study is motivated by the empirical findings of Mankiw and Zeldes (1991)

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on consumer's asset holdings. In their study, they find that among a sample of 2,998 families (1984 survey of Panel Study of Income Dynamics), only 27.6% of the households own stocks. Even for families with liquid assets of $ 100,000 or more, only 47.7% own stocks. Also, in the sample, 31.3% of the consumers held less than $ 1,000 in liquid assets. These numbers are adjusted for indirect investments channels such as pension funds. These facts are puzzling because the models with proportional transactions costs can not explain why households would not participate in asset markets and also stocks have prolonged, on average, better performance than riskless assets such as Treasury Bills.

Previous studies on the asset market participations have focused on the effect of liquidity, the ease at which an asset can be bought or sold, on consumer's participating decisions. Within this line of research Allen and Gale (1994) examine the relationship between liquidity preference *(early* versus *late* consumers) and volatility of asset prices in the limited market participation framework due to different entry costs. They find that an arbitrarily small aggregate liquidity shock can cause significant price volatility and there exist multiple equilibria with very different participation regimes and levels of asset price volatility. In a similar setup, Williamson (1994) shows that there is a tendency for underprovision of the liquid asset and overprovision of the illiquid asset. In his model, however, there exists a unique competitive equilibrium.

In a related study, Brennan (1975) shows that with fixed setup costs, it is only worth investing in a limited number of assets[2]. Haliassos and Bertaut (1995) point to minimum investment constraint, information costs, and inertia as possible explanations, among others, for why so few hold stocks.

In this paper we attempt to attack the participation issues more directly by investigating the quantitative implications of fixed costs. While various types of costs may give rise to similar effects on decisions of financial market participants, we do not, in this paper, distinguish between these sources but rather summarize them into a single type named fixed costs. This is a reasonable assumption for the purpose of quantitative analysis of the effects of fixed entry costs on the participation patterns. Although fixed costs might seem an obvious argument as to why people do not participate in certain markets, we want to ask in a positive manner whether introducing fixed costs to a computable general equilibrium model, coupled with endowment and preference heterogeneity, can account for participation patterns similar to the ones found in the cross-sectional data. In particular, we want to find, given some parameter restrictions, the magnitude of fixed costs required to explain the observed participation rates and their asset pricing implications.

Our model differs from the studies cited above in the following ways. First, in Allen and Gale (1994) and Williamson (1994), there exist two types of agents with different time preferences as a result of a preference shock. Here, a continuum of agents have the same time preferences but heterogeneous initial wealth and attitude toward risks. Second, there are three periods in their models but only two periods in our model. The extra period, in their model, allows a preference shock that is impinged on investors to divide them into *early* and *late* consumers. Third, our model is different from Allen and Gale (1994) in that the return of the liquid asset is endogenously determined and is different from Williamson (1994) in that there is also a fixed cost for entering the liquid asset market. Finally, concentrating on wealth heterogeneity allows us to base our calibration on income earnings profile data which none of the above mentioned papers are set to do.

Our main findings can be summarized as follows. First, under certain conditions there exists a unique equilibrium in which agents optimally choose to stay in autarky, participate in the riskless asset market or in both the riskless and risky asset markets. Second, for reasonable fixed entry costs to these asset markets, the model can replicate the participation rates documented in Mankiw and Zeldes (1991). For instance, for no more than $200 (in 1987 dollars) of fixed entry costs to asset markets, we find the proportion of agents who participate in the risky asset (such as stocks) market to be around 37% to 47% which are consistent with the existing empirical evidence. The entry costs also match the current average commission charged by brokers as reported in an article by Jeffrey (1995) on the Wall Street Journal. It quoted an average commission of $246 per trade for full service brokers, $102 per trade for the Big Three discount brokers (Fidelity Investment, Quick & Reilly Group, and Charles Schwab), and $50 per trade for the deep discount firms[3].

Third, the differential cost associated with participating in the risky asset market has a larger impact on agent's participation decisions than the levels of entry costs. Fourth, increasing the mean payoff of the risky asset increases the proportion of agents who participate in both the riskless and risky asset markets and slightly lowers the percentage of agents who participate only in the riskless asset market. A qualitatively similar effect on participation rates is also found when the variance of the risky asset payoff increases. Fifth, decreasing the mean of the wealth distribution while holding entry costs fixed as percentage of the mean wealth increases the participation rates. However, increasing the variance of the wealth distribution only has a small and nonmonotonic effect on participation rates. Finally, our results indicate that increasing fixed costs increases the equilibrium risk free rate - a fact that makes the risk free rate puzzle an even more tenuous task for general equilibrium dynamic asset pricing models.

This paper is organized as follows: Section II introduces the model with fixed costs. Section III discusses agents' participation decisions and the existence of equilibrium. Section IV describes calibration and numerical results. Section V provides concluding remarks.

II. The Model

In this section we present a two period model to demonstrate the effect of fixed costs on agents' participation decisions. The economy consists of a continuum of agents indexed by $i \in [0,1]$. Agents differ in both their endowments denoted w_0^i and risk aversion coefficients denoted γ_i. Each agent lives for two periods. In the first period, they make investment decisions including what asset markets to participate and how much to invest in each market. They then consume everything in the second period. Agents in the economy have access to two financial markets: a riskless asset market and a risky asset market. Purchasing one unit of riskless asset in the first period provides an agent a claim to a unit of consumption good in the second period. Holding a unit of risky asset, however, gives an agent a random payoff denoted by F in the second period. The riskless asset is in zero net supply while the risky asset has a fixed positive net supply. To gain access to the asset markets, agents have to pay certain fixed entry costs. Specifically, an agent can pay an amount of f_1 to *access* the riskless asset market or f_2 to participate in both the riskless and risky asset markets. We model f_2 as the fixed cost of participating in both the riskless and risky markets rather than the risky asset market only. This allows us to take into account any potential returns to scale in participating in more than one asset market. To facilitate tractability we make the following simplifying assumptions:

Assumption 1 *f_2 is greater than f_1.*

Assumption 2 *Agents' preferences take the exponential utility function form:*

$$U(c^i) = -exp(-\gamma_i c^i).$$

Assumption 3 *An agent's risk aversion coefficient is a decreasing function of his endowment. Define $\gamma_i = \Gamma(w_0^i)$ with $\frac{d\Gamma}{dw} < 0$.*

Assumption 4 *The payoff to the risky asset is normally distributed with mean μ_F and variance σ_F^2.*

All of the above assumptions are straightforward except assumption 3 which needs some elaboration. Since γ_i is the absolute risk aversion coefficient, the higher the value the more risk averse an agent is. Assumption 3 thus implies that rich consumers are less risk averse than poor consumers. Without this assumption, asset holdings will be independent of wealth distribution[4]. The objective of a typical agent can be represented as follows according to his decision of asset market participation:

Staying in autarky:

$$U(w_0^i) = -exp(-\gamma_i w_0^i) \tag{1}$$

Participation in riskless asset market:

$$U(c_0^i) = -exp\left(-\gamma_i \frac{w_0^i - f_1}{q}\right) \tag{2}$$

where q is the price of one unit of riskless asset.

Participation in both riskless and risky asset markets:

$$\max E\left(-exp\left(-\gamma_i c^i\right)/\mathcal{I}\right) \tag{3}$$

$$\text{such that } c^i = \frac{\left(w_0^i - f_2 - px^i\right)}{q} + Fx^i \tag{4}$$

where c^i is the consumption of agent i who participates in both asset markets, p is the price of risky asset, and x^i is the holding of risky asset by the agent, and $\mathcal{I}$ is the information set which consists of the wealth distribution, payoff distribution, and the asset prices. The information set is common knowledge to every agent in the economy.

Since there is no uncertainty involved in the first two cases, we only study the third case in which agents choose how much risky asset to purchase and how much riskless asset to hold. In this simple model, an agent who only participates in the riskless asset market holds a positive amount of the riskless asset, $\left(w_0^i - f_1\right)$.

Under the assumptions above, the utility maximization when an agent participates in both asset markets is equivalent to the following:

$$\max E\left(c^i/\mathcal{I}\right) - \frac{\gamma_i}{2} Var\left(c^i/\mathcal{I}\right) \tag{5}$$

and subject to the budget constraint (4). The first-order condition to the above problem gives the optimal risky asset holding:

$$x^i = \frac{\mu_F - p/q}{\gamma_i \sigma_F^2} \tag{6}$$

If agent i holds positive amount of the risky asset, then everyone who participates in the risky asset market holds positive amount of this asset. This result is obvious because $x^i > 0$ implies that $\mu_F - p/q > 0$ which is independent of i. Short sales are therefore effectively disallowed in this model. Asset prices for the riskless and risky assets are then determined jointly using the market clearing conditions which in turn are jointly determined by the participation rates in each market. However, before we do that, we have to find who participates and in which market. This is given in the next section.

III. Market Participation Decisions and Existence of Equilibrium

3.1 Market participation decisions

In the above section we laid out the model, we now discuss agents decisions of market participations under the assumption that equilibrium asset prices p and

q exist and leave the theoretical analysis on the existence of equilibrium for the next subsection.

Since no one will buy the riskless asset if its price (q) is greater than 1, then if equilibrium exists, the riskless asset price will be less than 1. It is straight forward to show that an agent will participate in the riskless asset market if his endowment w_0^i is greater than $f_1/(1-q)$. Holding endowment constant, if we lower the entry costs (f_1), the proportion of people who participate in the riskless asset market will increase and vice versa.

While the decision of participating in the riskless asset market is simple, an agent's decision of participating in the risky asset market is more complex. Under assumption 3 that rich people are more likely to take risks than poor people, we have the following result.

Lemma 1 *Let p and q be the equilibrium risky and riskless asset prices respectively. An agent participates in both asset markets if and only if his endowment w_0^i is greater than*

$$\Gamma^{-1}\left(\frac{q(\mu_F - p/q)^2}{2(f_2 - f_1)\sigma_F^2}\right) \tag{7}$$

Proof: see Appendix.

Under the assumption that $\Gamma(\cdot)$ is a decreasing function, we have the following partial equilibrium results. First, high riskless asset price (q) or low risky asset price (p) tends to lower the wealth threshold of participating in the risky asset market. Intuitively, a high riskless asset price means a low rate of return to the riskless asset and a lower risky asset price means a high rate of return to the risky asset given the payoff of the asset. This makes the risky asset market more attractive to traders whose objectives are to maximize the end of period wealth. Second, high payoffs to the risky asset lower the wealth threshold. For given asset prices, high payoffs mean high returns to the risky asset and makes it more attractive. Third, high entry costs to the risky asset market ($f_2 - f_1$) tend to raise the wealth threshold determining participation in the risky asset market. This is consistent with our intuition because high entry costs to the risky asset market make it less attractive compared to participating the riskless asset market only or staying in autarky.

Next, we examine two special cases for which explicit forms for the wealth threshold of participating in the risky asset market are found. The first uses a bilinear $\Gamma(\cdot)$ function and the second adopts a hyperbolic $\Gamma(\cdot)$ function. Let's first consider the bilinear case. Suppose that the relationship between an agent's risk aversion coefficient and its wealth is represented by the following function:

$$\Gamma(w_0^i) = \begin{cases} \alpha - \beta w_0^i & \text{if } w_0^i < \alpha/\beta \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

where α is nonnegative and β is positive. Applying the result in Lemma 1 and solving for w_0^i yield the following result on the wealth threshold of participating in the risky asset market:

$$w_0^i \geq \frac{\alpha}{\beta} - \frac{q(\mu_F - p/q)^2}{2(f_2 - f_1)\sigma_F^2}.$$

Since the second term is always positive, the wealth threshold is less than α/β. Thus, all the risk neutral agents ($\gamma_i = 0$) will participate in the risky asset market because their wealth is at least as high as α/β. The top panel of Figure 1 illustrates the risk aversion function, wealth distribution, and the wealth threshold of asset market participation for the bilinear $\Gamma(\cdot)$ function.

Now we consider another special case in which we set

$$\Gamma(w_0^i) = \alpha + \frac{\beta}{w_0^i + \theta} \tag{9}$$

where β and θ are strictly positive and α is nonnegative. Solving for the wealth threshold determining participation in the risky asset market yields the following result:

$$w_0^i \geq \frac{2\beta(f_2 - f_1)\sigma_F^2}{q(\mu_F - p/q)^2 - \alpha(f_2 - f_1)\sigma_F^2} - \theta. \tag{10}$$

In contrast to the first special case, there exist no risk neutral agents for finite wealth. The wealth threshold of participation increases smoothly with p and $(f_2 - f_1)\sigma_F^2$, and decreases with q. The bottom panel of Figure 1 illustrates the risk aversion function, wealth distribution, and the wealth threshold of asset market participation for the hyperbolic $\Gamma(\cdot)$ function. We will use this specification for our numerical simulation, mainly to avoid a mass of risk neutral agents.

3.2 Equilibrium

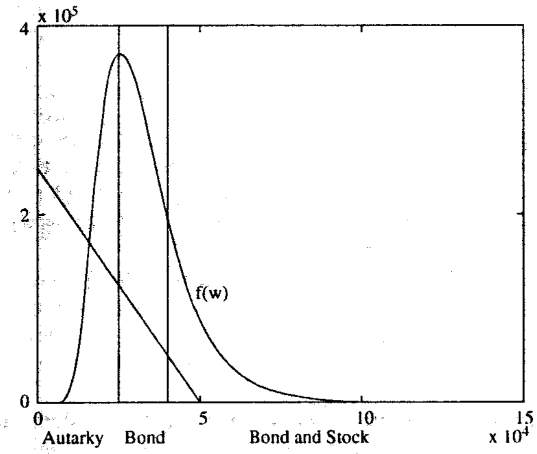
In the above subsection, we derived agents' participation decisions as a function of their wealth and preferences under the assumption that equilibrium exists. Now we show that under certain assumptions there indeed exists an equilibrium in which agents optimally decide on which market to participate such that their utilities are maximized.

Let's first define the following indicator functions:

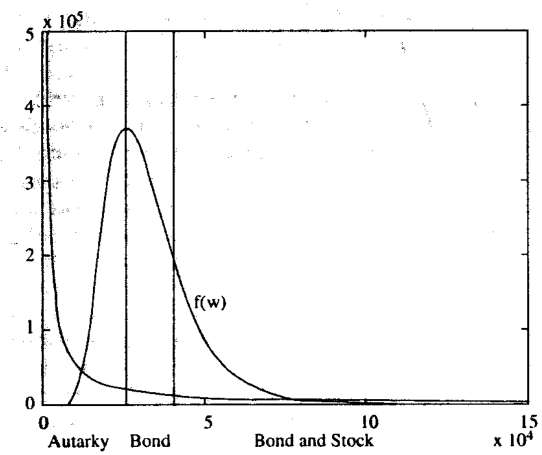
$$I\left(w_0^i > \Gamma^{-1}\left(\frac{q(\mu_F - p/q)^2}{2(f_2 - f_1)\sigma_F^2}\right)\right) \text{ and } I\left(w_0^i > \frac{f_1}{1-q}\right).$$

FIGURE 1

ASSET MARKET PARTICIPATION WITH BILINEAR $\Gamma(w_0)$



ASSET MARKET PARTICIPATION WITH HYPERBOLIC $\Gamma(w_0)$.



The vertical lines represent the wealth thresholds which separate agents into three groups: Autarky –do not participate either market, Bond–participate only bond market, Bond and Stock–participate both bond and stock markets. The area under $f(w_0)$ and to the left of the first vertical line is the proportion of agents who stay in autarky. The area under $f(w_0)$ and to the right of the second vertical line is the proportion of agents who participate both bond and stock markets. The area under $f(w_0)$ and between the two vertical lines is the proportion of agents who participate only bond market.

The first one represents the case in which agent i participates in both the riskless and risky asset markets and the second indicates that agent i participates in the riskless asset market. Without loss of generality we normalize the total supply of the risky asset to one share.

Definition 1 *An equilibrium for the economy with fixed costs and endogenous asset market participation is a pair of asset prices (p, q) and asset allocations such that every agent maximizes his utility and the asset markets clear, i. e, risky asset market clears:*

$$\int_0^1 x^i I\left(w_0^i > \Gamma^{-1}\left(\frac{q(\mu_F - p/q)^2}{2(f_2 - f_1)\sigma_F^2}\right)\right)di = 1, \tag{11}$$

and riskless asset market clears:

$$\begin{aligned}\int_0^1 (w_0^i - f_1) I\left(w_0^i > \frac{f_1}{1-q}\right) I\left(w_0^i \le \Gamma^{-1}\left(\frac{q(\mu_F - p/q)^2}{2(f_2 - f_1)\sigma_F^2}\right)\right)di \\ + \int_0^1 (w_0^i - f_2 - px^i) I\left(w_0^i > \Gamma^{-1}\left(\frac{q(\mu_F - p/q)^2}{2(f_2 - f_1)\sigma_F^2}\right)\right)di = 0.\end{aligned} \tag{12}$$

Now we introduce the following result on the existence of equilibrium for the model.

Proposition 1 *Under assumptions 1, 2, 3, and 4, there exists an equilibrium for the economy with fixed costs and endogenous asset market participation as long as the fixed costs are not too high such that no one participates in any asset market.*

Proof: see Appendix.

This result ensures that we can find an equilibrium using numerical simulation and in turn facilitates our numerical simulation analysis. In the next section we calibrate the model to the U.S. earnings profile and numerically solve for the equilibrium.

IV. Numerical Simulation

4.1 Parameters

Since equilibrium prices for both the risky and the riskless assets and agent's participation decision on asset markets are determined endogneously, it is difficult to find closed form solutions to the problem even for our relatively simple set-up. We resort to numerical simulation to obtain the solution instead. The lack of reliable data and strong identification schemes for the fixed costs prevents us, at this point in time, from formally estimating and testing the model. However, we can still calibrate the model based on our limited information and conduct numerical simulation analysis. We report results for a variety of fixed costs surfaces. Our goal is to see if there exists any plausible fixed cost combination that leads to participation rates that are similar to those found in the empirical studies.

For the numerical simulation conducted below, we assume that the individual endowment follows a log normal distribution. Most empirical studies characterize U.S. earnings profile using a log normal distribution (see Abowd and Card, 1989, Karoly, 1990; Heaton and Lucas, 1995, 1996; MaCurdy, 1982 among them). Let W be the random variable representing the wealth distribution. These empirical studies on the earnings distributions in the U.S. reveal that the mean of log W should be around 10.0 to 10.4 with standard deviation being around 0.39 (in particular see Gottschalk and Moffitt, 1992). This corresponds to mean earnings ranging from $23.767 to $35.456 with standard deviations being in the range of $9.633 to $14.371 (all in 1987 dollars). Also note that the log normal distribution still allows for a substantial right tail of high income.

After having selected the mean and the standard deviation for the endowment distribution, we can choose the range for the mean and the variance of return of the risky asset. Let r^s be the rate of return of the risky asset. It can then be expressed as

$$r^s = \frac{F-p}{p}$$

Solving for F yields $F = (1 + r^s)p$. Since p is a constant for a given set of structural parameters, the mean and variance of F can be obtained as

$$\mu_F = \left(1+\overline{r^s}\right)p, \quad \sigma_F^2 = \sigma_{r^s}^2 p^2.$$

where $\overline{r^s}$ is the mean rate of return of the risky asset. If we use the return on the value weighted NYSE market portfolio as the relevant benchmark for the risky asset in the model, then the rate of return r^s should have a mean around 8% and standard deviation around 20%. According to the market clearing condition for the riskless asset, the risky asset price p is approximately equal to the partial mean of the endowment W_0, i.e., $p \approx \int_{W_1}^{\infty} Wf(W)dW$, where W_1 is the threshold endowment level of participating in the riskless asset market. Mankiw and Zeldes (1991) have documented that around 30% to 40% of households hold zero or almost zero financial assets using PSID data. This allows us to back out $\overline{W_1}$ which is the endowment level corresponding to the probability $Prob(W < \overline{W_1}) = 0.3$ to 0.4 under the log normal assumption. We can therefore choose μ_F and σ_F^2 as follows:

$$\mu_F = \left(1+\overline{r^s}\right)\int_{\overline{W_1}}^{\infty} Wf(W)\,dW, \tag{13}$$

$$\sigma_F^2 = \sigma_{r^s}^2 \left[\int_{\overline{W_1}}^{\infty} Wf(W)\,dW\right]^2. \tag{14}$$

Finally, we choose the parameters in $\Gamma(\cdot)$. There are many possible functional forms that $\Gamma(\cdot)$ can take. For simplicity, we choose the form in the second special case to conduct our numerical simulation. There are three parameters involved, α, β, and θ. We also restrict the absolute risk aversion coefficient γ to be within the interval [0.5]. When $W = 0$, $\alpha + \beta/\theta = 5$; when $W = \infty$, $\alpha = 0$. Since the role of θ is to obtain numerical stability when W approaches 0, we assign θ a small value, $\theta = 0.01$. Since $\beta/\theta = 5$, we thus obtain $\beta = 0.05$.

4.2 Results

Tables 1 to 6 show the participation patterns, riskfree rates and equity premium for different levels of fixed entry costs and parameters governing the risky asset payoff and the wealth distribution. We measure ~~our~~ fixed costs as percentage of the mean of the wealth distribution (denoted as w in the tables). The tables show the following features. First, relatively small fixed entry costs can generate participation patterns that match observed ones. For instance, Table 1 shows that when the fixed costs of entering the markets are .4% ($142) and .55% ($195) respectively, the proportions of agents participating in only the riskless asset market and both the riskless and the risky markets are 35% and 30%, respectively. The participation proportions are 50% and 37% respectively as the fixed entry costs take values .4% ($116) and .55% ($160) respectively as shown in Table 3.

These results indicate that fixed entry costs can be important factors in agent's participation decisions and small fixed costs can have a large impact on asset market participation. However, this may be affected by the benefit associated with their participations which in turn depends on the number of periods that an agent lives. Therefore, if the model is extended to a multiperiod dynamic setting, it may entail higher entry costs to deter agents from participating than in a two period model. If the fixed costs are incurred whenever an agent changes his asset holdings, we anticipate that the trading frequency will be greatly reduced.

Second, the cost differential associated with entering into the risky asset market has a larger impact on agent's participation decisions than the levels of fixed entry costs f_1 and f_2. This is shown in Tables 1, 3, and 5. For example, in Table 1, holding the entry cost to the riskless asset market constant at .4% ($142) and letting the entry cost to both asset markets increase from .5% ($177) to .55% ($195), the proportion of agents participating in only the riskless asset market increases from 27.7% to 35.3% while the proportion of agents participating both the riskless and the risky asset market decreases from 37.6% to 30.1%. In Table 3, when the entry cost to both asset markets increases from .5% ($145) to .55%

TABLE 1

FIXED COSTS, MARKET PARTICIPATION, AND RETURNS: EXPERIMENT ONE

Specification f_1	f_2	% of bond market	% of both markets	R^f %	$E(R^m - R^f)$ %
Data		41	35	1	7
.1% ($35.46)	.2% ($70.91)	27.4	37.7	0.126	0.182
.2% ($70.91)	.3% ($106.37)	27.6	37.7	0.252	0.182
.3% ($106.37)	.4% ($141.82)	27.7	37.7	0.379	0.182
.4% ($141.82)	.5% ($177.28)	27.7	37.6	0.506	0.182
.4% ($141.82)	.55% ($195.01)	35.3	30.1	0.506	0.215
.75% ($265.92)	1.0% ($354.56)	43.1	22.5	0.956	0.267

$\mu_F = 19639$, $\sigma_F^2 = (0.2\mu_F/1.08)^2$, $\mu_w = 10.4$, $\sigma_w = 0.39$, $\overline{w} = \$35.456$, $\gamma \in [1,5]$.

TABLE 2

FIXED COSTS, MARKET PARTICIPATION, AND RETURNS: EXPERIMENT TWO

Specification f_1	f_2	% of bond market	% of both markets	R^f %	$E(R^m - R^f)$ %
Data		41	35	1	7
.1% ($32.08)	.2% ($64.16)	28.7	41.1	0.133	0.190
.2% ($64.16)	.3% ($96.25)	28.7	41.1	0.264	0.161
.3% ($96.25)	.4% ($128.33)	28.8	41.0	0.397	0.172
.4% ($128.33)	.5% ($160.41)	28.8	40.9	0.531	0.186

$\mu_F = 19639$, $\sigma_F^2 = (0.2\mu_F/1.08)^2$, $\mu_w = 10.3$, $\sigma_w = 0.39$, $\overline{w} = \$32.082$, $\gamma \in [1,5]$.

TABLE 3

FIXED COSTS, MARKET PARTICIPATION, AND RETURNS: EXPERIMENT THREE

Specification f_1	f_2	% of bond market	% of both markets	R^f %	$E(R^m - R^f)$ %
Data		41	35	1	7
.1% ($29.03)	.2% ($58.06)	29.5	47.1	0.143	0.191
.2% ($58.06)	.3% ($87.09)	29.6	47.0	0.287	0.191
.3% ($87.09)	.4% ($116.12)	29.6	47.0	0.431	0.191
.4% ($116.12)	.5% ($145.15)	29.6	47.0	0.575	0.192
.4% ($116.12)	.55% ($159.66)	49.9	37.2	0.676	0.223
.75% ($217.72)	1.0% ($290.29)	59.2	27.7	1.269	0.277

$\mu_F = 19639$, $\sigma_F^2 = (0.2\mu_F/1.08)^2$, $\mu_w = 10.2$, $\sigma_w = 0.39$, $\overline{w} = \$29.029$, $\gamma \in [1,5]$.

TABLE 4

FIXED COSTS, MARKET PARTICIPATION, AND RETURNS: EXPERIMENT FOUR

Specification f_1	f_2	% of bond market	% of both markets	R^f %	$E(R^m - R^f)$ %
Data		41	35	1	7
.1% ($32.61)	.2% ($65.22)	28.7	41.1	0.133	0.190
.2% ($65.22)	.3% ($97.84)	28.2	39.3	0.264	0.184
.3% ($97.84)	.4% ($130.45)	28.5	40.4	0.402	0.221
.4% ($130.45)	.5% ($163.06)	28.2	39.3	0.536	0.186

$\mu_F = 19639$, $\sigma_F^2 = (0.2\mu_F/1.08)^2$, $\mu_w = 10.3$, $\sigma_w = 0.39$, $\overline{w} = \$32.612$, $\gamma \in [1,5]$.

TABLE 5

FIXED COSTS, MARKET PARTICIPATION, AND RETURNS: EXPERIMENT FIVE

Specification f_1	f_2	% of bond market	% of both markets	R^f %	$E(R^m - R^f)$ %
Data		41	35	1	7
.1% ($35.46)	.2% ($70.91)	17.5	47.7	0.126	0.365
.2% ($70.91)	.3% ($106.37)	17.6	47.7	0.252	0.491
.3% ($106.37)	.4% ($141.82)	16.6	48.6	0.379	0.619
.4% ($141.82)	.5% ($177.28)	16.7	48.5	0.506	0.747
.4% ($141.82)	.55% ($195.01)	26.8	38.5	0.506	0.787
.75% ($265.92)	1.0% ($354.56)	36.9	28.6	0.954	1.301

$\mu_F = 19639$, $\sigma_F^2 = (0.25\mu_F/1.08)^2$, $\mu_w = 10.4$, $\sigma_w = 0.39$, $\overline{w} = \$35.456$, $\gamma \in [1,5]$.

TABLE 6

FIXED COSTS, MARKET PARTICIPATION, AND RETURNS: EXPERIMENT SIX

Specification f_1	f_2	% of bond market	% of both markets	R^f %	$E(R^m - R^f)$ %
Data		41	35	1	7
.1% ($35.46)	.2% ($70.91)	27.2	40.1	0.128	0.171
.2% ($70.91)	.3% ($106.37)	27.2	40.1	0.257	0.175
.3% ($106.37)	.4% ($141.82)	27.4	40.0	0.387	0.183
.4% ($141.82)	.5% ($177.28)	27.3	40.0	0.520	0.211

$\mu_F = 19639 * (1.05)$, $\sigma_F^2 = (0.2\mu_F/1.08)^2$, $\mu_w = 10.4$, $\sigma_w = 0.39$, $\overline{w} = \$35.456$, $\gamma \in [1,5]$.

($160) and the entry cost to the riskless asset market is held constant at .4% ($116), the proportion of agents participating in only the riskless asset market increases from 29.6% to 49.9% and the percentage of agents participating both asset markets decreases from 47.0% to 37.2%. Similarly, in Table 5 holding the entry cost to the riskless asset market constant at .4% ($142) and increasing the entry cost to both asset markets from .5% ($177) to .55% ($195), we find that the proportion of agents participating only in the riskless asset market increases from 16.7% to 26.8% and the rate of participation in both the riskless and the risky asset market decreases from 48.5% to 38.5%.

On the other hand, if the extra cost is held fixed, changing the levels of entry costs to the riskless and both markets does not have a significant impact on the participation patterns. For instance, in Table 1, holding the extra cost at .1% ($35) and letting the levels increase from .2% ($71) to .3% ($ 106) for the riskless asset market and from .3% ($106) to .4% ($142) for both asset markets, we only observe a tiny increase (~ 0.1%) in the participation in the riskless market and a negligible decrease in the participation in both markets. Similar results are obtained in Tables 2, 3, 4, 5 and 6. This finding has an important policy implication. It indicates that if we want to attract more people to participate in certain asset markets we should pay special attention to the cost differentials in the entry to various markets.

Third, lowering the mean of the wealth distribution while holding the entry costs at some fixed percentage of this mean wealth results in higher participation rates in both groups. This is made clear by examining Tables 1, 2, and 3. The participation rates for asset markets as reported in Table 1 (the first four rows) are lower than those reported in Table 2 which are further lower than those reported in 3. However, the levels of fixed entry costs in Table 1 are higher than that in Table 2 which are also higher than that in Table 3. This is consistent with the theoretical analysis discussed earlier. Intuitively, high entry costs in the asset markets reduce the net benefits from participation, therefore, making the asset market less attractive.

In the meantime, the results in Tables 2 and 4 indicate that higher variance of wealth distribution is associated with lower participation rates. For instance, increasing the variance of log W from 0.39^2 to 0.43^2 decreases, on average, the percentage of people who participate only in the riskless asset market by 0.35% and decreases, on average, the percentage of people who participate in both asset markets by 0.75%.

Fourth, the numerical analysis also confirms our conjecture on the effects of changing the mean payoff of the risky asset on participation rates. Tables 1 (the first four rows) and 6 show that increasing the mean payoff of the risky asset increases the percentage of people who participate in both asset markets and slightly lowers the percentage of people who participates only in the riskless asset market.

Tables 1 and 5 show the effect of changing the variance of the risky asset payoff. Increasing the variance of the risky asset payoff increases the percentage of people who participate in both asset markets and decreases the percentage of people who participate only in the riskless asset market. An intuition for this result is the following. As the variance increases, the risky asset price decreases. For a given mean payoff of the risky asset, lower risky asset price means a higher mean return. This attracts more investors to participate in both markets. However, increasing the variance of the risky asset payoff does not have a significant impact on the price of the riskless asset. Therefore, the percentage of consumers who participate in the riskless asset market does not change much. This leads to the decline of the percentage of consumers who participate only in the riskless asset market because it is the difference of the above two percentages.

Fifth, while the model generates reasonable riskfree rates, it fails to create sizable risk premium. For all the parameters that we have experimented the riskfree rates range from 0.13% to 1.27% per year and increase as the fixed entry costs increase. On the other hand, the risk premium is only in the range of 0.16% to 1.3% for the same set of parameters. This result indicates that fixed entry costs alone are unable to resolve the risk premium puzzle. Since standard asset pricing models tend to generate risk free rates that are substantially higher than observed ones, introducing fixed costs in such environments will make the task of matching that moment even more difficult because the riskfree rates are likely to be even higher than without fixed entry costs. Further, our findings on the asset market participations and the risk premium indicate that limited asset market participations and the risk premium are related yet not identical issues. While limited asset market participation may help increasing the risk premium, it may not be able to completely resolve the "equity premium puzzle"[5].

V. Concluding Remarks

In this study we show that with small fixed entry costs we can replicate the asset market participation rates reported in Mankiw and Zeldes (1991). The participation rates don't vary much across specifications (levels of fixed costs, mean and variance of income) in which the relative cost differentials (in percentage) are constant. They are, however, very sensitive to the relative cost differentials. We find this result to be interesting since it implies researchers should concentrate on modeling relative fixed costs. Another interesting issue is the sensitivity of risk free rates to the fixed costs levels. As the tables indicate the riskfree rate increases as the fixed costs level increases. Since most dynamic models result in risk free rates that are substantially higher than observed ones, fixed costs deepen the risk free rate puzzle and in turn the equity premium puzzle even further (see Mehra and Presscott, 1985, and Weil, 1989).

The study can be extended in the following ways. First, the model can be extended to a dynamic setting where an infinitely lived agent faces both fixed and proportional transaction costs for riskless and risky asset markets. The fixed costs create segmentation in market participation while the proportional costs allow us to generate a sizable equity premium. The presence of both factors is important, in our opinion, for the joint analysis of asset pricing and trading volume as implied by market participation. To do so, we will need to numerically solve for an equilibrium which will involve the laws of motion for the distributions of wealth and participation decisions. This set-up will allow us to analyze the participation pattern in conjunction with the business cycle.

Second, many households do invest in somewhat risky assets such as their houses and automobiles. To account for such issues we will have to explicitly deal with durables (see Grossman and Laroque, 1990). There is also much work left to be done in terms of analyzing the risky component of the durables that households obtain. The third environment where fixed costs might provide information on participation patterns is the case in which agents have asymmetric information because adverse selection of inside trading may further increase the wealth thresholds determining the asset market participation. We hope to deal with these issues in the future.

APPENDIX

Proof of Lemma 1

'If' part. For agent i to participate in both the riskless and risky asset markets, it has to be true that

$$-E\,exp\left(-\gamma_i\left(\frac{w_0^i - f_2 - px^i}{q} + Fx^i\right)\right) \geq -exp\left(-\gamma_i\left(\frac{w_0^i - f_1}{q}\right)\right). \tag{A1}$$

Straight manipulation yields

$$\gamma_i \leq \frac{q(\mu_F - p/q)^2}{2(f_2 - f_1)\sigma_F^2}. \tag{A2}$$

Since $d\Gamma/dw < 0$, we can invert the function and get the following result:

$$w_0^i \geq \Gamma^{-1}\left(\frac{q(\mu_F - p/q)^2}{2(f_2 - f_1)\sigma_F^2}\right). \tag{A3}$$

'Only if' part. Suppose that

$$w_0^i \geq \Gamma^{-1}\left(\frac{q(\mu_F - p/q)^2}{2(f_2 - f_1)\sigma_F^2}\right). \tag{A4}$$

Using assumption 3, we have

$$\gamma_i = \Gamma\left(w_0^i\right) \leq \frac{q(\mu_F - p/q)^2}{2(f_2 - f_1)\sigma_F^2}, \tag{A5}$$

which gives

$$\frac{\gamma_i f_2}{q} - \frac{(\mu_F - p/q)^2}{2(f_2 - f_1)\sigma_F^2} \leq \frac{\gamma_i f_1}{q}. \tag{A6}$$

Straightforward manipulation yields:

$$-exp\left(-\gamma_i\left(\frac{w_0^i-f_2}{q}+\frac{(\mu_F-p/q)^2}{2(f_2-f_1)\sigma_F^2}\right)\right)\geq -exp\left(-\gamma_i\left(\frac{w_0^i-f_1}{q}\right)\right). \tag{A7}$$

The left hand side (LHS) can be written as

$$-exp\left(-\gamma_i\left(\frac{w_0^i-f_2-px^i}{q}\right)\right)\times exp\left(-\gamma_i\left(\frac{px^i}{q}+\frac{(\mu_F-p/q)^2}{2(f_2-f_1)\sigma_F^2}\right)\right). \tag{A8}$$

Applying the normality assumption on F, we have

$$exp\left(-\gamma_i\left(\frac{px^i}{q}+\frac{(\mu_F-p/q)^2}{2(f_2-f_1)\sigma_F^2}\right)\right)=exp\left(-\frac{\mu_F^2-(p/q)^2}{2\sigma_F^2}\right)=E\left[exp\left(-\gamma_i Fx^i\right)\right]. \tag{A9}$$

The LHS can then be written as

$$-exp\left(-\gamma_i\left(\frac{w_0^i-f_2-px^i}{q}\right)\right)\times E\left[exp\left(-\gamma_i Fx^i\right)\right]=-E\,exp\left(-\gamma_i\left(\frac{w_0^i-f_2-px^i}{q}+Fx^i\right)\right). \tag{A10}$$

Q.E.D.

Proof of Proposition 1

Using the results in Lemma 1, we can write the market clearing conditions as follows:

For the risky asset market:

$$\int_0^1 x^i I\left(w_0^i>\Gamma^{-1}\left(\frac{q(E(F)-p/q)^2}{2(f_2-f_1)V}\right)\right)di=1, \tag{A11}$$

For riskless asset market:

$$\int_0^1 (w_0^i-f_1)I\left(w_0^i>\frac{f_1}{1-q}\right)I\left(w_0^i\leq\Gamma^{-1}\left(\frac{q(E(F)-p/q)^2}{2(f_2-f_1)V}\right)\right)di$$
$$+\int_0^1 (w_0^i-f_2-px^i)I\left(w_0^i>\Gamma^{-1}\left(\frac{q(E(F)-p/q)^2}{2(f_2-f_1)V}\right)\right)di=0. \tag{A12}$$

Combining the above two equations gives

$$p=\int_0^1 (w_0^i-f_1)I\left(\Gamma^{-1}\left(\frac{q(E(F)-p/q)^2}{2(f_2-f_1)V}\right)\geq w_0^i>\frac{f_1}{1-q}\right)di$$
$$+\int_0^1 (w_0^i-f_2)I\left(w_0^i>\Gamma^{-1}\left(\frac{q(E(F)-p/q)^2}{2(f_2-f_1)V}\right)\right)di. \tag{A13}$$

Recall that the optimal risky asset holding is given by

$$x^i=\frac{E(F)-p/q}{\gamma_i V}.$$

Denote $f(w_0^i)$ as the probability density function of the individual wealth w_0^i. Equations (A11) and (A13) can be written as (we omit the superscript for simplicity)

$$\int_{\Gamma^{-1}(\gamma^*)}^{\infty}\frac{\mu_F-p/q}{\gamma V}f(w_0)dw_0=1, \tag{A14}$$

$$p=\int_{f_1/(1-q)}^{\Gamma^{-1}(\gamma^*)}(w_0-f_1)f(w_0)dw_0$$
$$+\int_{\Gamma^{-1}(\gamma^*)}^{\infty}(w_0-f_2)f(w_0)dw_0. \tag{A15}$$

where $\gamma^*=\frac{q(E(F)-p/q)^2}{2(f_2-f_1)V}$. Differentiating (A14) with respect to p and treating q as a function of p yield

$$\frac{1}{2V}\left(\frac{1}{q}-\frac{p}{q^2}\frac{dq}{dp}\right)\int_{\Gamma^{-1}(\gamma^*)}^{\infty}\frac{f(w_0)}{\gamma}dw_0+\frac{\mu_F-p/q}{2V}\frac{f(\Gamma^{-1}(\gamma^*))}{\gamma^*}\frac{d\Gamma^{-1}(\gamma^*)}{d\gamma^*}\frac{d\gamma^*}{dp}=0. \tag{A16}$$

Lengthy derivation leads to

$$\frac{dq}{dp}=\frac{q\left(\int_{\Gamma^{-1}(\gamma^*)}^{\infty}\frac{f(w_0)}{\gamma}dw_0-2f(\Gamma^{-1}(\gamma^*))\frac{d\Gamma^{-1}(\gamma^*)}{d\gamma^*}\right)}{p\int_{\Gamma^{-1}(\gamma^*)}^{\infty}\frac{f(w_0)}{\gamma}dw_0-q\left(\mu_F+\frac{p}{q}\right)f\left(\Gamma^{-1}(\gamma^*)\right)\frac{d\Gamma^{-1}(\gamma^*)}{d\gamma^*}} \tag{A17}$$

According to Assumption 3, $\frac{d\Gamma^{-1}(\gamma^*)}{d\gamma^*} < 0$. Thus $\frac{dq}{dp} > 0$ or market clearing for the risky asset requires that the riskless asset price increases as the risky asset price increases.

Next, we define the operator $\Upsilon(p)$ such that

$$\Upsilon(p) = \int_{f_1/(1-q)}^{\Gamma^{-1}(\gamma^*)} (w_0 - f_1) f(w_0) dw_0 + \int_{\Gamma^{-1}(\gamma^*)}^{\infty} (w_0 - f_2) f(w_0) dw_0. \quad \text{(A18)}$$

Thus the proof of existence of equilibrium boils down to finding a fixed point for p such that $p = \Upsilon(p)$ under the condition that equation (A17) holds. Since $\Upsilon(p)$ is continuous and bounded from above by $E(w_0)$. As long as $d\Upsilon(p)/dp < 0$, there must exist a fixed point for p. Further, if f_1 and f_2 are small relative to $E(w_0)$,the fixed point p must be positive. Differentiating (A18) with respect to p yields

$$\frac{\Upsilon(p)}{dp} = (f_2 - f_1) f(\Gamma^{-1}(\gamma^*)) \frac{d\Gamma^{-1}(\gamma^*)}{d\gamma^*} \frac{d\gamma^*}{dp} - \frac{q f_1^2}{(1-q)^3} f\left(\frac{f_1}{1-q}\right)\frac{dq}{dp}. \quad \text{(A19)}$$

The above equation indicates that the sign of $d\Upsilon(p)/dp$ depends on the sign of $d\gamma^*/dp$. Differentiating γ^* with respect to p yields

$$\frac{d\gamma^*}{dp} = \frac{\mu_F - p/q}{2(f_2 - f_1)V}\left[(\mu_F + p/q)\frac{dq}{dp} - 2\right]. \quad \text{(A20)}$$

Substituting (A17) into the above equation yields

$$\frac{d\gamma^*}{dp} = \frac{\frac{q}{2(f_2-f_1)}\int_{\Gamma^{-1}(\gamma^*)}^{\infty} \frac{f(w_0)}{\gamma} dw_0}{p\int_{\Gamma^{-1}(\gamma^*)}^{\infty} \frac{f(w_0)}{\gamma} dw_0 - q\left(\mu_F + \frac{p}{q}\right) f\left(\Gamma^{-1}(\gamma^*)\right)\frac{d\Gamma^{-1}(\gamma^*)}{d\gamma^*}}. \quad \text{(A21)}$$

Since $\frac{d\Gamma^{-1}(\gamma^*)}{d\gamma^*} < 0$, we have $\frac{d\gamma^*}{dp} > 0$. Therefore, $d\Upsilon(p)/dp < 0$. Under Assumption 1, there must exist a fixed point for p. Since $dq/dp > O$, there also exists a q such that both markets are cleared.

Notes

1 We will use fixed costs and fixed entry costs interchangeably in this paper because we have only two periods. In a more general dynamic setting, it is important to distinguish the fixed costs per trade from the one time fixed entry costs.

2 King and Leape (1984) find empirical evidence that supports limited diversification in the presence of fixed set-up costs.

3 We notice that the trading costs using the internet are even lower. However, investors still have to pay to access the internet and also do their own research to make optimal investment decisions.

4 In a similar manner, Zeldes (1989) relates agents' risk aversion to their demographics.

5 Mankiw and Zeldes (1991) find, using the consumption growth of stockholders, that in order to match the mean and standard deviation of the observed equity premium, the required risk aversion coefficient for the CRRA utility function has to be 35. This value is a third of the value required when the consumption growth of all PSID families are used. However, such a risk aversion coefficient value is still way too high to be considered reasonable hy most economists.

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